ADVANCED HIGHER MATHEMATICS

## Exam Questions on Differential Equations 3

**Solve each second order differential equation by finding the complementary function and a particular integral (if necessary).**

**1.** Obtain the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0.$$

Hence obtain the solution for which $y = 3$ when $x = 0$ and $y = e^{-\pi}$ when $x = \frac{\pi}{2}$.

**2.** Find the solution $y = f(x)$ to the differential equation

$$4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0,$$

given that $y = 4$ and $\frac{dy}{dx} = 3$ when $x = 0$.

**3.** Solve the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0,$$

given that when $x = 0$, $y = 0$ and $\frac{dy}{dx} = 2$.

**4.** Obtain the general solution of the differential equation $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = e^{2x}$.

**5.** Find the general solution of the following differential equation:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 6x - 1.$$

**6.** Obtain the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2.$$

Given that $y = \frac{1}{2}$ and $\frac{dy}{dx} = 1$ when $x = 0$, find the particular solution.

**7.** Solve the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^x,$$

given that $y = 2$ and $\frac{dy}{dx} = 1$ when $x = 0$.

**8.** Solve the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 12x^2 + 2x - 5$$

given that $y = -6$ and $\frac{dy}{dx} = 3$ when $x = 0$.

**9.** Solve the second order differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 3e^{2x}$$

given that when $x = 0$, $y = 1$ and $\frac{dy}{dx} = 0$.

**10.** Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 8\sin x + 19\cos x$$

given that $y = 7$ and $\frac{dy}{dx} = \frac{1}{2}$ when $x = 0$.

**11.** Obtain the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 20\sin x.$$

Hence find the particular solution for which $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$.

**12.** Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^x + 12.$$

Find the particular solution for which $y = -\frac{3}{2}$ and $\frac{dy}{dx} = \frac{1}{2}$ when $x = 0$.

**13.** Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 4\cos x\,.$$

Hence determine the solution which satisfies $y(0) = 0$ and $y'(0) = 1$.