1.1 Applying Algebraic skills to matrices and systems of equations

- **Using Gaussian elimination to solve a 3X3 system of linear equations**

1. Solve the following sets of equations

(a) $\begin{aligned} x+y+z&=10\\ 2x+3y+z&=21\\ x+2y+4z&=19 \end{aligned}$ (b) $\begin{aligned} x+y+2z&=4\\ 3x+4y+z&=19\\ 2x+y+4z&=6 \end{aligned}$ (c) $\begin{aligned} x+2y-z&=8\\ x+3y+2z&=27\\ 2x-2y+5z&=2 \end{aligned}$ (d) $\begin{aligned} x+y-z&=11\\ 2x+3y+2z&=16\\ -x+2y+2z&=4 \end{aligned}$

(e) $\begin{aligned} x+2y+z&=4\\ 2x+5y-3z&=-9\\ 4x-2y-z&=21 \end{aligned}$ (f) $\begin{aligned} x+y-z&=3\\ x+2y+z&=1\\ 2x-3y+2z&=2 \end{aligned}$ (g) $\begin{aligned} x-3y+z&=9\\ 2x-5y+3z&=13\\ 3x+2y-2z&=-12 \end{aligned}$ (h) $\begin{aligned} -x+y+z&=0\\ x+2y+z&=5\\ 2x+y-z&=7 \end{aligned}$

1.1 Applying Algebraic skills to matrices and systems of equations

- **Performing matrix operations of addition, subtraction and multiplication**

2. Given the matrices

$$A=\begin{pmatrix} 1 & -1 & 0\\ 2 & 1 & 2 \end{pmatrix},\quad B=\begin{pmatrix} 0 & 1 & -1\\ 3 & 2 & 1 \end{pmatrix},\quad C=\begin{pmatrix} 4 & 1 & 2\\ -5 & 0 & 2 \end{pmatrix},\quad D=\begin{pmatrix} 1 & 3\\ 3 & 2\\ 3 & 1 \end{pmatrix}$$

$$E=\begin{pmatrix} 2 & 0\\ 4 & 1 \end{pmatrix},\quad F=\begin{pmatrix} 0 & -2\\ 1 & 2 \end{pmatrix},\quad G=\begin{pmatrix} -1 & 3 & 2\\ -2 & 0 & 1\\ 4 & 2 & 3 \end{pmatrix},\quad H=\begin{pmatrix} 0 & -1 & 2\\ 3 & 2 & 0\\ 1 & -1 & 4 \end{pmatrix}$$

find the matrix:

(a) $2A-3B-C$ (b) $3A-2B+C$ (c) $A-2B+3C$
(d) $2(A+B+C)$ (e) $4A-B-2C$ (f) $4B-A+2C$
(g) AD (h) BD (i) CD (j) EA (k) EB (l) EC
(m) FA (n) FB (o) FC (p) EF (q) DE (r) DF
(s) AG (t) BG (u) CG (v) GH (w) HG

1.1 Applying Algebraic skills to matrices and systems of equations

- **Calculating the determinant of a matrix**
- **Finding the inverse of a matrix**

3. Given the matrices

$$P=\begin{pmatrix} 2 & 0\\ 4 & a \end{pmatrix},\quad Q=\begin{pmatrix} 1 & -2\\ 0 & b \end{pmatrix},\quad R=\begin{pmatrix} -2 & 3\\ 1 & c \end{pmatrix},\quad S=\begin{pmatrix} -3 & 4\\ 1 & d \end{pmatrix},$$

$$T=\begin{pmatrix} 3 & 1\\ -2 & e \end{pmatrix},\quad U=\begin{pmatrix} 12 & 3\\ 8 & f \end{pmatrix},\quad V=\begin{pmatrix} 5 & -2\\ -1 & g \end{pmatrix},\quad W=\begin{pmatrix} -4 & -5\\ 2 & h \end{pmatrix},$$

$$A=\begin{pmatrix} k & 3 & 2\\ -2 & 0 & 1\\ 4 & 2 & 3 \end{pmatrix},\quad B=\begin{pmatrix} 0 & -1 & 2\\ m & 2 & 0\\ 1 & -1 & 4 \end{pmatrix}\quad C=\begin{pmatrix} -1 & 3 & 2\\ -2 & 0 & 1\\ p+1 & 2 & 3 \end{pmatrix},\quad D=\begin{pmatrix} 2 & 3 & -2\\ 1 & 2+q & 4\\ -3 & 1 & 2 \end{pmatrix}$$

(a) Find $P^{-1},Q^{-1},R^{-1},S^{-1},T^{-1},U^{-1},V^{-1},W^{-1}$

(b) Determine the value(s) of a,b,c,d,e,f,g,h,k,m,p,q for which each matrix is singular.

