SECTION A (MATHEMATICS 1 AND 2)

A1.

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 10 \\ 2 & -1 & 3 & 4 \\ 1 & 0 & 2 & 20 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 10 \\ 0 & -3 & 1 & -16 \\ 0 & -1 & 1 & 10 \end{array}\right) \quad \begin{array}{c} \\ r_2 - 2r_1 \\ r_3 - r_1 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 10 \\ 0 & -3 & 1 & -16 \\ 0 & 0 & \frac{2}{3} & 15\frac{1}{3} \end{array}\right) \quad \begin{array}{c} \\ \\ r_3' - \frac{1}{3}r_2' \end{array}$$

$z = 23;\ y = 13;\ x = -26$

1 method mark for Gaussian elimination
1 for middle row
1 for bottom row
1 for third row
1 for back substitution — Total 5

A2.

(*a*) $f(x) = (2+x)\ \tan^{-1}\sqrt{x-1}$

$$f'(x) = \tan^{-1}\sqrt{x-1} + \frac{(2+x)\frac{1}{2}(x-1)^{-\frac{1}{2}}}{1+(x-1)}$$

$$= \tan^{-1}\sqrt{x-1} + \frac{2+x}{2x\sqrt{x-1}}$$

1 method mark for product rule
1 for first term
1 for second term
1 mark

(*b*)

$g(x) = e^{\cot 2x}$

$g'(x) = -2\ \mathrm{cosec}^2 2x\ e^{\cot 2x}$

1 method mark for the chain rule
1E1 for the rest — Total 6

A3.

$\int_0^{\pi/4} 2x\sin 4x\,dx$	1 method mark for integration by parts
$= \left[2x\int \sin 4x\,dx - \int \left(2\int \sin 4x\,dx \right) dx \right]_0^{\pi/4}$	1 for first term 1 for second term
$= \left[2x\tfrac{1}{4}(-\cos 4x) + \tfrac{1}{2}\int \cos 4x\,dx \right]_0^{\pi/4}$	
$= \left[-\tfrac{1}{2}x\cos 4x + \tfrac{1}{8}\sin 4x \right]_0^{\pi/4}$	1 for reaching this stage
$= -\tfrac{1}{2}\tfrac{\pi}{4}(-1) = \tfrac{\pi}{8}$	1E1 for the rest Total 5

A4.

When $n = 1$, LHS $= 2$, RHS $= \tfrac{1}{2}\times 1\times 4 = 2$, thus true for $n = 1$.	1 for starting condition
Assume true for $n = k$ and consider the case when $n = k+1$	1 for stating the inductive hypothesis
$2 + 5 + 8 + \ldots + (3k-1) + (3(k+1)-1)$	
$= \tfrac{1}{2}k(3k+1) + 3k + 2$	1 for applying the inductive hypothesis
$= \tfrac{1}{2}(3k^2 + 7k + 4)$	
$= \tfrac{1}{2}(k+1)(3k+4)$	
$= \tfrac{1}{2}(k+1)(3(k+1)+1)$	1 for final form
ie since true for $n = k$ implies true for $n = k+1$ and true for $n = 1$, the result is true for all $n \geq 1$.	1 for statement Total 5

A5.

(*a*) $\dfrac{x}{x^2-1}=\dfrac{A}{x-1}+\dfrac{B}{x+1}$ — 1 method mark

$=\dfrac{\frac{1}{2}}{x-1}+\dfrac{\frac{1}{2}}{x+1}$ — 1 for accuracy

(*b*) $\dfrac{x^3}{x^2-1}=x+\dfrac{x}{x^2-1}$ — 1 for division

$=x+\dfrac{\frac{1}{2}}{x-1}+\dfrac{\frac{1}{2}}{x+1}$ — 1 for applying result of (*a*)

$$\int\frac{x^3}{x^2-1}dx=\int x+\frac{\frac{1}{2}}{x-1}+\frac{\frac{1}{2}}{x+1}\,dx$$

