Partial Fractions (Including Long Division)
Type 1

QUESTION

Find Partial Fractions for the following rational functions.

1. (a) $\dfrac{4x - 9}{(x - 2)(x - 3)}$ (b) $\dfrac{3 - 8x}{x(1 - x)}$

(c) $\dfrac{x + 24}{x^2 - x - 12}$ (d) $\dfrac{2(3x + 4)}{x^2 + 4x}$

ANSWERS

1. (a) $\dfrac{1}{x - 2} + \dfrac{3}{x - 3}$ (b) $\dfrac{3}{x} - \dfrac{5}{1 - x}$ (c) $\dfrac{4}{x - 4} - \dfrac{3}{x + 3}$ (d) $\dfrac{2}{x} + \dfrac{4}{x + 4}$

QUESTION

Express as partial fractions

(1) $\dfrac{5x+1}{(x+5)(x-3)}$ (2) $\dfrac{4x-19}{(x-1)(x-2)}$ (3) $\dfrac{14x}{x^2+x-12}$ (4) $\dfrac{8-x}{1+x-6x^2}$

(5) $\dfrac{x^2-6x-7}{(x-1)(x-2)(x+3)}$

Answers

(1) $\dfrac{3}{x+5} + \dfrac{2}{x-3}$ (2) $\dfrac{15}{x-1} - \dfrac{11}{x-2}$ (3) $\dfrac{8}{x+4} + \dfrac{6}{x-3}$ (4) $\dfrac{5}{3x+1} + \dfrac{3}{1-2x}$

(5) $\dfrac{3}{x-1} - \dfrac{3}{x-2} + \dfrac{1}{x+3}$

Partial Fractions (Including Long Division)
Type 1

QUESTION

Express in partial fractions (all are Type 1)

(a) $\dfrac{2x}{(x-1)(x+1)}$ (b) $\dfrac{20}{(x-3)(x+2)}$ (c) $\dfrac{5x-14}{(x-2)(x-3)}$ (d) $\dfrac{7x+5}{(x-1)(x+3)}$

(e) $\dfrac{3x+4}{x(x+2)}$ (f) $\dfrac{7x-11}{(5-x)(x+1)}$ (g) $\dfrac{5x-3}{x^2+x-30}$ (h) $\dfrac{2x^2-2x-6}{(x+1)(x+2)(x-1)}$

ANSWERS

(a) $\dfrac{1}{x-1}+\dfrac{1}{x+1}$ (b) $\dfrac{4}{x-3}-\dfrac{4}{x+2}$ (c) $\dfrac{4}{x-2}+\dfrac{1}{x-3}$ (d) $\dfrac{3}{x-1}+\dfrac{4}{x+3}$

(e) $\dfrac{2}{x}+\dfrac{1}{x+2}$ (f) $\dfrac{4}{5-x}-\dfrac{3}{x+1}$ (g) $\dfrac{3}{x+6}+\dfrac{2}{x-5}$

(h) $\dfrac{1}{x+1}+\dfrac{2}{x+2}-\dfrac{1}{x-1}$

Partial Fractions (Including Long Division)
Type 2

QUESTION

Find Partial Fractions for the following rational functions.

1. (a) $\dfrac{3x^2 + 1}{x(x + 1)^2}$ (b) $\dfrac{3x^2 + 2}{x(x - 1)^2}$

(c) $\dfrac{x^2 - 2x + 10}{(x + 2)(x - 1)^2}$ (d) $\dfrac{5x^2 - 6x - 21}{(2x - 3)(x - 4)^2}$

ANSWERS

1. (a) $\dfrac{1}{x} + \dfrac{2}{x + 1} - \dfrac{4}{(x + 1)^2}$ (b) $\dfrac{2}{x} + \dfrac{1}{x - 1} + \dfrac{5}{(x - 1)^2}$

(c) $\dfrac{2}{x + 2} - \dfrac{1}{x - 1} + \dfrac{3}{(x - 1)^2}$ (d) $-\dfrac{3}{2x - 3} + \dfrac{4}{x - 4} + \dfrac{7}{(x - 4)^2}$

QUESTION

Express in partial fractions (all are Type 2)

(a) $\dfrac{3x^2 - 11x + 5}{(x-2)(x-1)^2}$ (b) $\dfrac{6x^2 + x - 7}{(x+2)^2(x-3)}$ (c) $\dfrac{x^2 - x - 1}{x^2(x-1)}$ (d) $\dfrac{x^2 + 6x - 3}{x(x-1)^2}$

(e) $\dfrac{25}{(2-x)(2x+1)^2}$ (f) $\dfrac{x-2}{x^2(3x-2)}$ (g) $\dfrac{1}{x^2 - 3x^3}$ (h) $\dfrac{16}{(x^2 - 2x - 3)(x-3)}$

ANSWERS

(a) $-\dfrac{5}{x-2} + \dfrac{8}{x-1} + \dfrac{3}{(x-1)^2}$ (b) $\dfrac{4}{x+2} - \dfrac{3}{(x+2)^2} + \dfrac{2}{x-3}$

(c) $\dfrac{2}{x} + \dfrac{1}{x^2} - \dfrac{1}{x-1}$ (d) $-\dfrac{3}{x} + \dfrac{4}{x-1} + \dfrac{4}{(x-1)^2}$ (e) $\dfrac{1}{2-x} + \dfrac{2}{2x+1} + \dfrac{10}{(2x+1)^2}$

(f) $\dfrac{1}{x} + \dfrac{1}{x^2} - \dfrac{3}{3x-2}$ (g) $\dfrac{3}{x} + \dfrac{1}{x^2} + \dfrac{9}{1-3x}$ (h) $\dfrac{-1}{x-3} + \dfrac{4}{(x-3)^2} + \dfrac{1}{x+1}$

Partial Fractions (Including Long Division)
Type 3

QUESTIONS

Find Partial Fractions for the following rational functions.