Advanced Higher Mathematics - Geometry, Proof and Systems of Equations
Unit Assessment Preparation - Further Practice Questions

Answers:

1 **(a)** $x=5, y=3, z=2$ **(b)** $x=4, y=2, z=-1$ **(c)** $x=-2, y=7, z=4$

(d) $x=2, y=6, z=-3$ **(e)** $x=5, y=-2, z=3$ **(f)** $x=2, y=0, z=-1$

(g) $x=-2, y=-4, z=-1$ **(h)** $x=1, y=3, z=-2$

2 (a) $\begin{pmatrix} -2 & 0 & -5 \\ 18 & 8 & 5 \end{pmatrix}$ **(b)** $\begin{pmatrix} 7 & -4 & 4 \\ -5 & -1 & 6 \end{pmatrix}$ **(c)** $\begin{pmatrix} 13 & 0 & 8 \\ -19 & -3 & 6 \end{pmatrix}$

(d) $\begin{pmatrix} 10 & 2 & 2 \\ 0 & 6 & 10 \end{pmatrix}$ **(e)** $\begin{pmatrix} -4 & -7 & -3 \\ 15 & 2 & 3 \end{pmatrix}$ **(f)** $\begin{pmatrix} 7 & 7 & 0 \\ 0 & 7 & 6 \end{pmatrix}$

(g) $\begin{pmatrix} -2 & -1 \\ 11 & 10 \end{pmatrix}$ **(h)** $\begin{pmatrix} 0 & 1 \\ 0 & 14 \end{pmatrix}$ **(i)** $\begin{pmatrix} 13 & 16 \\ 1 & -13 \end{pmatrix}$

(j) $\begin{pmatrix} 2 & -2 & 0 \\ 6 & -3 & 2 \end{pmatrix}$ **(k)** $\begin{pmatrix} 0 & 2 & -2 \\ 3 & 6 & -3 \end{pmatrix}$ **(l)** $\begin{pmatrix} 8 & 2 & 4 \\ 11 & 4 & 10 \end{pmatrix}$

(m) $\begin{pmatrix} -4 & -2 & -4 \\ 5 & 1 & 4 \end{pmatrix}$ **(n)** $\begin{pmatrix} -6 & -4 & -2 \\ 6 & 5 & 1 \end{pmatrix}$ **(o)** $\begin{pmatrix} 10 & 0 & -4 \\ -6 & 1 & 6 \end{pmatrix}$

(p) $\begin{pmatrix} 0 & -4 \\ 1 & -6 \end{pmatrix}$ **(q)** $\begin{pmatrix} 14 & 3 \\ 14 & 2 \\ 10 & 1 \end{pmatrix}$ **(r)** $\begin{pmatrix} 3 & 4 \\ 2 & -2 \\ 1 & -4 \end{pmatrix}$

(s) $\begin{pmatrix} 1 & 3 & 1 \\ 4 & 10 & 11 \end{pmatrix}$ **(t)** $\begin{pmatrix} -6 & -2 & -2 \\ -3 & 11 & 11 \end{pmatrix}$ **(u)** $\begin{pmatrix} 2 & 16 & 15 \\ 13 & -11 & -4 \end{pmatrix}$

(v) $\begin{pmatrix} 11 & 6 & 6 \\ 1 & 1 & 0 \\ 9 & -3 & 20 \end{pmatrix}$ **(w)** $\begin{pmatrix} 10 & 4 & 5 \\ -7 & 9 & 8 \\ 9 & 11 & 13 \end{pmatrix}$

3 (a) $P^{-1}=\frac{1}{2a}\begin{pmatrix} a & 0 \\ -4 & 2 \end{pmatrix}$ $Q^{-1}=\frac{1}{b}\begin{pmatrix} b & 2 \\ 0 & 1 \end{pmatrix}$ $R^{-1}=\frac{1}{-2c-3}\begin{pmatrix} c & -3 \\ -1 & -2 \end{pmatrix}$

$S^{-1}=\frac{1}{-3d-4}\begin{pmatrix} d & -4 \\ -1 & -3 \end{pmatrix}$ $T^{-1}=\frac{1}{3e+2}\begin{pmatrix} e & -1 \\ 2 & 3 \end{pmatrix}$ $U^{-1}=\frac{1}{12f-24}\begin{pmatrix} f & -3 \\ -8 & 12 \end{pmatrix}$

$V^{-1}=\frac{1}{5g-2}\begin{pmatrix} g & 2 \\ 1 & 5 \end{pmatrix}$ $W^{-1}=\frac{1}{10-4h}\begin{pmatrix} h & 5 \\ -2 & -4 \end{pmatrix}$

(b) $a=0$ $b=0$ $c=-\frac{3}{2}$ $d=-\frac{4}{3}$ $e=-\frac{2}{3}$ $f=2$ $g=\frac{2}{5}$ $h=\frac{5}{2}$

$\det A=22-2k,\ k=11$ $\det B=2m-4,\ m=2$ $\det C=19+3p,\ p=\frac{19}{3}$

$\det D=-2q-49, q=-\frac{49}{2}$