ADVANCED HIGHER MATHEMATICS

## Exam Questions on Differential Equations 2

**Solve each first order linear differential equation by using an integrating factor.**

**1.** Find the general solution of the following differential equation

$$\frac{dy}{dx} + \frac{y}{x} = x, \quad x > 0.$$

**2.** (a) Use integration by parts to find $\int x \sin 3x dx$.

(b) Hence find the particular solution of

$$\frac{dy}{dx} - \frac{2}{x} y = x^3 \sin 3x, \quad x \neq 0$$

given that $y = 0$ when $x = \pi$.

Express your answer in the form $y = f(x)$.

**3.** At any point $(x, y)$ on a curve where $x \neq 0$, the gradient of the tangent is given by

$$\frac{dy}{dx} = 4 - \frac{3y}{x}.$$

Given that the point (1, 3) lies on the curve, obtain an equation for the curve in the form $y = f(x)$.

**4.** Obtain the solution of the differential equation

$$x\frac{dy}{dx} - y = x^2 e^x$$

for which $y = 2$ when $x = 1$.

**5.** A mathematical biologist believes that the differential equation $x\frac{dy}{dx} - 3y = x^4$ models a process. Find the general solution of the differential equation.

Given that $y = 2$ when $x = 1$, find the particular solution, expressing $y$ in terms of $x$.

**6.** Find the general solution of the differential equation

$$\frac{1}{x}\frac{dy}{dx} + 2y = 6, \quad x \neq 0$$