Differential Equations
Variables Separable

QUESTIONS

Find the general solutions of :-

1. $(1+x)\frac{dy}{dx} = xy$

2. $\frac{dy}{dx} = x(1-y)^2$

3. $\frac{dy}{dx} = e^x y^2$

4. $x(y-1)\frac{dy}{dx} = 2y$

5. $\sin x \cos y = \sin y \cos x \frac{dy}{dx}$

6. $y - x\frac{dy}{dx} = 1 + x^2 \frac{dy}{dx}$

ANSWERS

1. $y = \frac{e^{x+c}}{x+1} = \frac{Ae^x}{x+1}$

2. $y = 1 - \frac{2}{x^2+C}$

3. $y = \frac{-1}{e^x + c}$

4. $\frac{e^y}{y} = cx^2$ or $y = \frac{e^y}{cx^2}$

5. $\frac{\cos x}{\cos y} = c$

6. $y = \frac{(c+1)x+1}{(x+1)}$

QUESTIONS

Find the particular solutions of :-

1. $(1-\cos 2x)\frac{dy}{dx} = 2\sin 2x$ when $x = \frac{\pi}{4}$ and $y = 1$.

2. $(1+x^2)\frac{dy}{dx} = 1 + y^2$ when $x = 0$ and $y = 1$.

3. $\frac{dy}{dx} = x(y-2)$ when $x = 0$ and $y = 5$.

4. $\frac{dy}{dx} = \sqrt{(1-y^2)}$ when $x = \frac{\pi}{6}$ and $y = 0$.

5. $\frac{dy}{dx} = y\cos x$ when $x = 0$ and $y = 1$.

6. $\frac{dy}{dx} = \tan x \tan y$ when $x = \frac{\pi}{4}$ and $y = \frac{\pi}{4}$

ANSWERS

1. $y = \ln(1 - \cos 2x) + 1$

2. $\tan^{-1} y = \tan^{-1} x + \frac{\pi}{4}$

3. $y = 3e^{\frac{1}{2}x^2} + 2$

4. $y = \sin\left(x - \frac{\pi}{6}\right)$

5. $y = e^{\sin x}$

6. $\sin y = \frac{1}{2\cos x}$

Differential Equations
Variables Separable

QUESTION 1

Solve the following

(1) $\frac{dy}{dx} = x^3$ (2) $\frac{dy}{dx} = y^3$ (3) $\frac{dy}{dx} = ax$, where a is a constant

(4) $\frac{dy}{dx} = ay$, where a is a constant (5) $\frac{dy}{dx} = y^2 - 1$ (6) $\frac{dy}{dx} = \sec^2 x$

Answers

(1) $y = \frac{x^4}{4} + C$ (2) $y = \frac{1}{\sqrt{C - 2x}}$ (3) $y = \frac{ax^2}{2} + C$

(4) $y = Ae^{ax}$ (5) $y = \frac{2}{1 - Ae^{2x}} - 1$ (6) $y = \tan x + C$

QUESTION

(1) $\frac{dy}{dx} + y = 0$; $y = 2$ when $x = 0$

(2) $\frac{dy}{dx} + y = 3$; $y = 0$ when $x = 0$

(3) $\frac{dy}{dx} = \sin x$; $y = 0$ when $x = 0$

Answers

(1) $y = 2e^{-x}$ (2) $y = 3 - 3e^{-x}$ (3) $y = \cos x - 1$

Differential Equations
Variables Separable

QUESTION

Find solution of

(1) $(1+x)\dfrac{dy}{dx} = xy$ (2) $\dfrac{dy}{dx} = x(1-y)^2$ (3) $(1+y)^2\dfrac{dy}{dx} = e^{x+y}$

(4) $\cos^2 x\dfrac{dy}{dx} = xy^2$

(5) $\dfrac{dy}{dx} = (1-y)\cos x$, and the particular solution for which $y = 2$ when $x = \dfrac{1}{2}\pi$

(6) $y - x\dfrac{dy}{dx} = 1 + x^2\dfrac{dy}{dx}$ (7) $\dfrac{dy}{dx} = \dfrac{xy}{\sqrt{x^2+C}}$, where C is a constant

(8) $\dfrac{dy}{dx} = y^{\frac{3}{2}}$ (9) $\dfrac{dy}{dx} = \dfrac{x}{y}$ (10) $\dfrac{dy}{dx} = \tan x \tan y$

