Exam Questions on Matrices

1. Given that $A = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}$, show that $A^2 - A = kI$ for a suitable value of k, where I is the 2×2 identity matrix.

2. Given that A, B, C and D are square matrices where:

$$A = \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix} \qquad B = \begin{pmatrix} 4 & 6 \\ 0 & -3 \end{pmatrix} \qquad C = \begin{pmatrix} x & 2 \\ 0 & y \end{pmatrix} \qquad D = \begin{pmatrix} 2 & 7 \\ 12 & -1 \end{pmatrix}$$

(a) Find AB.

(b) Express $4C + D$ as a single matrix.

(c) Given that $AB = 4C + D$, find the values of x and y.

3. (a) Obtain the inverse of the matrix $\begin{pmatrix} 2 & x \\ -1 & 3 \end{pmatrix}$.

(b) For what value of x is this matrix singular?

4. Let A be the matrix $A = \begin{pmatrix} t+4 & 3t \\ 3 & 5 \end{pmatrix}$.

(a) Find A^{-1} in terms of t when A is non-singular.

(b) Write down the value of t such that A is singular.

(c) Given that the transpose of A is $\begin{pmatrix} 6 & 3 \\ 6 & 5 \end{pmatrix}$, find t.

5. Matrices are given as

$$A = \begin{pmatrix} 4 & x \\ 0 & 2 \end{pmatrix}, \; B = \begin{pmatrix} 5 & 1 \\ 0 & 1 \end{pmatrix}, \; C = \begin{pmatrix} y & 3 \\ -1 & 2 \end{pmatrix}.$$

(a) Write $A^2 - 3B$ as a single matrix.

(b)(i) Given that C is non-singular, find C^{-1}, the inverse of C.

(ii) For what value of y would matrix C be singular?

6. Let A be the matrix $A = \begin{pmatrix} 1 & x \\ x & 4 \end{pmatrix}$.

(a) Obtain the value(s) of x for which A is singular.

(b)(i) When $x = 2$, show that $A^2 = pA$ for some constant p.

(ii) Determine the value of q such that $A^4 = qA$.

7. (a) For the matrix $A = \begin{pmatrix} \lambda & 2 \\ 2 & \lambda - 3 \end{pmatrix}$, find the values of λ such that the matrix is singular.

(b) Write down the matrix A^{-1} when $\lambda = 3$.

8. Matrices P and Q are $P = \begin{pmatrix} x & 2 \\ -5 & -1 \end{pmatrix}$ and $Q = \begin{pmatrix} 2 & -3 \\ 4 & y \end{pmatrix}$, where $x,\ y \in \mathbf{R}$.

(a) Given the determinant of P is 2, obtain:

(i) the value of x

(ii) P^{-1}

(ii) $P^{-1}Q'$, where Q' is the transpose of Q.

(b) The matrix R is defined by $R = \begin{pmatrix} 5 & -2 \\ z & -6 \end{pmatrix}$, where $z \in \mathbf{R}$.

Determine the value of z such that R is singular.

9. Matrices A and B are defined by $A = \begin{pmatrix} 4 & p \\ -2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} x & -6 \\ 1 & 3 \end{pmatrix}$.

(a) Find A^2.

(b) Find the value of p for which A^2 is singular.

(c) Find the values of p and x if $B = 3A'$.

10. A is the matrix $\begin{pmatrix} 2 & 0 \\ \lambda & -1 \end{pmatrix}$.

(a) Find the determinant of matrix A.

(b) Show that A^2 can be expressed in the form $pA + qI$, stating the values of p and q.

(c) Obtain a similar expression for A^4.

11. (a) Given $A = \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$, obtain A^{-1} .

(b) Given $AB = \begin{pmatrix} -4 & -3 \\ 6 & -3 \end{pmatrix}$, hence or otherwise find the matrix *B*.

12. Matrices *A*, *B* and *C* are given by

$$A = \begin{pmatrix} 1 & 3 & 4 \\ k & 0 & -1 \\ 5 & 3 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & -10 & 2 \\ -3 & 9 & 0 \\ 0 & -2 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 3 & 2 & -6 \\ 1 & 1 & -2 \\ 2 & 2 & -1 \end{pmatrix}.$$

(a) Find $A + B$.

(b) Find the determinant of *A*.

(c) Find BC .

(d) Describe the relationship between *B* and *C*.

13. Obtain the value(s) of *p* for which the matrix $A = \begin{pmatrix} p & 2 & 0 \\ 3 & p & 1 \\ 0 & -1 & -1 \end{pmatrix}$ is singular.

