AH

National
Qualifications
SPECIMEN ONLY

S847/77/12

Mathematics
Paper 2

Date — Not applicable

Duration — 2 hours 30 minutes

Total marks — 80

Attempt ALL questions.

You may use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

S847712

FORMULAE LIST

Standard derivatives

$f(x)$	$f'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Standard integrals

$f(x)$	$\int f(x)\,dx$
$\sec^2(ax)$	$\frac{1}{a}\tan(ax)+c$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c$
e^{ax}	$\frac{1}{a}e^{ax}+c$

Summations

(Arithmetic series) $$S_n = \frac{1}{2}n\left[2a+(n-1)d\right]$$

(Geometric series) $$S_n = \frac{a(1-r^n)}{1-r}, r \neq 1$$

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}, \quad \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

Binomial theorem

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r \quad \text{where} \quad \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Maclaurin expansion

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!} + \ldots$$

De Moivre's theorem

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

Vector product

$$\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Matrix transformation

Anti-clockwise rotation through an angle, θ, about the origin, $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

[Turn over

MARKS

Total marks — 80

Attempt ALL questions

1. Express $\dfrac{x^2-6x+20}{(x+1)(x-2)^2}$ in partial fractions. **4**

2. (a) Given $f(x)=\sin^{-1}3x$, find $f'(x)$. **2**

(b) For $y\cos x + y^2 = 6x$, use implicit differentiation to find $\dfrac{dy}{dx}$. **4**

3. (a) Write down and simplify the general term in the binomial expansion of $\left(\dfrac{3}{x}-2x\right)^{13}$. **3**

(b) Hence, or otherwise, find the term in x^9. **2**

4. On a suitable domain, a curve is defined parametrically by $x=t^2+1$ and $y=\ln(3t+2)$.

Find the equation of the tangent to the curve where $t=-\dfrac{1}{3}$. **5**

5. (a) Obtain the matrix, A, associated with an anticlockwise rotation of $\dfrac{\pi}{3}$ radians about the origin. **1**

(b) Find the matrix, B, associated with a reflection in the x-axis. **1**

(c) Hence obtain the matrix, P, associated with an anticlockwise rotation of $\dfrac{\pi}{3}$ radians about the origin followed by reflection in the x-axis, expressing your answer using exact values. **2**

(d) Explain why matrix P is not associated with rotation about the origin. **1**

6. Solve $\dfrac{dy}{dx}=e^{2x}\left(1+y^2\right)$ given that when $x=0$, $y=1$.

Express y in terms of x. **5**

7. Let $z = \sqrt{3} - i$.

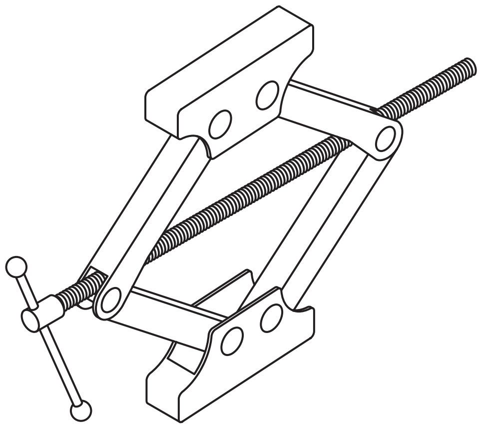
(a) Plot z on an Argand diagram. **1**

(b) Let $w = az$ where $a > 0$, $a \in \mathbb{R}$.

Express w in polar form. **2**

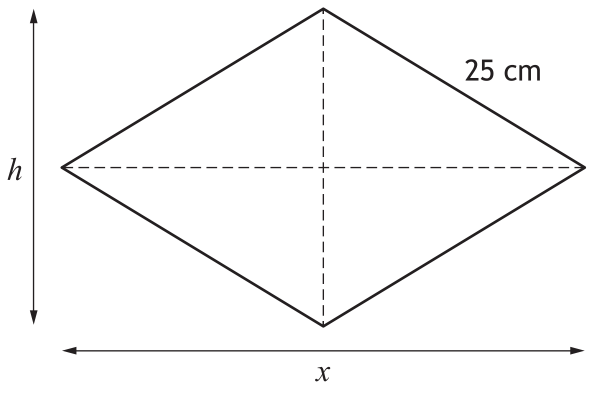
(c) Express w^8 in the form $ka^n\left(x + i\sqrt{y}\right)$ where $k, x, y \in \mathbb{Z}$. **3**

8. An engineer has designed a lifting device. The handle turns a screw which shortens the horizontal length and increases the vertical height.



The device is modelled by a rhombus, with each side 25 cm.