Answers

(1) $y = \dfrac{Ae^x}{x+1}$ (2) $y = 1 - \dfrac{2}{x^2+A}$ (3) $(1+y)^3 = 3e^x + C$

(4) $\dfrac{1}{y} = -x\tan x - \ln(\cos x) + C$ (5) $\dfrac{1}{y-1} = -e^{\sin x - 1}$

(6) $y = \dfrac{Ax}{1+x} + 1$ (7) $y = Ae^{\sqrt{x^2+C}}$ (8) $y = \dfrac{4}{(x+C)^2}$

(9) $y = Ae^{\frac{1}{2}x^2}$ (10) $\sin y = \dfrac{A}{\cos x}$

Differential Equations
Variables Separable

QUESTIONS

1. Find a general solution for each of the following differential equations

(a) $\dfrac{dy}{dx} = 2x$ (b) $\dfrac{dy}{dx} = x^2 - 1$ (c) $\dfrac{dy}{dx} = \dfrac{1}{x+8}$ (d) $\dfrac{dy}{dx} = x^2 + e^x$

2. Find the particular solution of

$$\frac{dy}{dx} = e^{3x} + 2 \text{ given that } y = e^2 \text{ when } x = 0.$$

3. Using the method of variables separable find the general solution for the following differential equations

(a) $\dfrac{dy}{dx} = \dfrac{1}{xy}$ (b) $3y\dfrac{dy}{dx} = 4x$ (c) $2y = (x+1)\dfrac{dy}{dx}$ (d) $\dfrac{dy}{dx} = x(y-1)$

(e) $3(y+2) = 2(x+3)\dfrac{dy}{dx}$ (f) $x^2\dfrac{dy}{dx} + y^2 = 0$

4. Find the particular solution for the differential equations (variables separable)

(a) $\dfrac{dy}{dx} = \dfrac{3x^2+1}{4y+2}, \quad y(1) = 1$

(b) $x\dfrac{dy}{dx} = (1+x)y^2, \quad y(1) = 1$

5. Use integration by parts to help solve the differential equation (variables separable)

$$e^x \frac{dy}{dx} = xy^2, \quad y(0) = 1$$

6. Use partial fractions to help solve the differential equation (variables seperable)

$$y - x\frac{dy}{dx} = 1 + x^2\frac{dy}{dx}$$

Differential Equations
Variables Separable

ANSWERS

1. (a) $y = x^2 + C$ (b) $y = \frac{x^3}{3} - x + C$ (c) $y = \ln|x+8| + C$

   (d) $y = \frac{x^3}{3} + e^x + C$

2. $y = \frac{e^{3x}}{3} + 2x - \frac{1}{3} + e^2$

3. (a) $y = \pm\sqrt{2\ln|x| + C}$ (b) $y = \pm\sqrt{\frac{4}{3}x^2 + C}$ (c) $y = C(x+1)^2$

   (d) $y = 1 + Ce^{x^2/2}$ (e) $y = -2 + C(2x+6)^{3/2}$ (f) $y = \frac{x}{Cx - 1}$

4. (a) $y = -\frac{1}{2} + \frac{1}{2}\sqrt{5 + 2x^3 + 2x}$ (b) $y = \frac{1}{2 - x - \ln x}$

5. $y = \frac{1}{e^{-x}(x+1)}$

6. $y = \frac{1 + Cx}{1 + x}$

Differential Equations
Variables Separable

QUESTION 1

1. The rate at which a radioactive material decays at any instant is proportional to the mass remaining at that instant. Given that there are x grams present after t days, the differential equation in x is modelled by :- $\frac{dx}{dt} = -kx$.
The half–life if a radioactive material is roughly 25 days.
(This means that a given mass will decay to half its mass in 25 days)

Find the time taken for 100 g of the material to decay to 20 g.

2. A pan of milk is heated in a kitchen where the temperature is 15°.
When the milk reaches boiling point it is left to cool and after t minutes the temperature of the milk is ϕ degrees.

The rate of cooling is propertional to $\phi - 15$.
The differential equation modelling this situation is $\frac{d\phi}{dt} = -k(\phi - 15)$.

(a) Find the general solution of the differential equation.