$=\frac{1}{2}x^2+\frac{1}{2}\ln(x-1)+\frac{1}{2}\ln(x+1)+c$ — 2E1 for the rest

$=\frac{1}{2}\left(x^2+\ln\left(x^2-1\right)\right)+c$

Total 6

A6.

$$\left(x^2-\frac{2}{x}\right)^4$$

$=x^8-4x^6.\dfrac{2}{x}+6x^4.\dfrac{4}{x^2}-4x^2\dfrac{8}{x^3}+\dfrac{16}{x^4}$ — 1 for binomial coefficients; 2E1 for accuracy

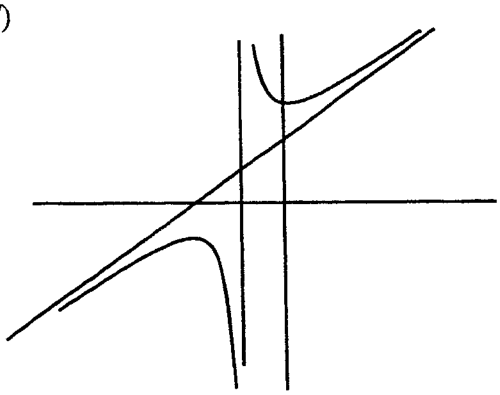
$=x^8-8x^5+24x^2-\dfrac{32}{x}+\dfrac{16}{x^4}$ — 2E1 for accuracy

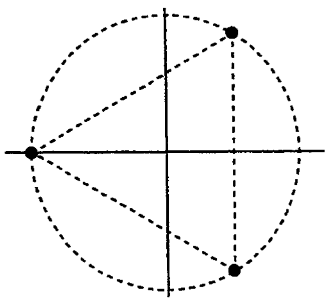
Total 5

A7. (*a*)

$xy + y^2 = 2$	
$x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$	1 method mark 1 for accuracy
$(x+2y)\frac{dy}{dx} = -y$	
$\frac{dy}{dx} = \frac{-y}{x+2y}$	1E1
(*b*) When $x = 1$ and $y = 1$,	
$\frac{dy}{dx} = \frac{-1}{1+2} = -\frac{1}{3}$	1E1
Equation is $(y-1) = -\frac{1}{3}(x-1)$.	1E1 Total 5

A8.

(*a*) $f(x)=\dfrac{x^2+6x+12}{x+2}=x+4+\dfrac{4}{x+2}$	2E1
(*b*) Vertical asymptote $x=-2$	1 for vertical asymptote
Slant asymptote $y=x+4$	1 for slant asymptote
(*c*) $f'(x)=1-\dfrac{4}{(x+2)^2}=0$ at S.V.	1 for derivative
$(x+2)^2=4$	
$x+2=\pm 2$	
$x=0$ or $x=-4$.	1 for coordinates
$f''(x)=\dfrac{8}{(x+2)^3}$	
When $x=0, f(0)=6$ and $f''(0) > 0$ so $(0,6)$ is a minimum turning point. When $x=-4, f(-4)=-2$ and $f''(-4)<0$ so $(-4,-2)$ is a maximum turning point	1 for nature 1 for y coordinates
(*d*) 	1 mark
(*e*) $-2<k<6$	1 mark Total 10