1. (a) $\dfrac{8x - 1}{(x - 2)(x^2 + 4)}$ (b) $\dfrac{7x^2 - x + 14}{(x - 2)(x^2 + 4)}$

(c) $\dfrac{x(3x + 2)}{(x - 2)(x^2 + 4)}$ (d) $\dfrac{x^3 + 2x^2 + 61}{(x + 3)^2(x^2 + 4)}$

ANSWERS

1. (a) $\dfrac{3}{x - 2} - \dfrac{3x - 2}{x^2 + 1}$ (b) $\dfrac{5}{x - 2} + \dfrac{2x + 3}{x^2 + 4}$

(c) $\dfrac{2}{x - 2} + \dfrac{x + 4}{x^2 + 4}$ (d) $\dfrac{3}{x + 3} + \dfrac{4}{(x + 3)^2} - \dfrac{2x - 1}{x^2 + 4}$

QUESTION

Express in partial fractions (all are Type 3)

(a) $\dfrac{3}{(x+1)(x^2 - x+1)}$ (b) $\dfrac{2x^2 + 3x + 1}{(x-1)(x^2 + x+1)}$ (c) $\dfrac{16x^2}{(x-3)(2x^2 - x+1)}$

(d) $\dfrac{3x-2}{x^3 + 2x}$ (e) $\dfrac{4x^2 - 4x+1}{(x^2 - x+1)(x-2)}$

ANSWERS

(a) $\dfrac{1}{x+1} + \dfrac{2-x}{x^2 - x+1}$ (b) $\dfrac{2}{x-1} + \dfrac{1}{x^2 + x+1}$ (c) $\dfrac{9}{x-3} + \dfrac{3-2x}{2x^2 - x+1}$

(d) $-\dfrac{1}{x} + \dfrac{x+3}{x^2+2}$ (e) $\dfrac{3}{x-2} + \dfrac{x+1}{x^2 - x+1}$

Partial Fractions (Including Long Division)

Mixed

QUESTION

(1) Express in partial fractions (you decide what type)

(a) $\dfrac{2}{(x-1)^2(x+1)}$ (b) $\dfrac{4}{x^2-4}$ (c) $\dfrac{1}{x(x^2+4)}$ (d) $\dfrac{2x-7}{(x^2+4)(x-1)^2}$

(e) $\dfrac{4x-1}{(x-2)(x+5)}$ (f) $\dfrac{5x+4}{x^3-x^2-x-2}$

(2) Use long division to help turn each of the improper functions into a polynomial and partial fractions

(a) $\dfrac{x^2-x+6}{x^2+x-2}$ (b) $\dfrac{x^3+4x^2-x+2}{x^2+x}$ (c) $\dfrac{x^2}{(x-1)^2}$

HINT: to divide in (c) you need to multiply out the brackets first of all.

ANSWER

(1) (a) $-\dfrac{1}{2(x-1)}+\dfrac{1}{(x-1)^2}+\dfrac{1}{2(x+1)}$ (b) $\dfrac{1}{x-2}-\dfrac{1}{x+2}$ (c) $\dfrac{1}{4x}-\dfrac{x}{4(x^2+4)}$

(d) $\dfrac{4}{5(x-1)}-\dfrac{1}{(x-1)^2}+\dfrac{1-4x}{5(x^2+4)}$ (e) $\dfrac{1}{x-2}+\dfrac{3}{x+5}$ (f) $\dfrac{2}{x-2}-\dfrac{2x+1}{x^2+x+1}$

(2) (a) $1+\dfrac{8-2x}{(x+2)(x-1)}=1+\dfrac{2}{x-1}-\dfrac{4}{x+2}$

(b) $x+3+\dfrac{2-4x}{x(x+1)}=x+3+\dfrac{2}{x}-\dfrac{6}{x+1}$

(c) $1+\dfrac{2x-1}{(x-1)^2}=1+\dfrac{2}{x-1}+\dfrac{1}{(x-1)^2}$

Partial Fractions (Including Long Division)

Mixed

QUESTION

Express as partial fractions:

(1) $\dfrac{x^2+8x+1}{(2-x)(1+x+x^2)}$ (2) $\dfrac{3x^2+x+1}{x(x+1)^2}$ (3) $\dfrac{x^2-2x+10}{(x+2)(x-1)^2}$

(4) $\dfrac{x^2+4x+7}{(x+2)(x+3)^2}$ (5) $\dfrac{3x^2-x-2}{(1+2x)(x+2)^2}$ (6) $\dfrac{3x^2+92x}{(x^2+1)(x+6)}$

(7) $\dfrac{x^3-2x-13}{x^2-2x-3}$

Answers

(1) $\dfrac{3}{2-x}+\dfrac{2x-1}{x^2+x+1}$ (2) $\dfrac{1}{x}+\dfrac{2}{x+1}-\dfrac{3}{(x+1)^2}$ (3) $\dfrac{2}{x+2}-\dfrac{1}{x-1}+\dfrac{3}{(x-1)^2}$

(4) $\dfrac{3}{x+2}-\dfrac{2}{x+3}-\dfrac{4}{(x+3)^2}$ (5) $\dfrac{-3}{2x+1}+\dfrac{7}{x+2}-\dfrac{4}{(x+2)^2}$ (6) $\dfrac{-12}{x+6}+\dfrac{15x+2}{x^2+1}$

(7) $x+2+\dfrac{9}{2(x-3)}+\dfrac{1}{2(x+1)}$