(b) After 10 minutes the temperature of the milk was 50°.

(i) Calculate the temperature of the milk 5 minutes after it boiled.

(ii) The milk is required when the temperature is 45°.
Calculate how long this takes after the milk boiled.

3. The population of a small town was 468 in 1980 and 534 in 1990.
Assuming that the rate of increase of the population p, is proportional to p,

(a) write down a differential equation representing the above information and find the solution to the equation.

(b) calculate the population in 2000.

4. The surface area of a pool is 10000 m^2 and is partially covered with weeds. At any instant, the weeds are increasing in area at a rate proportional to its area at that instant.

(a) If the area of the weed is x m^2 formed in t days, form a differential equation.

(b) Initially, the area is 100 m^2 and after 7 days, the area is 1000 m^2. Show that :-

$$\ln\left(\frac{x}{100}\right) = \frac{1}{7}t\ln(10)$$

(c) Find the area of the pool not covered by weeds after 10·5 days.

(d) Find the time t, when the weeds cover half the surface of the pool.

5. A man is given a drug which causes an initial level of 2 mg of the drug per litre of his blood. After t hours there are x mg of the drug per litre and it is known that the rate of decrease of x is proportional to x. (i.e. $\frac{dx}{dt} = kx$). After 1 hour, $x = 1{\cdot}6$.

(a) Calculate the value of x after 3 hours.

(b) Calculate the time after which $x = 0{\cdot}5$.

Differential Equations
Variables Separable

6. A container is shaped so that when the depth of water is x cm, the volume of water in the container is $(x^2 + 3x)$ cm^3. Water is poured into the container so that, when the depth of water is x cm, the rate of increase is $(x^2 + 4)$ cm^3/sec.

   (a) Show that the differential equation for this situation can be modelled by:-

   $$\frac{dx}{dt} = \frac{x^2+4}{2x+3}, \text{ where } t \text{ is the time in seconds.}$$

   (b) Solve the differential equation to obtain t in terms of x, given that initially the container is empty.

7. The motion of a particle is such that its speed at time t is given by:-

   $$\frac{dv}{dt} = \frac{1}{2}\left(v - v^2\right)$$

   When $t = 0$, $v = 0{\cdot}2$.

   Express v in terms of t.

8. Heat is supplied to an electric kettle at a constant rate of 2000 watts, but heat is lost to the surroundings at a rate of 20 watts for every °C difference between the kettle and that of the surroundings. One watt causes the temperature of the kettle to rise at a rate of 0·02°C per minute.

   If the temperature of the surroundings is 15°C and θ°C is the temperature of the the kettle after t minutes, the differential equation modelling this is :-

   $$\frac{d\theta}{dt} = 40 - \frac{2}{5}(\theta - 15)$$

   How long will it take for the temperature to rise from 15°C to 100°C?

ANSWERS

1. 58 days 2. (a) $\theta = Ae^{-kt} + 15$, (b) (i) 69·5° (ii) 11·7 minutes

3. (a) $p = 468e^{0{\cdot}0132t}$ (b) 609 4. (a) $\frac{dx}{dt} = kx$ (b) Proof (c) 6837 m^2 (d) 12 days

5. (a) 1·024 mg (b) 6·22 hours 6. Proof and $t = \ln\left(\frac{x^2+4}{4}\right) + \frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)$

7. $v = \frac{1}{1+4e^{-\frac{1}{2}t}}$ 8. 4·74 mins

Differential Equations
Variables Separable

QUESTION

1. Population models come in various guises. One theory states that

$$\frac{dP}{dt} = \frac{P}{2}$$

where $P\ (>0)$ denotes the population after time t. If $P = P_0$ at time $t = 0$, express P in terms of t.

2. A learner driver has to learn 400 facts for her test. The rate at which the number of facts she can recall grows is given by

$$\frac{dF}{dt} = k(400 - F)$$

where $F\ (< 400)$ is the number of facts memorised, t is the time measured in days since she started the task and k is a positive constant. Initially she knew no facts. After five days she could memorise 250 facts.

(a) Express F explicitly in terms of t.

(b) When she can remember 80% of the facts she can claim to have mastery of the topic. For how many days will she have to study to claim mastery?