A9. (*a*) $-1 = \cos\ \pi + i\ \sin\ \pi$	1 mark
(*b*) Let $z\ = \cos\ \theta\ + i\ \sin\ \theta$, then	1 for applying de Moivre
$z^3 \cos\ 3\theta + i\ \sin\ 3\theta$	
$\cos\ 3\theta = -1 \Rightarrow 3\theta = \pi$ or 3π or 5π	
$\Rightarrow \theta = \dfrac{\pi}{3}$ or π or $\dfrac{5\pi}{3}$	2E1 for the values of θ
The roots are	
$\cos\frac{\pi}{3} + i\ \sin\frac{\pi}{3} = \frac{1}{2}\left(1 + i\sqrt{3}\right)$	
$\cos 3\pi + i\ \sin\ 3\pi = -1$	2E1 for the roots
$\cos\frac{5\pi}{3} + i\ \sin\ \frac{5\pi}{3} = \frac{1}{2}\left(1 - i\sqrt{3}\right)$	
$\left[\frac{1}{2}\left(1 + i\sqrt{3}\right)^2\right] = \frac{1}{4}\left(1 + 2i\sqrt{3} - 3\right)$	
$= -\frac{1}{2}\left(1 - i\sqrt{3}\right)$	1 mark
$\left[\frac{1}{2}\left(1 - i\sqrt{3}\right)^2\right] = \frac{1}{4}\left(1 - 2i\sqrt{3} - 3\right)$	
$= -\frac{1}{2}\left(1 + i\sqrt{3}\right)$	1 mark
(*c*) 	1 for diagram
The points are on the unit circle	1 mark
and are equally spaced.	1 mark
OR	**OR**
The solutions form an equilateral triangle.	2 marks
	Total 11

A10. (*a*) $\frac{dM}{dt} = kM$	
$\int \frac{dM}{M} = \int k\,dt$	1 method
$\ln M = kt + c$	1 mark
$t = 0,\ M = M_0 \Rightarrow c = \ln M_0$	
$M = M_0 e^{kt}$	1 mark
(*b*) When $t = 30$, $M = \frac{1}{2}M_0$ so	1 mark
$e^{30k} = 0 \cdot 5$	1 mark
$k = \frac{1}{30}\ln 0 \cdot 5 \approx -0 \cdot 0231$	1 mark
(*c*) *When* t = 35	
$\frac{M}{M_0} = e^{35k} \approx 0 \cdot 4454 \approx 45\%$	2E1
(*d*) When $\frac{M}{M_0} = 0 \cdot 25$	1 mark
$e^{kt} = 0 \cdot 25$	
$t = \frac{1}{k}\ln 0 \cdot 25 = \frac{30 \ln 0 \cdot 25}{\ln 0 \cdot 5} = 60$	1 mark
The manufacturer is justified.	Total 10

SECTION B (MATHEMATICS 3)

B1.

$$149 = 1 \times 139 + 10$$
$$139 = 13 \times 10 + 9$$
$$10 = 1 \times 9 + 1$$
$$1 = 10 - 9$$
$$= 10 - (139 - 13 \times 10)$$
$$= 14 \times (149 - 139) - 139$$
$$= 14 \times 149 - 15 \times 139$$

ie $x = 14$ and $y = -15$.

1 method

2E1 accuracy

1 mark

Total 4

B2. $\frac{dy}{dx} + \frac{y}{x} = x$

Integrating factor is $e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

1 method
1 accuracy

$\therefore x\frac{dy}{dx} + y = x^2$

$\frac{d}{dx}(xy) = x^2$

Hence $xy = \int x^2 dx = \frac{1}{3}x^3 + c$

1 mark

Thus $y = \frac{1}{3}x^2 + \frac{c}{x}$.

1 mark

Total 4

B3.

$$AB = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & -1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ 4 & -2 & -2 \\ -3 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} = 2I$$

1 mark

$$A^{-1} = \tfrac{1}{2}B$$

1 mark

$$= \tfrac{1}{2}\begin{pmatrix} 1 & 0 & 1 \\ 4 & -2 & -2 \\ -3 & 2 & 1 \end{pmatrix}$$

1 mark

$$A^2B = A.AB = A.21 = 2A$$

$$= \begin{pmatrix} 2 & 2 & 2 \\ 2 & 4 & 6 \\ 2 & -2 & -2 \end{pmatrix}$$

1 mark

Total 4

B4.

