ADVANCED HIGHER MATHEMATICS

Exam Questions on Maclaurin Series

1. (a) Find the Maclaurin series for $f(x) = \cos x$ as far as the term in x^4.

(b) Deduce the Maclaurin series for $g(x) = \frac{1}{2}\cos 2x$ as far as the term in x^4.

(c) Hence write down the first three non-zero terms of the series for $g(3x)$.

2. (a) Write down the Maclaurin expansion of e^x as far as the term in x^3.

(b) Hence, or otherwise, obtain the Maclaurin expansion of $(1+e^x)^2$ as far as the term in x^3.

3. Find Maclaurin expansions for $\sin 3x$ and e^{4x} up to and including the term in x^3.

Hence obtain an expansion for $e^{4x}\sin 3x$ up to and including the term in x^3.

4. Give that first three non-zero terms in the Maclaurin series for $\cos 3x$.

Write down the first four terms of the Maclaurin series for e^{2x}.

Hence determine the Maclaurin series for $e^{2x}\cos 3x$ up to, and including, the term in x^3.

5. Obtain the first four terms in the Maclaurin series of $\sqrt{1+x}$ and hence write down the first four terms in the Maclaurin series of $\sqrt{1+x^2}$.

Hence obtain the first four terms in the Maclaurin series of $\sqrt{(1+x)(1+x^2)}$.

6. (a) Find the Maclaurin expansion, up to and including the term in x^3, for each of the following functions:

(i) $f(x) = e^{3x}$

(ii) $g(x) = (x+2)^{-2}$

(b) Given that $h(x) = \dfrac{xe^{3x}}{(x+2)^2}$, use the expansions from (a) to find the Maclaurin expansion of $h(x)$ up to and including the term in x^4.

7. Write down the Maclaurin expansion of e^x as far as the term in x^4.

Deduce the Maclaurin expansion of e^{x^2} as far as the term in x^4.

Hence, or otherwise, find the Maclaurin expansion of e^{x+x^2} as far as the term in x^4.

8. Obtain the first three non-zero terms in the Maclaurin expansion of $x\ln(2+x)$.

Hence deduce the first three non-zero terms in the Maclaurin expansion of $x\ln(2-x)$.

Hence obtain the first **two** non-zero terms in the Maclaurin expansion of $x\ln(4-x^2)$.

[*Throughout this question, it can be assumed that* $-2<x<2$.]

9. Find the Maclaurin expansion of $f(x)=e^x\sin x$ up to and including the term in x^3.

10. Obtain the Maclaurin series for $f(x)=\sin^2 x$ up to the term in x^4.

Hence write down the series for $\cos^2 x$ up to the term in x^4.

11. (a) Given $f(x)=e^{2x}$, obtain the Maclaurin expansion for $f(x)$ up to, and including, the term in x^3.

(b) On a suitable domain, let $g(x)=\tan x$.

(i) Show that the third derivative of $g(x)$ is given by $g'''(x)=2\sec^4 x+4\tan^2 x\sec^2 x$.

(ii) Hence obtain the Maclaurin expansion for $g(x)$ up to and including the term in x^3.

(c) Hence obtain the Maclaurin expansion for $e^{2x}\tan x$ up to, and including, the term in x^3.

(d) Write down the first three non-zero terms in the Maclaurin expansion for $2e^{2x}\tan x+e^{2x}\sec^2 x$.

12. Express $\dfrac{x^2+6x-4}{(x+2)^2(x-4)}$ in partial fractions.

Hence obtain the first three non-zero terms in the Maclaurin expansion of

$$f(x)=\frac{x^2+6x-4}{(x+2)^2(x-4)}.$$