3. The motion of a body falling due to gravity and under the influence of air resistance can be modelled by $\frac{dv}{dt} = -g - 0.2v$ where g is the gravitational constant. At $t = 0$ a body is released from rest. Find an expression for the velocity v in terms of t. (N.B. Throughout the motion $g + 0.2v > 0$)

4. In 1845 Otto Verhulst introduced the logistic equation for population models. In one particular country the population is modelled by $\frac{dP}{dt} = \frac{P}{625}(200 - P)$ where $P\ (0 < P < 200)$ is the population measured in millions and t is the time measured in units of 10 years. At time $t = 0$, $P = 4$. Find, with the aid of partial fractions, the solution to the differential equation.

Differential Equations
Variables Separable

5. In a chemical reaction, two substances X and Y combine to form a third substance Z. Let $Q(t)$ (<15) denote the number of grams of Z formed t minutes after the reaction begins. The rate at which $Q(t)$ varies is governed by the differential equation

$$\frac{dQ}{dt} = \frac{(30-Q)(15-Q)}{900}.$$

(a) Show that $60\ln\left(\frac{30-Q}{15-Q}\right) = t + C$, where C is the constant of integration.

(b) Given that $Q(0) = 0$ find the value of C.

(c) Find, correct to 2 decimal places,

(i) the time taken to form 5 grams of Z.

(ii) the number of grams of Z formed 45 minutes after the reaction begins.

ANSWERS

1. $P = P_0 e^{t/2}$

2. (a) $F = 400(1 - e^{-kt})$, where $k \approx 0.2$ (from $F(5) = 200$).

   (b) when $t \approx 8.04$ i.e. after 9 days.

3. $v = 5g(e^{-t/5} - 1)$

4. $P = \dfrac{200}{1 + 49e^{-8t/25}}$

5. (a) Proof (b) $C = 60\ln 2$ (c) (i) 13.39 minutes (ii) 10.36 grams

Differential Equations
First-Order Linear

QUESTION

(1) $(x+1)\dfrac{dy}{dx} - y = (x+1)^2$

(2) $\dfrac{dy}{dx} - y\tan x = \sin x \cos x$

(3) $\tan x \dfrac{dy}{dx} + 2y = x \operatorname{cosec} x$

(4) $\dfrac{dy}{dx} + \dfrac{2y}{1-x^2} = 1 - x$

(5) $(1+x^2)\dfrac{dy}{dx} + xy = 1$

(6) $\dfrac{dy}{dx} + \dfrac{x+1}{x}y = e^{-x}$

(7) $(1-x)\dfrac{dy}{dx} + xy = (1-x)^2 e^{-x}$

(8) $\dfrac{dy}{dx} + y = 5\cos 2x$

Answers

(1) $y = x^2 + x + Cx + C$

(2) $y = -\dfrac{1}{3}\cos^2 x + \dfrac{C}{\cos x}$

(3) $y = \dfrac{x}{\sin x} + \dfrac{\cos x}{\sin^2 x} + \dfrac{C}{\sin^2 x}$

(4) $y = \left(\dfrac{1-x}{1+x}\right)(x + \dfrac{1}{2}x^2 + C)$

(5) $y = \dfrac{x+C}{1+x^2}$

(6) $y = \dfrac{1}{2}xe^{-x} + Cxe^{-x}$

(7) $y = -\dfrac{1}{2}(1-x)e^{-x} + \dfrac{Ce^{-x}}{1-x}$

(8) $y = \cos 2x + 2\sin 2x + C$

Differential Equations
First-Order Linear

QUESTIONS

1. Find the general solutions of these linear differential equations:

(a) $(x + 1)\frac{dy}{dx} - y = (x + 1)^2$

(b) $\frac{dy}{dx} - y\tan x = \sin x\cos x$

(c) $\tan x\frac{dy}{dx} + 2y = x \operatorname{cosec} x$

(d) $\frac{dy}{dx} + \frac{2y}{1-x^2} = 1 - x$

(e) $x(x + 1)\frac{dy}{dx} - y = x^3e^x$

(f) $\frac{dy}{dx} + y = 5\cos 2x$

(g) $(1 - x)\frac{dy}{dx} + xy = (1 - x)^2e^{-x}$

(h) $\frac{dy}{dx} + \frac{x+1}{x}y = e^{-x}$

(i) $x(x + 1)\frac{dy}{dx} + y = x(x + 1)^2e^{-x}$

(j) $\frac{dy}{dx} + y\cot x = \cos x$

2. Find the particular solutions of these linear differential equations:

(a) $x\frac{dy}{dx} + 2y = x^3$ when $x = 1$ and $y = 2$.