Let $f(x) = (x+2)\ \ln\ (2+x)$

1 mark

then $f(0) = 1 + \ln 2$

$f'(x) = 1 + \ln(x+2) \qquad f'(0) = 1 + \ln 2$

$f''(x) = (2+x)^{-1} \qquad f''(0) = \tfrac{1}{2}$

1 for derivatives

$f'''(x) = -(2+x)^{-2} \qquad f'''(0) = -\tfrac{1}{4}$

1 for evaluating derivatives

$$(x+2)\ \ln\ (2+x)$$

$$= 2\ \ln\ 2\ + (1 + \ln 2)\,x\ + \frac{1}{2}\frac{x^2}{2!} - \frac{1}{4}\frac{x^3}{3!} + \ldots$$

$$= 2\ \ln\ 2\ +\ (1 + \ln 2)x\ + \frac{x^2}{4} - \frac{x^3}{24} + \ldots$$

1 mark

Total 4

B5.

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 6x - 1$	
AE $m^2 + 2m - 3 = 0$	1 for auxiliary equation
$\Rightarrow m = 1$ or $m = -3$	1 for the roots
Complementary function $y = \mathrm{A}e^{x} + \mathrm{B}e^{-3x}$	1 for complementary function
Particular integral $y = ax + b$	1 for trial integral
$\frac{dy}{dx} = a; \frac{d^2y}{dx^2} = 0$	
$0 + 2a - 3ax - 3b = 6x - 1$ $a = -2;\ b = -1$	1 for values
General solution $y = Ae^{x} + \mathrm{B}e^{-3x} - 2x - 1$	Total 5

B6.

(*a*) (i) In parametric form, L_2 can be written as $x = -2s$; $y = -2 - s$; $z = 9 + 2s$ Solving x and y $8 - 2t = -2s \quad \Rightarrow s = t - 4$ $-4 + 2t = -2 - s \quad s = 2 - 2t$ $t - 4 = 2 - 2t \Rightarrow t = 2$ and $s = -2$ from $L_{1,}$ $z = 3 + t = 5$. From L_2, $z = 9 + 2s = 9 - 4 = 5$. So the lines intersect and do so at (4, 0, 5).	1 for parametric form 3E1 for working out the point of intersection
(ii) Representing an angle between L_1 and L_2 by θ $\cos\theta = \dfrac{(-2)^2 + 2(-1) + 1(2)}{\sqrt{2^2 + 2^2 + 1^2}\sqrt{2^2 + 1^2 + 2^2}} = \dfrac{4}{9}$	1 for method 1 for correct calculation
(*b*) (i) Direction of L_2 is $-2\mathrm{i} - \mathrm{j} + 2\mathrm{k}$. Equation of Π is of the form $-2x - y + 2z = k$. Using (1, −4, 2) gives $k = -2 + 4 + 4 = 6$ so an equation is $-2x - y + 2z = 6$.	1 for direction vector 1 for equation 1 for final equation
(ii) Substituting $x = 8 - 2t, y = -4 + 2t, z = 3 + t$ into Π gives $-16 + 4t + 4 - 2t + 6 + 2t = 6$ $4t = 12$ $t = 3$ The point of intersection is (2,2,6).	 1 for parameter 1 for point Total 11

	Mark
C1. $P(R) = 1 - P(W) = 1 - 0\cdot6 = 0\cdot4$ $P(\text{Error}) = P(\text{Error} \mid W).P(W) + P(\text{Error} \mid R)P(R)$ $= 0\cdot02 \times 0\cdot6 + 0\cdot01 \times 0\cdot4$ $= 0\cdot016$	3
$P(R \mid Error)$ $= \dfrac{P(Error \mid R).P(R)}{P(Error \mid W).P(W) + P(Error \mid R).P(R)}$ $\dfrac{0\cdot004}{0\cdot016} = 0\cdot25$	2
C2. $Y = 1\cdot8X + 32$ $\Rightarrow \mu_y = 1\cdot8\mu_x + 32 = 1\cdot8 \times 30 + 32 = 86$ Also $\sigma_y = 1\cdot8\sigma_x = 1\cdot8 \times 5 = 9$	4
C3. In stratified sampling a population is divided up into sub-populations or strata and samples selected from each of the strata. When population members belong to identifiable clusters a sample from the population can be obtained by first taking a sample of clusters and subsequently taking samples of population members from within the selected clusters. In sampling businesses it would be possible to use classification of the businesses as large, medium and small as a basis for stratified sampling. In taking a sample of theatre nurses employed in Scottish hospitals it would be possible to treat the hospitals as clusters.	4

