Advanced Higher Mathematics – Methods in Algebra and Calculus
Unit Assessment Preparation - Further Practice Questions

1. Use implicit differentiation to find $\dfrac{dy}{dx}$ for each equation below

(a) $x^2 - y^2 = 4$ (b) $x^2 + 2y^2 = 9$ (c) $xy = x^2 + y^2$ (d) $x^2y + xy^2 = 2$

(e) $x^2 + 4xy - y^2 = 16$ (f) $y^3 + 3xy = 3x^2 + 2$ (g) $x^3 - 2y^3 = 3xy$ (h) $y^2 + 2xy = x^3 + 7$

2.

(a) A curve is defined by the parametric equations $x = t^2$ and $y = 2t$
Find the gradient of the curve at the point with parameter $t = 3$

(b) Find the gradient of the curve defined by the following pair of parametric equations
when $t = 3$: $x = t^2 + 1$ and $y = 2t^3$

(c) A curve is given by the parametric equations $x = q + \sin q$ and $y = q - \cos q$
Find $\dfrac{dy}{dx}$ in terms of q.

(d) The position of an object at time, t, is given by: $x = t^2$ and $y = 2t$
Find $\dfrac{dy}{dx}$ in terms of t.

(e) The position of an object at time, t, is given by: $x = 2t^2 + t - 5$ and $y = t^2 + 3t + 1$
Find the rate of change of the object when $t = 1$.

Answers:

1   (a) $\dfrac{dy}{dx} = \dfrac{x}{y}$ (b) $\dfrac{dy}{dx} = -\dfrac{x}{2y}$ (c) $\dfrac{dy}{dx} = \dfrac{2x - y}{x - 2y}$ (d) $\dfrac{dy}{dx} = -\dfrac{y(2x + y)}{x(x + 2y)}$

    (e) $\dfrac{dy}{dx} = -\dfrac{(x + 2y)}{(2x - y)} = \dfrac{2y + x}{y - 2x}$ (f) $\dfrac{dy}{dx} = \dfrac{2x - y}{x + y^2}$ (g) $\dfrac{dy}{dx} = \dfrac{x^2 - y}{x + 2y^2}$ (h) $\dfrac{dy}{dx} = \dfrac{3x^2 - 2y}{2(x + y)}$

2   (a) $\dfrac{dy}{dx} = \dfrac{1}{3}$ (b) $\dfrac{dy}{dx} = 3$ (c) $\dfrac{dy}{dx} = \dfrac{1 + \sin q}{1 + \cos q}$ (d) $\dfrac{dy}{dx} = \dfrac{2}{2t}$ (e) $\dfrac{dy}{dx} = \dfrac{2t + 3}{4t + 1} = 1$