(b) $(1 + x)\frac{dy}{dx} + 2y = x^2$ when $x = 0$ and $y = 0$.

(c) $\sin x\frac{dy}{dx} - y\cos x = 1$ when $x = \frac{\pi}{2}$ and $y = 3$.

(d) $x(x + 1)\frac{dy}{dx} + y = (x + 1)^2e^x$ when $x = 1$ and $y = 0$.

(e) $x\frac{dy}{dx} = y + x^2(\sin x + \cos x)$ when $x = \frac{\pi}{2}$ and $y = 0$.

ANSWERS

1.

(a) $y = (x + 1)(x + c)$

(b) $y = -\frac{1}{3}\cos^2 x + c\sec x$

(c) $y = \operatorname{cosec} x(x + \cot x + c\operatorname{cosec} x)$

(d) $y = \left(\frac{1-x}{1+x}\right)\left(\frac{1}{2}x^2 + x + c\right)$

(e) $y = \left(\frac{x}{x+1}\right)\left(xe^x - e^x + c\right)$

(f) $y = (2\sin 2x + \cos 2x) + ce^{-x}$

(g) $y = ce^x(1 - x) - \frac{1}{2}e^{-x}(1 - x)$

(h) $y = e^{-x}\left(\frac{x}{2} + \frac{c}{x}\right)$

(i) $y = \frac{c(x+1)}{x} - \frac{(x+1)^2}{x}e^{-x}$

(j) $y = c\operatorname{cosec} x - \frac{1}{4}\frac{\cos 2x}{\sin x}$

2.

(a) $y = \frac{1}{5x^2}\left(x^5 - 9\right)$

(b) $y = \frac{x^3(3x+4)}{12(1+x)^2}$

(c) $y = 4\sin x - \operatorname{cosec} x\cot x$

(d) $\left(\frac{x+1}{x}\right)\left(e^x - e\right)$

(e) $y = x^2(\sin x - \cos x) + x(\sin x + \cos x) - x(\frac{\pi}{2} + 1)$

Differential Equations
First-Order Linear

QUESTIONS

1. Find the general solution for the following first order linear differential equations

(a) $\frac{dy}{dx}+2y=5$ (b) $\frac{dy}{dx}+2xy=x$ (c) $\frac{dy}{dx}-2y=1-2x$ (d) $\frac{dy}{dx}-y=e^x$

(e) $\frac{dy}{dx}+\frac{2}{x}y=\frac{\cos x}{x^2}$ (f) $\frac{dy}{dx}+\frac{2}{x+1}y=(x+1)^{5/2}$

2. Write the following first order linear differential equations in the correct form and then find their general solutions

(a) $x\frac{dy}{dx}-2y=-3x$ (b) $3x\frac{dy}{dx}+3y=2x$ (c) $\frac{dy}{dx}+2x=1+2y$

(d) $x^2\frac{dy}{dx}-x^3+xy=0$

3. Find the particular solution of the following first order linear differential equations

(a) $\frac{dy}{dx}-3y=x, \qquad y(1)=2$

(b) $x^2\frac{dy}{dx}+2xy=1, \qquad y(1)=2$

(c) $x\frac{dy}{dx}+2y=xe^{-x}, \qquad y(1)=-1$

4. For a coil of self-inductance L connected to an alternating electromotive force E, the formula connecting the current I and time t is

$$E\sin wt - L\frac{di}{dt}=iR$$

where E, L, R and w are constants relating to the electrical circuit. Show that

$$i=\frac{ER\sin wt-ELw\cos wt}{R^2+L^2w^2}+\frac{C}{e^{\frac{R}{L}t}}$$

Differential Equations
First-Order Linear

ANSWERS

1. (a) $y = \frac{5}{2} + Ce^{-2x}$ (b) $y = \frac{e^{x^2} + 2C}{2e^{x^2}}$ (c) $y = x + Ce^{2x}$

   (d) $y = xe^x + Ce^x$ (e) $y = \frac{\sin x + C}{x^2}$ (f) $y = \frac{2}{11}(x+1)^{7/2} + \frac{C}{(x+1)^2}$