The horizontal length is x cm, and the vertical height is h cm as shown.



(a) Show that $h = \sqrt{2500 - x^2}$. **1**

(b) The horizontal length decreases at a rate of 0·3 cm per second as the handle is turned.

Find the rate of change of the vertical height when $x = 30$. **5**

MARKS

9. For each of the following statements, decide whether it is true or false.

If true, give a proof; if false, give a counterexample.

A. If a positive integer p is prime, then so is $2p+1$.

B. If a positive integer n has remainder 1 when divided by 3, then n^3 also has remainder 1 when divided by 3. **4**

10. Given $y = x^{2x^3+1}$ where $x > 0$, find $\frac{dy}{dx}$.

Write your answer in terms of x. **5**

11. A geometric sequence has first term 80 and common ratio $\frac{1}{3}$.

(a) For this sequence calculate

(i) the 7th term **2**

(ii) the sum to infinity of the associated geometric series. **2**

The first term of this geometric sequence is equal to the first term of an arithmetic sequence.

The sum of the first five terms of this arithmetic sequence is 240.

(b) Find the common difference of this sequence. **2**

Let S_n represent the sum to n terms of this arithmetic sequence.

(c) Find the values of n for which $S_n = 144$. **3**

12. Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 6e^{2x}$$

given $y = 4$ and $\frac{dy}{dx} = 7$ when $x = 0$. **10**

MARKS

13. A line, L, has equation $\frac{x+1}{2}=\frac{y-2}{1}=\frac{z}{-1}$.

(a) Find the Cartesian equation of the plane, perpendicular to the line L, which passes through the point P(1,1,0). **3**

(b) Find the shortest distance from P to L and explain why this is the shortest distance. **7**

[END OF SPECIMEN QUESTION PAPER]

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Mathematics Paper 2

Marking Instructions

These marking instructions have been provided to show how SQA would mark this specimen question paper.

General marking principles for Advanced Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

The marking instructions for each question are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

$$x^2 + 5x + 7 = 9x + 4$$

$$x - 4x + 3 = 0$$

This is a transcription error and so the mark is not awarded.

$$x = 1$$

This is no longer a solution of a quadratic equation, so the mark is not awarded.

The following example is an exception to the above

$$x^2 + 5x + 7 = 9x + 4$$

$$x - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 1 \text{ or } 3$$

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

(i) **Horizontal/vertical marking**
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	•5	•6
•5	$x = 2$	$x = -4$
•6	$y = 5$	$y = -7$

Horizontal: •5 $x = 2$ and $x = -4$
•6 $y = 5$ and $y = -7$

Vertical: •5 $x = 2$ and $y = 5$
•6 $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $(x^3 + 2x^2 + 3x + 2)(2x + 1)$ written as

  $(x^3 + 2x^2 + 3x + 2) \times 2x + 1$

  $= 2x^4 + 5x^3 + 8x^2 + 7x + 2$

  gains full credit

- repeated error within a question, but not between questions or papers

(l) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(m) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(n) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(o) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

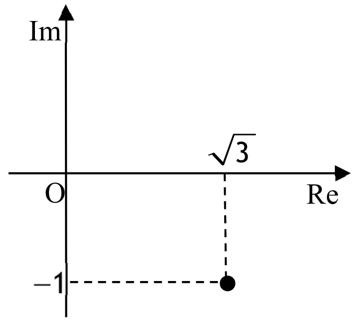
Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•1 state expression •2 form equation •3 obtain two of A, B and C •4 obtain final constant and state expression	•1 $\frac{x^2-6x+20}{(x+1)(x-2)^2}=\frac{A}{(x+1)}+\frac{B}{(x-2)}+\frac{C}{(x-2)^2}$ •2 $x^2-6x+20$ $=A(x-2)^2+B(x+1)(x-2)+C(x+1)$ •3 $A=3,\ B=-2,\ C=4$ •4 $\frac{3}{(x+1)}-\frac{2}{(x-2)}+\frac{4}{(x-2)^2}$	4
2.	(a)		•1 start differentiation •2 apply chain rule and complete differentiation	•1 $\frac{1}{\sqrt{1-(3x)^2}}\times\ldots$ •2 $\frac{3}{\sqrt{1-9x^2}}$	2
	(b)		•3 start to differentiate product with one term correct •4 complete differentiation of product •5 differentiate remaining terms •6 express derivative explicitly in terms of x and y	•3 $\frac{dy}{dx}\cos x+\ldots$ **OR** $-y\sin x+\ldots$ •4 $\frac{dy}{dx}\cos x\ldots$ **OR** $-y\sin x$ •5 $+2y\frac{dy}{dx}=6$ •6 $\frac{dy}{dx}=\frac{6+y\sin x}{\cos x+2y}$	4
3.	(a)		•1 state general term •2 simplify powers of x **OR** coefficients and signs •3 state simplified general term	•1 $\binom{13}{r}\left(\frac{3}{x}\right)^{13-r}(-2x)^r$ •2 $(3)^{13-r}(-2)^r$ or x^{2r-13} •3 $\binom{13}{r}(3)^{13-r}(-2)^r x^{2r-13}$	3
	(b)		•4 determine value of r •5 evaluate coefficient and state term	•4 11 •5 $-1437696x^9$	2

