1.1 Applying algebraic and geometric skills to vectors

- **Calculating the vector product**

1. (a) Given the vectors $a = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ and $b = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}$, calculate $a \times b$

(b) Given the vectors $c = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$ and $d = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$, calculate $c \times d$

(c) Given the vectors $e = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $f = \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix}$, calculate $e \times f$

(d) Given the vectors $g = i - k$ and $h = 6j + 5k$, calculate $g \times h$

(e) Given the vectors $m = -2i - 3j + k$ and $n = -i + 3j + 4k$, calculate $m \times n$

(f) Given the vectors $p = i + j + 2k$ and $r = 2j - k$, calculate $p \times r$

1.2 Applying algebraic and geometric skills to vectors

- **Finding the equation of a line in three dimensions**

2. Obtain the equation of each line in Parametric form, Symmetric form and Vector form. Each line passes through the pair of points as detailed below:

(a) $A(0, 1, 3)$ and $B(-1, 2, -4)$

(b) $C(5, -1, 0)$ and $D(6, 2, -7)$

(c) $E(3, 11, -2)$ and $F(6, -1, 0)$

(d) $G(2, 1, 0)$ and $H(3, 7, 10)$

(e) $J(0, 0, 0)$ and $K(1, 2, 3)$

(f) $L(1, -2, -1)$ and $M(2, 3, 1)$

(g) $N(3, -1, 6)$ and $P(0, -3, -1)$

(h) $R(1, 2, -1)$ and $S(-1, 0, 1)$

3. Obtain the equation of each line in Parametric form, Symmetric form and Vector form.

(a) A line passes through the point $S(1, -2, 3)$ and is parallel to the line $\begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$

(b) A line passes through the point $T(-1, 2, -2)$ and is parallel to the line $\begin{pmatrix} 5 \\ 2 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

(c) A line passes through the point $U(4,2,-1)$ and is parallel to the line $\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}$

(d) A line passes through the point $V(1,-1,1)$ and is parallel to the line $\frac{x-3}{2}=\frac{y-7}{2}=\frac{z-10}{1}$

(e) A line passes through the point $W(3,4,5)$ and is parallel to the line $\frac{x+1}{2}=\frac{y-2}{1}=\frac{z+3}{-3}$

(f) A line passes through the point $Z(-1,3,0)$ and is parallel to the line $\frac{x-2}{-2}=\frac{y+1}{-1}=\frac{z-2}{4}$

1.2 Applying algebraic and geometric skills to vectors

- **Finding the equation of a plane**

4. Obtain the equation of each plane in Parametric form, Cartesian form and Vector form.

(a) The plane has normal vector $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ and passes through the point $A(0,2,6)$.

(b) The plane has normal vector $\begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix}$ and passes through the point $B(2,1,-1)$.

(c) The plane has normal vector $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and passes through the point $C(5,3,-2)$.

(d) The plane which passes through the point $D(-4,6,7)$ and is perpendicular to $-4i+6j+7k$

(e) The plane which passes through the point $E(-1,2,1)$ and is perpendicular to $i-3j+2k$.

(f) The plane which passes through the point $F(1,-3,1)$ and is perpendicular to $i+2j-2k$.

(g) The plane which is parallel to the vectors $3i+2j-k$ and $4i-2k$ and passes through the point $G(1,1,0)$.

(h) The plane which is parallel to the vectors $i+j+2k$ and $2j-k$ and passes through the point $H(1,2,-1)$.

(i) The plane which is parallel to the vectors $i+j+k$ and $-2i-3j+4k$ and passes through the origin.