2. (a) $y = 3x + Cx^2$ (b) $y = \frac{x^2 + 3C}{3x}$ (c) $y = x + Ce^{2x}$ (d) $y = \frac{x^3 + 3C}{3x}$

3. (a) $y = -\frac{x}{3} - \frac{1}{9} + \frac{22}{e^3}e^{3x}$ (b) $y = \frac{x+1}{x^2}$ (c) $y = \frac{5e^{-1} - 1 - e^{-x}(x^2 + 2x + 2)}{x^2}$

4. Proof.

Differential Equations

Second Order Linear Homogeneous Constant Coefficients

QUESTIONS

Type 1

(1) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$ (2) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0$ (3) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$

(4) $2\frac{d^2y}{dx^2} - 9\frac{dy}{dx} + 9y = 0$ (5) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = 0$

Type 2

(1) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$ (2) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$ (3) $4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = 0$

(4) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$ (5) $\frac{d^2y}{dx^2} + 2n\frac{dy}{dx} + n^2y = 0$

Type 3

(1) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$ (2) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = 0$ (3) $8\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = 0$

(4) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 13y = 0$ (5) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$

Answers

Type 1:

(1) $y = Ae^{2x} + Be^{x}$ (2) $y = Ae^{3x} + Be^{x}$ (3) $y = Ae^{-3x} + Be^{-2x}$

(4) $y = Ae^{3x} + Be^{3/2x}$ (5) $y = Ae^{3x} + B$

Type 2:

(1) $y = e^{2x}(A + Bx)$ (2) $y = e^{3x}(A + Bx)$ (4) $y = e^{-3x}(A + Bx)$ (3) $y = e^{-1/2x}(Ax+B)$

(5) $y = e^{-nx}(A + Bx)$

Type 3:

(1) $y = e^{-x}(A\cos x + B\sin x)$ (2) $y = e^{-2x}(A\cos 2x + B\sin 2x)$

(3) $y = e^{-\frac{1}{4}x}(A\cos\frac{1}{4}x + B\sin\frac{1}{4}x)$ (4) $y = e^{-3x}(A\cos 2x + B\sin 2x)$

(5) $y = e^{-1/2x}\left(A\cos\frac{\sqrt{3}}{2}x + B\sin\frac{\sqrt{3}}{2}x\right)$

Differential Equations

Second Order Linear Non Homogeneous Constant Coefficients

QUESTIONS

1. Find the general solutions of these linear differential equations:

(a) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = x^3$ (b) $\frac{d^2y}{dx^2} - y = 2 - 5x$

(c) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = 32x^2$ (d) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 1 + x^2$

2. Find the general solutions of these linear differential equations:

(a) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 10e^{2x}$ (b) $4\frac{d^2y}{dx^2} + 13\frac{dy}{dx} + 9y = 7e^{-2x}$

(c) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^x$ (d) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 2e^{-2x}$

3. Find the general solutions of these linear differential equations:

(a) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \sin x$ (b) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 10\cos 2x$

(c) $4\frac{d^2y}{dx^2} + y = 4\sin x$ (d) $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 25y = 26\cos 3x$

4. Find the general solutions of these linear differential equations:

(a) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = x - e^{2x}$ (b) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = e^x + e^{-x}$

(c) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = x - e^x$ (d) $4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = e^x - 2\cos 2x$

5. (a) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 3e^{-2x}$ (b) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2e^x$

6. Find the particular solution of

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = \sin x \text{ for which } y = 0,\ \frac{dy}{dx} = 0 \text{ when } x = 0$$

7. Find the particular solution of

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 5e^{3x} \text{ for which } y = 1,\ \frac{dy}{dx} = -6 \text{ when } x = 0$$

Differential Equations
Second Order Linear Non Homogeneous Constant Coefficients

ANSWERS

1. (a) $y = Ae^{x} + Be^{3x} + \frac{1}{3}x^3 + \frac{4}{3}x^2 + \frac{26}{9}x + \frac{80}{27}$

   (b) $y = Ae^{x} + Be^{-x} + 5x - 2$

   (c) $y = Ae^{-x} + Be^{-4x} + 8x^2 - 20x + 21$

   (d) $y = e^{-x}(A\cos x + B\sin x) + \frac{1}{2}x^2 - x + 1$

2. (a) $y = Ae^{-3x} + Be^{x} + 2e^{2x}$ (b) $y = Ae^{-x} + Be^{-\frac{9}{4}x} - 7e^{-2x}$