	Mark
C4. $p = \frac{320}{400} = 0 \cdot 8$ As 95% confidence interval for the population proportion is given by:- $p \pm 1 \cdot 96\sqrt{\frac{p(1-p)}{n}}$ $= 0 \cdot 8 \pm 1 \cdot 96\sqrt{\frac{0 \cdot 8 \times 0 \cdot 2}{400}}$ $= 0 \cdot 8 \pm 0 \cdot 04$ $= 0 \cdot 76,\ 0 \cdot 84$ Since the confidence interval does not include 0.9 the claim made by the action group is not supported by the data.	5
C5. $\bar{x} = 157 \cdot 5$ $z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{157 \cdot 5 - 160}{\frac{6}{\sqrt{36}}} = -2 \cdot 5$ The critical region for testing H_0 : $\mu = 160$ against $H_1 : \mu < 160$ is $z < -1 \cdot 64$. Since $-2.5 < -1 \cdot 64$ the null hypothesis would be rejected	5

	Mark
C6.	
(*a*) (i) $P(X \geq 3) = 1 - F(2) = 1 - 0 \cdot 6767 = 0 \cdot 3233$	2
(ii) $P(X = 3) = F(3) - F(2) = 0 \cdot 8571 - 0 \cdot 6767 = 0 \cdot 1804$	2
(iii) $P(Y = 0) = F(0) = 0 \cdot 0498$	1
(*b*) The total number of calls will have the Poisson distribution with parameter 2+3=5.	2
(*c*) $P(T > 5) = 1 - F(4) = 1 - 0 \cdot 4405 = 0 \cdot 5595$	2

SECTION D (NUMERICAL ANALYSIS 1)

D1. $f(x) = e^{1-2x}$ $\quad f'(x) = -2e^{1-2x}$ $\quad f''(x) = 4e^{1-2x}$ $\quad f'''(x) = -8e^{1-2x}$

Taylor polynomial is $p(x) = p(1+h) = e^{-1} - 2e^{-1}h + 2e^{-1}h^2 - 4/3e^{-1}h^3$ **3**

For $f(0\cdot98)$, $h = -0\cdot02$; $\quad p(0\cdot98) = e^{-1}(1 - 2(-0\cdot02) + 2(-0\cdot02)^2)$

$$= e^{-1}(1 + 0\cdot04 + 0\cdot0008) = 0\cdot3829$$ **2**

Principal truncation error term is $-4/3\ e^{-1}\ h^3 = 3\cdot9 \times 10^{-6}$

Hence second order estimate should be accurate to 4D. **2**

D2.

$$L(2\cdot5) = \frac{(2\cdot5-3)(2\cdot5-4)}{(-2)(-3)}\ 1\cdot7831 + \frac{(2\cdot5-1)(2\cdot5-4)}{(2)(-1)}\ 2\cdot0226 + \frac{(2\cdot5-1)(2\cdot5-3)}{(3)(1)}\ 1\cdot9308$$

$= 1\cdot7831/8 + 9 \times 2\cdot0226/8 - 1\cdot9308/4 = 2\cdot016$ **3**

D3.

$\Delta^2 f_0 = \Delta f_1 - \Delta f_0 = (f_2 - f_1) - (f_1 - f_0) = f_2 - 2f_1 + f_0$

$\Delta^3 f_0 = (f_3 - 2f_2 + f_1) - (f_2 - 2f_1 + f_0) = f_3 - 3f_2 + 3f_1 - f_0$

2

Maximum error is $\in + 3\in + 3\in + \in \ = \ 8\in$ **1**

D4.