Question			Generic scheme	Illustrative scheme	Max mark
4.			•1 find $\frac{dy}{dt}$ •2 complete differentiation and relate derivatives •3 evaluate gradient •4 find coordinates •5 state equation of tangent	•1 $\frac{dy}{dt}=\frac{3}{3t+2}$ •2 $\frac{dx}{dt}=2t$ and $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ stated or implied at •3 •3 $\frac{dy}{dx}=-\frac{9}{2}$ •4 $x=\frac{10}{9}$, $y=0$ •5 $y=-\frac{9}{2}x+5$	**5**
5.	(a)		•1 obtain A	•1 $\begin{pmatrix}\cos\frac{\pi}{3} & -\sin\frac{\pi}{3}\\ \sin\frac{\pi}{3} & \cos\frac{\pi}{3}\end{pmatrix}$	**1**
	(b)		•2 obtain B	•2 $\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$	**1**
	(c)		•3 correct order for multiplication ($P=BA$) •4 multiplication completed and appearance of exact values	•3 $\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\frac{1}{2}\begin{pmatrix}1 & -\sqrt{3}\\ \sqrt{3} & 1\end{pmatrix}$ •4 $\frac{1}{2}\begin{pmatrix}1 & -\sqrt{3}\\ -\sqrt{3} & -1\end{pmatrix}$	**2**
	(d)		•5 valid explanation	•5 for example compare the elements of P with the general form of a rotation matrix	**1**

Question			Generic scheme	Illustrative scheme	Max mark
6.			•[1] separate variables and write down integral equation •[2] integrate LHS •[3] integrate RHS •[4] evaluate constant of integration •[5] express y in terms of x	•[1] $\int \frac{dy}{1+y^2}=\int e^{2x}\,dx$ •[2] $\tan^{-1}y$ •[3] $\frac{1}{2}e^{2x}+c$ •[4] $c=\frac{\pi}{4}-\frac{1}{2}$ •[5] $y=\tan\left(\frac{1}{2}e^{2x}+\frac{\pi}{4}-\frac{1}{2}\right)$	**5**
7.	(a)		•[1] correctly plot z on Argand diagram	•[1] 	**1**
	(b)		•[2] find modulus or argument •[3] express in polar form	•[2] $\lvert w\rvert=2a$ or $\arg(w)=-\frac{\pi}{6}$ •[3] $w=2a\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)$	**2**
	(c)		**Method 1** •[4] process modulus •[5] process argument •[6] evaluate and express in form $ka^n\left(x+i\sqrt{y}\right)$	•[4] $256a^8$ •[5] $\ldots\left(\cos\left(-\frac{8\pi}{6}\right)+i\sin\left(-\frac{8\pi}{6}\right)\right)$ •[6] $w^8=128a^8\left(-1+i\sqrt{3}\right)$	**3**

Question			Generic scheme	Illustrative scheme	Max mark
7.	(c)		**Method 2** •[4] find w^2 and attempt to find a higher power of w •[5] obtain w^4 •[6] complete expansion and express in form $ka^n\left(x+i\sqrt{y}\right)$	 •[4] eg $w^2=a^2\left(2-2i\sqrt{3}\right)$ and $w^3=a^2\left(2-2i\sqrt{3}\right)\times a\left(\sqrt{3}-i\right)$. •[5] $w^4=a^4\left(-8-8i\sqrt{3}\right)$ •[6] $w^8=128a^8\left(-1+i\sqrt{3}\right)$	**3**