   (c) $y = Ae^{-x} + Be^{2x} - \frac{1}{2}e^{x}$ (d) $y = (A + Bx)e^{2x} + \frac{1}{8}e^{-2x}$

3. (a) $y = Ae^{x} + Be^{2x} + \frac{3}{10}\cos x + \frac{1}{10}\sin x$

   (b) $y = (A + Bx)e^{x} - \frac{6}{5}\cos 2x - \frac{8}{5}\sin 2x$

   (c) $y = A\cos\frac{1}{2}x + B\sin\frac{1}{2}x - \frac{4}{3}\sin x$

   (d) $y = e^{-4x}(A\cos 3x + B\sin 3x) + \frac{1}{2}\cos 3x + \frac{3}{4}\sin 3x$

4. (a) $y = (A + Bx)e^{-2x} + \frac{1}{4}x - \frac{1}{4} - \frac{1}{16}e^{2x}$

   (b) $y = Ae^{-3x} + Be^{2x} - \frac{1}{4}e^{x} - \frac{1}{6}e^{-x}$

   (c) $y = e^{-2x}(A\cos 2x + B\sin 2x) + \frac{1}{8}x - \frac{1}{16} - \frac{1}{13}e^{x}$

   (d) $y = (A + Bx)e^{-\frac{1}{2}x} + \frac{1}{9}e^{x} + \frac{30}{289}\cos 2x - \frac{16}{289}\sin 2x$

5. (a) $y = Ae^{-2x} + Be^{-3x} + 3xe^{-2x}$

   (b) $y = Ae^{x} + Be^{3x} - xe^{x}$

6. $y = e^{-x}\left(\frac{2}{5}\cos x + \frac{6}{5}\sin x\right) - \frac{2}{5}\cos x + \frac{1}{5}\sin x$

7. $y = 2e^{-2x} - e^{3x} + xe^{3x}$

Differential Equations
Second Order Linear Non Homogeneous Constant Coefficients

QUESTION

Type 1

(1) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = x^3$

(2) $\frac{d^2y}{dx^2} - y = 2 - 5x$

(3) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4(x+1)$

(4) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = 32x^2$

(5) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 1 + x^2$

Type 2

(6) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 10e^{2x}$

(7) $4\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 9y = 7e^{-2x}$

(8) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^x$

(9) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 2e^{-2x}$

Type 3

(10) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \sin x$

(11) $8\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = \sin x - 2\cos x$

(12) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 10\cos 2x$

(13) $4\frac{d^2y}{dx^2} + y = 4\sin x$

(14) $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 25y = 26\cos 3x$

Type 4

(15) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = x - e^{2x}$

(16) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = e^x + e^{-x}$

(17) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = x - e^x$

(18) $4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = e^x - 2\cos 2x$

(19) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2 + e^{-x}$

(20) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^x + \sin x$

Differential Equations
Second Order Linear Non Homogeneous Constant Coefficients

Answers

(1) $y = Ae^{3x} + Be^{x} + \frac{1}{3}x^3 + \frac{4}{3}x^2 + \frac{26}{9}x + \frac{80}{27}$

