Advanced Higher Maths
Partial Fractions

1. Express as partial fractions

   (a) $\frac{3x-1}{(x+2)(x+3)}$

   (b) $\frac{2x^2+5}{x^3+x^2+x}$

   (c) $\frac{x^2+5}{(x-1)(x-2)(x-3)}$

2. Express $\frac{2x^2}{x+3}$ in the form $Ax + B + \frac{C}{x+3}$

3. Express $\frac{4x^3-6x+2}{x^2+3x+1}$ in the form $Ax + B + \frac{Cx+D}{x^2+3x+1}$

4. Express as partial fractions

   (a) $\frac{x^2+5}{(x+2)(x+3)}$

   (b) $\frac{x^3-5x}{x^3+x^2+x}$

   (c) $\frac{x^3+3x^2+x-1}{x^2+2x}$

Advanced Higher Maths
Partial Fractions – Solutions

1. Express as partial fractions

(a) $\frac{3x-1}{(x+2)(x+3)}$

$$\frac{3x-1}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$$

Multiply both sides by $(x+2)(x+3)$

$$3x-1 = A(x+3) + B(x+2)$$

Try certain values of x

$$x = -2: \; -7 = A$$
$$x = -3: \; -10 = -B \rightarrow B = 10$$

State solution and tidy it up

$$\frac{3x-1}{(x+2)(x+3)} = \frac{-7}{x+2} + \frac{10}{x+3}$$
$$\frac{3x-1}{(x+2)(x+3)} = \frac{10}{x+3} - \frac{7}{x+2}$$

(b) $\frac{2x^2+5}{x^3+x^2+x}$

$$\frac{2x^2+5}{x^3+x^2+x} = \frac{2x^2+5}{x(x^2+x+1)}$$
$$\frac{2x^2+5}{x^3+x^2+x} = \frac{A}{x} + \frac{Bx+C}{x^2+x+1}$$
$$2x^2+5 = A(x^2+x+1) + (Bx+C)x$$

Count amount of x^2, x and number:

$$x^2: 2 = A + B$$
$$x: \; 0 = A + C$$
$$1: \; 5 = A$$

Solve

$$A = 5, B = -3, C = -5$$

State solution and tidy up

$$\frac{2x^2+5}{x^3+x^2+x} = \frac{5}{x} + \frac{-3x-5}{x^2+x+1}$$
$$\frac{2x^2+5}{x^3+x^2+x} = \frac{5}{x} - \frac{3x+5}{x^2+x+1}$$

(c) $\frac{x^2+5}{(x-1)(x-2)(x-3)}$

$$\frac{x^2+5}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

Multiply both sides by $(x-1)(x-2)(x-3)$

$$x^2+5 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

Try particular values of x

$$x = 1: 6 = 2A \rightarrow A = 3$$
$$x = 2: -9 = B$$
$$x = 3: 14 = 2C \rightarrow C = 7$$

State solution

$$\frac{x^2 + 5}{(x-1)(x-2)(x-3)} = \frac{3}{x-1} - \frac{9}{x-2} + \frac{7}{x-3}$$

2. Express $\frac{2x^2}{x+3}$ in the form $Ax + B + \frac{C}{x+3}$

$$\frac{2x^2}{x+3} = 2x - 6 + \frac{18}{x+3}$$

3. Express $\frac{4x^3-6x+2}{x^2+3x+1}$ in the form $Ax + B + \frac{Cx+D}{x^2+3x+1}$

$$\frac{4x^3 - 6x + 2}{x^2 + 3x + 1} = 4x - 12 + \frac{26x + 14}{x^2 + 3x + 1}$$

4. Express as partial fractions

(a) $\frac{x^2+5}{(x+2)(x+3)}$

First do algebraic division

$$\frac{x^2 + 5}{(x+2)(x+3)} = 1 - \frac{5x + 1}{(x+2)(x+3)}$$

Now partial fractions on the fraction

$$\frac{5x + 1}{(x+2)(x+3)} = -\frac{9}{x+2} + \frac{14}{x+3}$$

Put it together

$$\frac{x^2 + 5}{(x+2)(x+3)} = 1 + \frac{9}{x+2} - \frac{14}{x+3}$$

Part (b) and (c) are similar:

(b) $\frac{x^3-5x}{x^3+x^2+x}$

$$\frac{x^3 - 5x}{x^3 + x^2 + x} = 1 - \frac{x + 6}{x^2 + x + 1}$$

(c) $\frac{x^3+3x^2+x-1}{x^2+2x}$

$$\frac{x^3 + 3x^2 + x - 1}{x^2 + 2x} = x + 1 - \frac{1}{2x} - \frac{1}{2(x+2)}$$