

ADVANCED HIGHER MATHEMATICS

Exam Questions on Basic Differentiation

1. Given that $y = \sin(e^{5x})$, find $\frac{dy}{dx}$.
2. Given $f(x) = \frac{x-1}{1+x^2}$, show that $f'(x) = \frac{1+2x-x^2}{(1+x^2)^2}$.
3. Given $f(x) = \frac{x^2-1}{x^2+1}$, obtain $f'(x)$ and simplify your answer.
4. Given $f(x) = \frac{1-x^2}{1+4x^2}$, find $f'(x)$, simplifying your answer.
5. Given the curve $y = \frac{x}{x^2+4}$, calculate the gradient when $x = 2$.
6. Differentiate $y = \frac{e^{5x}}{7x+1}$.
7. On a suitable domain, a function is defined by $f(x) = \frac{e^{x^2-1}}{x^2-1}$.
Find $f'(x)$, simplifying your answer.
8. Find the gradient of the tangent to the curve
$$y = 2x\sqrt{x-1}$$
at the point where $x = 10$.
9. Given $f(x) = x(1+x)^{10}$, obtain $f'(x)$ and simplify your answer.
10. Given $f(x) = (x+1)(x-2)^3$, obtain the values of x for which $f'(x) = 0$.
11. A function f is defined by

$$f(x) = \frac{2x}{x-1}, \quad x \neq 1.$$

Find $f'(x)$ and deduce that the derivative is always negative.

12. (a) Differentiate $f(x) = e^{x^2+3}$.
- (b) Differentiate $g(x) = \ln \sqrt{x^2 + 3}$.
13. Given that $f(x) = \ln\left(\frac{1+x}{1-x}\right)$, find $f'(x)$, expressing your answer as a single fraction.
14. Obtain the derivative of $f(x) = \exp(\sin 2x)$.
15. Obtain the derivative of the function
- $$f(x) = x \ln x \quad (x > 0).$$
16. Find the equation of the tangent to the curve $y = x \ln x$ at the point where $x = e$.
17. Differentiate the function $\frac{\sin x}{x}$, $x \neq 0$.
18. Differentiate and simplify where possible:
- (a) $f(x) = \ln(1 + x^2)$
- (b) $g(x) = \frac{2 + \sin x}{2 + \cos x}$
19. Differentiate the following functions, simplifying where possible.
- (a) $f(x) = \frac{1 + \sin x}{1 + 2 \sin x}$, $0 \leq x \leq \pi$
- (b) $g(x) = \ln(1 + e^{2x})$
20. Differentiate the following functions with respect to x :
- (a) $y = x^3 e^{-x^2}$
- (b) $f(x) = \frac{x^2}{\cos x}$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$
21. Differentiate $\frac{1 + \ln x}{3x}$, where $x > 0$, and simplify your answer.
22. Given that $y = e^{2x} \cos x$, find $\frac{dy}{dx}$.

23. (a) Given $f(x) = x^3 \tan 2x$, where $0 < x < \frac{\pi}{4}$, obtain $f'(x)$.
- (b) For $y = \frac{1+x^2}{1+x}$, where $x \neq -1$, determine $\frac{dy}{dx}$ in simplified form.
24. Given that $y = e^{5x} \tan 2x$, find $\frac{dy}{dx}$.
25. Differentiate the following functions.
- (a) $f(x) = e^x \sin(x^2)$
- (b) $g(x) = \frac{x^3}{(1 + \tan x)}$
26. Differentiate the following, simplifying your answers as appropriate.
- (a) $f(x) = e^{2x} \tan x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$
- (b) $g(x) = \frac{\cos 2x}{x^3}$
27. Differentiate $f(x) = e^{\cos x} \sin^2 x$.
28. Given $f(x) = \sin x \cos^3 x$, obtain $f'(x)$.
29. (a) For $y = \frac{5x+1}{x^2+2}$, find $\frac{dy}{dx}$. Express your answer as a single simplified fraction.
- (b) Given $f(x) = e^{2x} \sin^2 3x$, obtain $f'(x)$.
30. (a) Given $f(x) = \frac{3x+1}{x^2+1}$, obtain $f'(x)$.
- (b) Let $g(x) = \cos^2 x \exp(\tan x)$.
Obtain an expression for $g'(x)$ and simplify your answer.
31. (a) If $f(x) = \frac{\ln x}{2x^2}$, $x \neq 0$, find $f'(x)$. Fully simplify your answer.
- (b) If $y = \operatorname{cosec}^2 3x$, show that $\frac{dy}{dx} + 6y \cot 3x = 0$.

32. Differentiate the function $y = \frac{\ln x}{x^3 + 1}$, $x > 0$, with respect to x .

33. Differentiate the following functions with respect to x , simplifying your answers where possible.

(a) $h(x) = \sin(x^2)\cos(3x)$

(b) $y = \frac{\ln(x+3)}{(x+3)}$, $x > -3$

34. Differentiate with respect to x

$$g(x) = e^{\cot 2x}, \quad 0 < x < \frac{\pi}{2}.$$

35. A function is defined as $f(x) = e^{\sec^2 x}$ where $0 \leq x < \frac{\pi}{2}$.

Find the exact value of $f'\left(\frac{\pi}{4}\right)$.

36. Differentiate and simplify as appropriate:

(a) $f(x) = \exp\left(\tan \frac{1}{2}x\right)$, where $-\pi < x < \pi$

(b) $g(x) = (x^3 + 1)\ln(x^3 + 1)$, where $x > 0$

37. Given that $f(x) = \sqrt{x}e^{-x}$, $x > 0$, obtain and simplify $f'(x)$.

38. Differentiate

$$g(x) = \frac{\sin x}{1 + \cos x}, \quad -\pi < x < \pi,$$

and simplify your answer.

39. Given that $y = \ln(1 + \sin x)$, where $0 < x < \frac{\pi}{2}$, show that $\frac{d^2y}{dx^2} = \frac{-1}{1 + \sin x}$.

40. A curve is defined by

$$y = \frac{\sin x}{2 - \cos x} \quad \text{for } 0 \leq x \leq \pi.$$

Find the exact values of the coordinates of the stationary point of this curve.

41. Let $f(x) = \frac{x}{\ln x}$ for $x > 1$.

- (a) Derive expressions for $f'(x)$ and $f''(x)$, simplifying your answers.
- (b) Obtain the coordinates and nature of the stationary point of the curve $y = f(x)$.
- (c) Obtain the coordinates of the point of inflexion.