ADVANCED HIGHER MATHEMATICS

Exam Questions on Applications of Calculus

1. A car is travelling along a straight road.
Its acceleration at time t seconds is given by

$$a(t) = 20 + 6t + 4t^2,$$

measured in metres per second per second.

The car started from rest at time $t = 0$ from a point O on the road.
Find expressions for the velocity of the car and its displacement from O after t seconds.

2. The performance of a prototype of a surface-to-air missile was measured on a horizontal test bed at a firing range and it was found that, until its fuel was exhausted, its acceleration (measured in metres per second per second) t seconds after firing was given by

$$a(t) = 8 + 10t - \frac{3}{4}t^2.$$

(a) Assuming that the missile was fired from rest, obtain a formula for its velocity t seconds after firing.

(b) The missile contained enough fuel to fire for 10 seconds.
Find the displacement of the missile from its firing point when its fuel was exhausted.

3. The acceleration of a particle travelling in a straight line is given by $\frac{1}{1+t^2}$ ms^{-2}, where t is the time in seconds since the particle started moving.

Given that the velocity is zero when $t = 1$, find the velocity when $t = \sqrt{3}$.

4. A body moves along a straight line with velocity $v = t^3 - 12t^2 + 32t$ at time t.

(a) Obtain the value of its acceleration when $t = 0$.

(b) At time $t = 0$, the body is at the origin O. Obtain a formula for the displacement of the body at time t.

Show that the body returns to O and obtain the time, T, when this happens.

5. The velocity, v, of a particle P at time t is given by

$$v = e^{3t} + 2e^{t}\,.$$

(a) Find the acceleration of P at time t.

(b) Find the distance covered by P between $t = 0$ and $t = \ln 3$.

6. A car is travelling along a straight road at a constant velocity of 19 ms^{-1}.
Spotting that the road is blocked some distance ahead by a falling tree, the driver brakes immediately and brings the car to a complete stop T seconds later.

The car's acceleration t seconds after the brakes are applied is $a(t)$ ms^{-2}, where

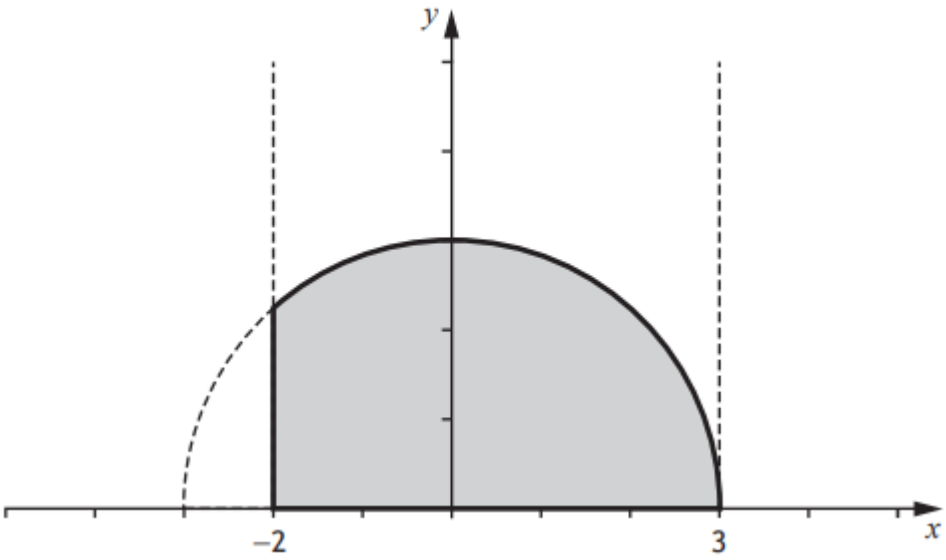
$$a(t) = -\frac{3}{2}\sqrt{4+t}\,, \qquad 0 \le t \le T\,.$$

(a) Show that $T = 5$.