(2) $y = Ae^{-x} + Be^{x} + 5x - 2$

(3) $y = Ae^{-2x} + Be^{-x} + 2x - 2$

(4) $y = Ae^{-4x} + Be^{-x} + 8x^2 - 20x + 21$

(5) $y = e^{-x}(A\cos x + B\sin x) + \frac{1}{2}x^2 - x + 1$

(6) $y = Ae^{-3x} + Be^{x} + 2e^{2x}$

(7) $y = e^{-\frac{3}{2}x}(A + Bx) + 7e^{-2x}$

(8) $y = Ae^{2x} + Be^{-x} + \frac{1}{2}e^{x}$

(9) $y = e^{2x}(A + Bx) + \frac{1}{8}e^{2x}$

(10) $y = Ae^{2x} + Be^{2x} + \frac{3}{10}\cos x + \frac{1}{10}\sin x$

(11) $y = e^{-x}(A\cos 2x + B\sin 2x) + \frac{2}{13}\cos x - \frac{3}{13}\sin x$

(12) $y = e^{x}(A + Bx) - \frac{6}{5}\cos 2x - \frac{8}{5}\sin 2x$

(13) $y = A\cos\frac{1}{4}x + B\sin\frac{1}{4}x - \frac{4}{3}\sin x$

(14) $y = e^{4x}(A\cos 3x + B\sin 3x) + \frac{1}{2}\cos 3x + \frac{3}{4}\sin 3x$

(15) $y = e^{-2x}(A + Bx) - \frac{1}{16}e^{2x} + \frac{1}{4}x$

(16) $y = Ae^{-3x} + Be^{2x} - \frac{1}{4}e^{x} - \frac{1}{6}e^{-x}$

(17) $y = e^{2x}(A\cos 2x + B\sin 2x) + \frac{1}{8}x - \frac{1}{13}e^{x} - \frac{1}{16}$

(18) $y = e^{-\frac{1}{2}x}(A + Bx) + \frac{1}{9}e^{x} + \frac{30}{289}\cos 2x - \frac{16}{289}\sin 2x$

(19) $y = Ae^{2x} + Be^{x} + \frac{1}{6}e^{-x} + 1$

(20) $y = e^{3x}(A + Bx) + \frac{1}{4}e^{x} + \frac{3}{50}\cos x + \frac{2}{25}\sin x$

Differential Equations

Second Order Linear Non Homogeneous Constant Coefficients

QUESTION

Find the general solutions and any particular solutions mentioned.

(1) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2x^2 + 1$

(2) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 3\,e^{-2x}$

(3) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2\,e^{x}$

(4) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = x$

(5) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = \sin x$, and the particular solution for which $y = 0$, $\frac{dy}{dx} = 1$ when $x = 0$.

(6) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 25y = 30\sin 5x$, and the particular solution for which $y = 0$, $\frac{dy}{dx} = 1$ when $x = 0$

(7) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 5\,e^{3x}$, and the particular solution for which $y = 1$, $\frac{dy}{dx} = -6$ when $x = 0$.

(8) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 5\sin x$

(9) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = x + \sin x$

(10) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = x + \cos 2x$

(11) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = x + e^{2x}$

(12) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 8e^{x} - 50\sin x$, and the particular solution for which $y = 0$, $\frac{dy}{dx} = 1$ when $x = 0$.

Differential Equations
Second Order Linear Non Homogeneous Constant Coefficients

Answers

(1) $y = Ae^{-2x} + Be^{-x} + x^2 - 3x + 4$

(2) $y = Ae^{-3x} + Be^{-2x} + 3xe^{-2x}$

(3) $y = Ae^{3x} + Be^{x} - xe^{x}$

(4) $y = Ae^{-2x} + Be^{x} - \frac{1}{2}x - \frac{1}{4}$

(5) $y = e^{x}(A\cos x + B\sin x) - \frac{2}{5}\cos x + \frac{1}{5}\sin x$;

$y = e^{x}(\frac{2}{5}\cos x - \frac{3}{5}\sin x) - \frac{2}{5}\cos x + \frac{1}{5}\sin x$

(6) $y = e^{3x}(A\cos 4x + B\sin 4x) - \cos 5x$; $y = e^{3x}(\cos 4x - \frac{1}{2}\sin 4x) - \cos 5x$

(7) $y = Ae^{3x} + Be^{-2x} + xe^{3x}$; $y = -e^{3x} + 2e^{-2x} + xe^{3x}$

(8) $y = Ae^{2x} + Be^{x} + \frac{3}{2}\cos x + \frac{1}{2}\sin x$

(9) $y = e^{-x}(A\cos x + B\sin x) + \frac{2}{5}\cos x + \frac{1}{5}\sin x + \frac{1}{2}x + \frac{1}{2}$

(10) $y = e^{x}(A\cos 2x + B\sin 2x) + \frac{1}{17}\cos 2x + \frac{4}{17}\sin 2x + \frac{1}{5}x - \frac{2}{25}$

(11) $y = Ae^{4x} + Be^{2x} - \frac{1}{2}xe^{x} + \frac{1}{8}x + \frac{3}{32}$

(12) $y = Ae^{-3x} + Be^{2x} + \cos x + 7\sin x - e^{x}$