Difference table is:

i	x	f(x)	diff1	diff2	diff3
0	0.5	−0.623	471	136	24
1	0.6	−0.152	607	160	27
2	0.7	0.455	767	187	23
3	0.8	1.222	954	210	
4	0.9	2.176	1164		
5	1.0	3.340			

3

$\Delta^3 f_1 = 0.027$ **1**

$p = 0.3$

$$f(0.574) = -0.152 + 0.3\ (0.607)\ + \frac{(0.3)(-0.7)}{2}(0.160) + \frac{(0.3)(-0.7)(-1.7)}{6}(0.027)$$

$$= -0.152 + 0.182 - 0.017 + 0.002$$

$$= 0.015$$

3

D5. Consider fitting $y = Ax^2 + Bx + C$ through $(-h, y_0)$, $(0, y_1)$ and (h, y_2).

$$\text{Then } y_0 = Ah^2 - Bh + C$$
$$y_1 = C \qquad\qquad y_0 + y_2(Ah^2 + C)$$
$$y_2 = Ah^2 + Bh + C$$

Approximation to $\int_{-h}^{h} f(x)dx$ is: $\int_{-h}^{h} (Ax^2 + Bx + C)dx = \left[\frac{Ax^3}{3} + \frac{Bx^2}{2} + Cx\right]_{-h}^{h}$

$$= 2(Ah^3/3 + Ch) = h\{2(Ah^2 + C) + 4C\}/3$$
$$= h(y_0 + 4y_1 + y_2)/3$$

4

$f^{(4)}(0) = -12$ which gives maximum numeric value of $f^{(4)}(x)$ on interval since $f^{(4)}(1) = 0.79$.

Then $|E| = 0.125^4 \times 12/180 = 0.00016$

Hence $I_8 = 0.2391$ is a suitable estimate. **4**

Richardson extrapolation gives

$I = (16 \times 0.2391235 - 0.2389685)/15 = 0.2391338)$ **2**

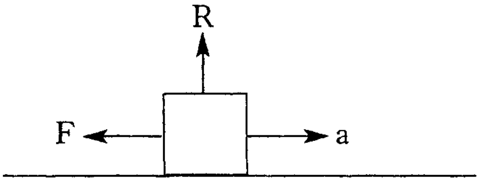
The principal truncation term is now in h^6 giving a likely reduction of the order of 0.125^2, ie two orders of magnitude.

Hence estimate would be 0.239134

2

SECTION E (MECHANICS 1)

E1.



Using $v^2 = u^2 + 2as$ with $(u = 0.2, v = 0, s = 1)$ the deceleration is (1 mark)

$$a = -\frac{u^2}{2x} = -0.02ms^{-2}.$$ (1 mark)

The reaction force is $R = mg$ and the frictional force $F = \mu mg$ in the usual notation. By Newton II

$ma = -\mu mg$

$\Rightarrow \mu = -\frac{a}{g} = 0.002$ (1 mark)

(1 mark)

E2. (*a*) Here $v = 2t(1-3t) + 1 = -6t^2 + 2t + 1$ so

$$a = 0 \Leftrightarrow \frac{dv}{dt} = 0 \Leftrightarrow -12t + 2 = 0 \Leftrightarrow t = \frac{1}{6}$$ (1 mark)

(*b*) Noting that $\frac{dx}{dt} = v$

$$x = t^2 - 2t^3 + t + c$$

and with $x(0) = 0$ then $c = 0$. (1 mark)

The particle is at the origin when

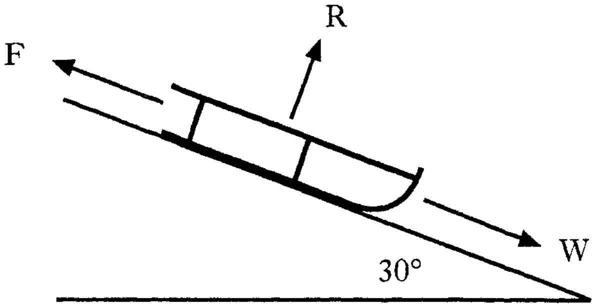
$x = 0 \Leftrightarrow t(2t+1)(t-1) = 0 \Leftrightarrow t = 0, -\frac{1}{2}, 1$ (1 mark)

When the particle returns to the origin when $t > 0$ so we must have $t = 1$. (1 mark)

At $t = 1$ the acceleration is $a = -10\ ms^{-2}$. (1 mark)

E3.