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		•1 Determine the relationship between x and h	•1 $x^2+h^2=2500$ $h=\sqrt{2500-x^2}$	**1**
	(b)		•2 interpret rate of change of x •3 find $\frac{dh}{dx}$ •4 form relationship •5 multiply by $\frac{dx}{dt}$ •6 evaluate $\frac{dh}{dt}$	•2 $\frac{dx}{dt}=-0\cdot 3$ •3 $\frac{dh}{dx}=-2x\cdot\frac{1}{2}\left(2500-x^2\right)^{-\frac{1}{2}}$ •4 $\frac{dh}{dt}=\frac{dh}{dx}\cdot\frac{dx}{dt}$ stated or implied at •5 •5 $\frac{dh}{dt}=\frac{0\cdot 3x}{\sqrt{2500-x^2}}$ •6 $\frac{dh}{dt}=\frac{9}{40}\text{cms}^{-1}$	**5**
9.			•1 give counterexample •2 set up n •3 consider expansion of n^3 •4 complete proof with conclusion	•1 for example choose $p=7$ $2(7)+1=15$, which is not prime. ∴ statement is false. •2 $n=3a+1$, $a\in\mathbb{W}$ •3 $n^3=27a^3+27a^2+9a+1$ •4 $=3\left(9a^3+9a^2+3a\right)+1$ and statement such as “so n^3 has remainder 1 when divided by 3 ∴ statement is true”.	**4**
10.			•1 take logarithms of both sides and apply rule •2 differentiate LHS •3 evidence use of product rule and one term correct •4 complete differentiation •5 write $\frac{dy}{dx}$ in terms of x	•1 $\ln y=\left(2x^3+1\right)\ln x$ •2 $\frac{1}{y}\frac{dy}{dx}$ •3 $6x^2\ln x$ or $\frac{2x^3+1}{x}$ •4 $6x^2\ln x+\frac{2x^3+1}{x}$ •5 $\frac{dy}{dx}=x^{2x^3+1}\left(6x^2\ln x+\frac{2x^3+1}{x}\right)$	**5**

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)	(i)	•¹ multiply first term by a power of the common ratio •² find term	•¹ $80\left(\frac{1}{3}\right)^{\cdots}$ •² $\frac{80}{729}$	**2**
		(ii)	•³ substitute •⁴ find sum to infinity	•³ $\frac{80}{1-\frac{1}{3}}$ •⁴ 120	**2**
	(b)		•⁵ start process •⁶ find common difference	•⁵ for example $400+10d=240$ •⁶ -16	**2**
	(c)		•⁷ set up equation •⁸ obtain quadratic equation in general form •⁹ find values of n	•⁷ $\frac{n}{2}\left[160+(n-1)(-16)\right]=144$ •⁸ $16n^2-176n+288=0$ •⁹ $n=2,\ n=9$	**3**

Question			Generic scheme	Illustrative scheme	Max mark
12.			•¹ solve auxiliary equation	•¹ $m=2$ twice	10
			•² state complementary function	•² $y=Ae^{2x}+Bxe^{2x}$	
			•³ state form of particular integral	•³ $y=Cx^2e^{2x}$	
			•⁴ find first derivative of particular integral	•⁴ $\frac{dy}{dx}=2Cx^2e^{2x}+2Cxe^{2x}$	
			•⁵ find second derivative	•⁵ $\frac{d^2y}{dx^2}=4Cx^2e^{2x}+8Cxe^{2x}+2Ce^{2x}$	
			•⁶ determine coefficient of particular integral	•⁶ $C=3$	
			•⁷ state general solution	•⁷ $y=Ae^{2x}+Bxe^{2x}+3x^2e^{2x}$	
			•⁸ find derivative of general solution	•⁸ $\frac{dy}{dx}=2Ae^{2x}+Be^{2x}+2Bxe^{2x}+6x^2e^{2x}+6xe^{2x}$	
			•⁹ find one constant	•⁹ $A=4$ or $B=-1$	
			•¹⁰ find second constant and state particular solution	•¹⁰ $y=4e^{2x}-xe^{2x}+3x^2e^{2x}$	

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		•[1] find normal vector •[2] substitute into equation of the plane •[3] find the equation of plane	•[1] $2\mathbf{i}+\mathbf{j}-\mathbf{k}$ •[2] $2x+y-z=d$ •[3] $2x+y-z=3$	**3**
	(b)		•[4] find parametric equations for the line •[5] substitute into equation of plane •[6] solve for t •[7] calculate coordinates •[8] components of *PQ* •[9] find shortest distance •[10] explanation	•[4] $x=-1+2t,\ y=2+t,\ z=-t$ •[5] $2(-1+2t)+(2+t)-(-t)=3$ •[6] $\frac{1}{2}$ •[7] $\left(0,\frac{5}{2},-\frac{1}{2}\right)$ •[8] $\begin{pmatrix}1\\ \frac{3}{2}\\ \frac{-1}{2}\end{pmatrix}$ •[9] $\sqrt{\frac{7}{2}}$ •[10] *PQ* is perpendicular to *L*.	**7**

[END OF SPECIMEN MARKING INSTRUCTIONS]