(b) Calculate the distance travelled by the car during the 5 seconds that it takes to come to a stop.

7. A solid is formed by rotating the curve $y = e^{-2x}$ between $x = 0$ and $x = 1$ through $360°$ about the x-axis. Calculate the volume of the solid that is formed.

8. A part of the graph of $x^2 + y^2 = 9$ is shown in the diagram below.

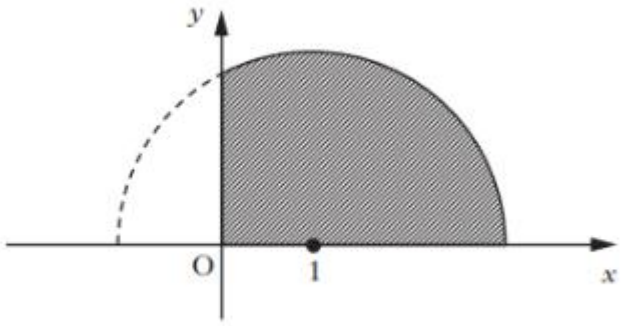


The shaded area is bounded by the graph, the x-axis and the lines $x = -2$ and $x = 3$.
This area is rotated $360°$ about the x-axis.

Calculate the volume of the solid formed by this rotation.

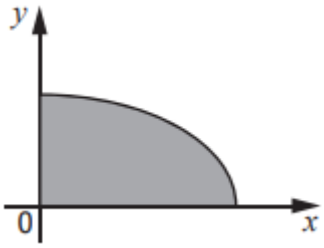
9. A semi-circle with centre (1, 0) and radius 2 lies on the *x*-axis as shown.

Find the volume of the solid of revolution formed when the shaded region is rotated completely about the *x*-axis.



10. On a suitable domain, a curve is defined by the equation $4x^2 + 9y^2 = 36$.

A section of the curve in the first quadrant, illustrated in the diagram below, is rotated $360°$ about the *y*-axis.



Calculate the exact value of the volume generated.

11. (a) Express $\dfrac{1}{x^2 + x}$ in partial fractions.

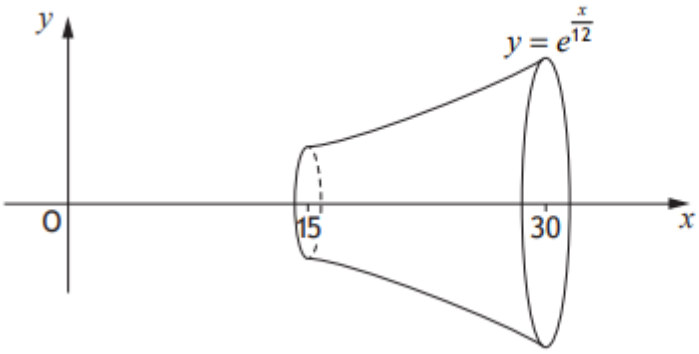
(b) A region is enclosed by the curve with equation $y = \dfrac{1}{\sqrt{x^2 + x}}$, the *x*-axis and the lines $x = 1$ and $x = 3$.

Calculate the volume of the solid of revolution formed by rotating this region through $360°$ about the *x*-axis.

12. (a) Use the substitution $u = 1 + x^2$ to obtain $\int_0^1 \frac{x^3}{(1+x^2)^4}\,dx$.

(b) A solid is formed by rotating the curve $y = \frac{x^{\frac{3}{2}}}{(1+x^2)^2}$ between $x = 0$ and $x = 1$ through $360°$ about the *x*-axis. Write down the volume of this solid.

13. A glass bowl is modelled by rotating the curve $y = e^{\frac{x}{12}}$ between $x = 15$ and $x = 30$ through 2π radians about the *x*-axis as shown in the diagram below, where 1 unit = 1 cm.



(a) Find the volume of the bowl correct to 3 significant figures.

(b) A line is to be put on the bowl to indicate when it is half full.
How far above the base of the bowl should this line be marked?