ADVANCED HIGHER MATHEMATICS

Exam Questions on Basic Integration

1. Determine $\int e^{-2t}dt$.

2. Obtain the exact value of $\int_0^{\frac{\pi}{4}} (\sec x - x)(\sec x + x)dx$.

3. Find $\int \frac{5x}{x^2+3}dx$.

4. Find $\int \frac{12x^3-6x}{x^4-x^2+1}dx$.

5. Integrate $\frac{\sec^2 3x}{1+\tan 3x}$ with respect to x.

6. Obtain $\int_0^{\frac{\pi}{3}} \cos^5 x \sin x dx$ by using the substitution $u = \cos x$ or otherwise.

7. Using the substitution $u = \sin\theta$, or otherwise, evaluate

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2\sin^4\theta\cos\theta d\theta .$$

8. Use the substitution $x = 1+\sin\theta$ to evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{(1+\sin\theta)^3}d\theta$.

9. Use the substitution $u = x+1$ to obtain $\int_0^3 x\sqrt{x+1}dx$.

10. Use the substitution $u = 1+x$ to obtain $\int_0^3 \frac{x}{\sqrt{1+x}}dx$.

11. Use the substitution $u = t+1$ to find $\int_0^8 \frac{t+2}{\sqrt{t+1}}dt$.

12. Use the substitution $u = \ln x$ to obtain $\int \frac{2}{x \ln x} dx$, where $x > 1$.

13. Use the substitution $u = 1 + x^2$ to obtain $\int \frac{x^3}{\sqrt{1+x^2}} dx$.

14. Find the exact value of the integral $\int_0^{\sqrt{5}} \frac{2x^3}{\sqrt{x^2+4}} dx$ using the substitution $u = x^2 + 4$.

15. By means of the substitution $u = x^2 - 8$, find $\int_3^4 x^3 (x^2 - 8)^{\frac{1}{3}} dx$.

16. Use the substitution $x = (u-1)^2$ to obtain $\int \frac{1}{(1+\sqrt{x})^3} dx$.

17. Use the substitution $x = 4\sin\theta$ to evaluate the definite integral

$$\int_0^2 \frac{x+1}{\sqrt{16-x^2}} dx.$$

18. Use the substitution $x = 1 - \sin\theta$ to evaluate the definite integral

$$\int_{\frac{1}{2}}^1 \frac{dx}{\sqrt{2x-x^2}}.$$

19. Use the substitution $x = 4\sin\theta$ to evaluate $\int_0^2 \sqrt{16-x^2} dx$.

(Note that $\cos 2A = 2\cos^2 A - 1$.)

20. Use the substitution $x = 2\sin\theta$ to obtain the exact value of $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} dx$.

(Note that $\cos 2A = 1 - 2\sin^2 A$.)

21. (a) Show that $1 + \tan^2 x = \sec^2 x$.

(b) Hence obtain $\int \tan^2 x dx$.

22. (a) Show that $1+\tan^2 x = \sec^2 x$.

(b) Hence use the substitution $x = \tan\theta$ to determine the exact value of

$$\int_0^1 \frac{dx}{(1+x^2)^{\frac{3}{2}}}.$$