14. Find the values of *m* for which the matrix

$$A = \begin{pmatrix} m & 1 & 1 \\ 0 & m & -2 \\ 1 & 0 & 1 \end{pmatrix}$$

is singular.

15. Find the values of the constant *k* for which the matrix $\begin{pmatrix} 3 & k & 2 \\ 3 & -4 & 2 \\ k & 0 & 1 \end{pmatrix}$ is singular.

16. Determine *k* such that the matrix $\begin{pmatrix} 1 & 1 & 0 \\ 0 & k-2 & -1 \\ 1 & 2 & k \end{pmatrix}$ does not have an inverse.

17. Matrices C and D are given by:

$$C = \begin{pmatrix} -2 & 1 & 2 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} 1 & 1 & 2 \\ k+3 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix}, \quad \text{where } k \in \mathbf{R}.$$

(a) Obtain $2C' - D$ where C' is the transpose of C.

(b)(i) Find and simplify an expression for the determinant of D.

(ii) State the value of k such that D^{-1} does not exist.

18. (a) For what value of λ is $\begin{pmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ -1 & \lambda & 6 \end{pmatrix}$ singular?

(b) For $A = \begin{pmatrix} 2 & 2\alpha - \beta & -1 \\ 3\alpha + 2\beta & 4 & 3 \\ -1 & 3 & 2 \end{pmatrix}$, obtain values of α and β such that

$$A' = \begin{pmatrix} 2 & -5 & -1 \\ -1 & 4 & 3 \\ -1 & 3 & 2 \end{pmatrix},$$

where A' denotes the transpose of the matrix A.

19. Let

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & -1 & -1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 & 1 \\ 4 & -2 & -2 \\ -3 & 2 & 1 \end{pmatrix}.$$

Show that $AB = kI$ for some constant k, where I is the 3×3 identity matrix.

Hence obtain: (i) the inverse matrix A^{-1}

(ii) the matrix A^2B.

20. The matrix A is such that $A^2 = 4A - 3I$ where I is the corresponding identity matrix. Find integers p and q such that

$$A^4 = pA + qI .$$

21. Given that matrix $A = \begin{pmatrix} 0 & 4 & 2 \\ 1 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix}$, show that $A^2 + A = kI$ for some constant *k*, where *I* is the 3×3 identity matrix.

Obtain the values of *p* and *q* for which $A^{-1} = pA + qI$.

22. Matrices *A* and *B* are defined by

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} x+2 & x-2 & x+3 \\ -4 & 4 & 2 \\ 2 & -2 & 3 \end{pmatrix}.$$

(a) Find the product AB .

(b) Obtain the determinants of *A* and of AB .
Hence, or otherwise, obtain an expression for $\det B$.

23. A non-singular $n \times n$ matrix *A* satisfies the equation $A + A^{-1} = I$, where *I* is the $n \times n$ identity matrix.

Show that $A^3 = kI$ and state the value of *k*.

24. (a) Write down the 2×2 matrix M_1 associated with an anti-clockwise rotation of $\frac{\pi}{2}$ radians about the origin.

(b) Write down the matrix M_2 associated with reflection in the *x*-axis.

(c) Find the matrix $M_2 M_1$ and describe geometrically the effect of the transformation represented by $M_2 M_1$.

25. (a) Write down the 2×2 matrix, M_1 , associated with a reflection in the *y*-axis.

(b) Write down a second 2×2 matrix, M_2 , associated with an anticlockwise rotation through an angle of $\frac{\pi}{2}$ radians about the origin.

(c) Find the 2×2 matrix, M_3 , associated with an anticlockwise rotation through $\frac{\pi}{2}$ radians about the origin followed by a reflection in the *y*-axis.

(d) What single transformation is associated with M_3 ?

26. Write down the 2×2 matrix *A* representing a reflection in the *x*-axis and the 2×2 matrix *B* representing an anti-clockwise rotation of $30°$ about the origin.

Hence show that the image of a point (*x*, *y*) under the transformation *A* followed by *B* is $\left(\frac{kx+y}{2}, \frac{x-ky}{2}\right)$, stating the value of *k*.

27. (a) Obtain the matrix, *A*, associated with an anticlockwise rotation of $\frac{\pi}{3}$ radians about the origin.

(b) Find the matrix, *B*, associated with a reflection in the *x*-axis.

(c) Hence obtain the matrix, *P*, associated with an anticlockwise rotation of $\frac{\pi}{3}$ radians about the origin followed by reflection in the *x*-axis, expressing your answer using exact values.

(c) Explain why matrix *P* is not associated with rotation about the origin.

28. Obtain the 2×2 matrix *M* associated with an enlargement, scale factor 2, followed by a **clockwise** rotation of $60°$ about the origin.