(*a*)



The reaction force is $R = mg\cos 30 = \frac{\sqrt{3}}{2}mg$, where m is the combined mass of John and the sledge. (1 mark for resolving perp to plane)

The frictional force is $F = \frac{\sqrt{3}}{2}\mu mg$. (1 mark)

The component of weight down the hillside is

$$W = mg\sin 30 = \tfrac{1}{2}mg$$

By Newton II

$ma = W - F$ (1 mark)

$\Rightarrow a = \frac{1}{2}(1 - \sqrt{3}\mu)g$ (1 mark)

(*b*) If $\mu > \frac{1}{\sqrt{3}}$ then $1 - \sqrt{3}\mu < 0$, (1 mark)

which implies that the sledge decelerates on the lower part of the hill. (1 mark)

E4. The velocity of the wind (relative to the ground) is $\underline{v}_W = u\underline{i} + v\underline{j}$ where $\underline{i}$ is the unit vector in the easterly direction, $\underline{j}$ the unit vector in the northerly direction.

Joan has velocity $\underline{v}_j = 20\underline{j}$ and the velocity of the wind relative to Joan is

$\underline{v}_w - \underline{v}_j = u\underline{i} + (v - 20)\underline{j}$ (1 mark)

Since the wind appears to come from the east, $v = 20$. (1 mark)

If Joan is cycling at 30km/hr, $\underline{v}_j = 30\underline{j}$, and

$\underline{v}_w - \underline{v}_j = u\underline{i} - 10\underline{j}$ (1mark)

Since the wind appears to come from the north-east, $u = -10$. (1 mark)

The wind speed is

$|\underline{v}_w| = \sqrt{100 + 400} \approx 22.4\ km/hr$ (1 mark)

By trig the wind directionis 26.6° west of north. (1 mark)

E5.

(*a*) From the equations of motion the position of the particle at time t is

$\ddot{x} = 0 \; \dot{x} = V\cos\alpha \Rightarrow x = (V\cos\alpha)t,$ (1 mark)

and

$$\ddot{y} = -g$$

$\Rightarrow \dot{y} = gt + V\sin\alpha$ (1 mark)

$$\Rightarrow y = (V\sin\alpha)t - \tfrac{1}{2}gt^2.$$

Setting $y = 0$ gives the time of flight to be

$t = \dfrac{2V\sin\alpha}{g},$ (1 mark)

and the range

$R = \dfrac{V^2}{g}(2\sin\alpha\cos\alpha) = \dfrac{V^2}{g}\sin 2\alpha.$ (1 mark)

(*b*)(i) Using (a) with $\alpha = 15°$ gives

$$D - d = \frac{V^2}{2g} \quad (*)$$

and when $\alpha = 30°$

$D + d = \dfrac{\sqrt{3V^2}}{2g} \quad . \; (**)$ (1 mark)

Eliminating the velocity between (*) and (**) gives

$D - d = \dfrac{1}{\sqrt{3}}(D + d)$ (1 mark)

which rearranges to give

$d = \left(\dfrac{\sqrt{3}-1}{\sqrt{3}+1}\right)D.$ (1 mark)

From equation (*)

$V^2 = 2gD\left(1 - \left(\dfrac{\sqrt{3}-1}{\sqrt{3}-1}\right)\right)$ (1 mark)

and simplifying

$V^2 = \dfrac{4gD}{\sqrt{3}+1}.$ (1 mark)

(ii) If θ is the required angle of projection, then from (a)

$\sin 2\theta = \dfrac{Dg}{V^2} = \dfrac{1+\sqrt{3}}{4}$ (1 mark)

and hence $\theta = 21.5°$ and $68.5°$. (1 mark)

[END OF MARKING INSTRUCTIONS]