Answers:

1 **(a)** $-4i+2j-8k$ **(b)** $-2i-18j-8k$ **(c)** $7i-6j-k$ **(d)** $6i-5j+6k$
(e) $-15i+7j-9k$ **(f)** $-5i+j+2k$

2 **Equation of each line in Parametric Form:**

(a) $x=t-1,\quad y=-t+2,\quad z=7t-4$
OR $x=t,\quad y=-t+1,\quad z=7t+3$

(b) $x=t+5,\quad y=3t-1,\quad z=-7t$
OR $x=t+6,\quad y=3t+2,\quad z=-7t-7$

(c) $x=3t+3,\quad y=-12t+11,\quad z=2t-2$
OR $x=3t+6,\quad y=-12t-1,\quad z=2t$

(d) $x=t+2,\quad y=6t+1,\quad z=10t$
OR $x=t+3,\quad y=6t+7,\quad z=10t+10$

(e) $x=t,\quad y=2t,\quad z=3t$
OR $x=t+1,\quad y=2t+2,\quad z=3t+3$

(f) $x=t+1,\quad y=5t-2,\quad z=2t-1$
OR $x=t+2,\quad y=5t+3,\quad z=2t+1$

(g) $x=-3t+3,\quad y=-2t-1,\quad z=-7t+6$
OR $x=-3t,\quad y=-2t-3,\quad z=-7t-1$

(h) $x=-2t+1,\quad y=-2t+2,\quad z=2t-1$
OR $x=-2t-1,\quad y=-2t,\quad z=2t+1$

2 **Equation of each line in Symmetric Form:**

(a) $\frac{x+1}{1}=\frac{y-2}{-1}=\frac{z+4}{7}$ **OR** $\frac{x}{1}=\frac{y-1}{-1}=\frac{z-3}{7}$

(b) $\frac{x-5}{1}=\frac{y+1}{3}=\frac{z}{-7}$ **OR** $\frac{x-6}{1}=\frac{y-2}{3}=\frac{z+7}{-7}$

(c) $\frac{x-3}{3}=\frac{y-11}{-12}=\frac{z+2}{2}$ **OR** $\frac{x-6}{3}=\frac{y+1}{-12}=\frac{z}{2}$

(d) $\frac{x-2}{1}=\frac{y-1}{6}=\frac{z}{10}$ **OR** $\frac{x-3}{1}=\frac{y-7}{6}=\frac{z-10}{10}$

(e) $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ **OR** $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$

(f) $\frac{x-1}{1}=\frac{y+2}{5}=\frac{z+1}{2}$ **OR** $\frac{x-2}{1}=\frac{y-3}{5}=\frac{z-1}{2}$

(g) $\frac{x-3}{-3}=\frac{y+1}{-2}=\frac{z-6}{-7}$ **OR** $\frac{x}{-3}=\frac{y+3}{-2}=\frac{z+1}{-7}$

(h) $\frac{x-1}{-2}=\frac{y-2}{-2}=\frac{z+1}{2}$ **OR** $\frac{x+1}{-2}=\frac{y}{-2}=\frac{z-1}{2}$

2 **Equation of each line in Vector Form:**

(a) $\begin{pmatrix}0\\1\\3\end{pmatrix}+t\begin{pmatrix}1\\-1\\7\end{pmatrix}$ **OR** $\begin{pmatrix}-1\\2\\-4\end{pmatrix}+t\begin{pmatrix}1\\-1\\7\end{pmatrix}$

(b) $\begin{pmatrix}5\\-1\\0\end{pmatrix}+t\begin{pmatrix}1\\3\\-7\end{pmatrix}$ **OR** $\begin{pmatrix}6\\2\\-7\end{pmatrix}+t\begin{pmatrix}1\\3\\-7\end{pmatrix}$

(c) $\begin{pmatrix} 3 \\ 11 \\ -2 \end{pmatrix} + t\begin{pmatrix} 3 \\ -12 \\ 2 \end{pmatrix}$ **OR** $\begin{pmatrix} 6 \\ -1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 3 \\ -12 \\ 2 \end{pmatrix}$ **(d)** $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 6 \\ 10 \end{pmatrix}$ **OR** $\begin{pmatrix} 3 \\ 7 \\ 10 \end{pmatrix} + t\begin{pmatrix} 1 \\ 6 \\ 10 \end{pmatrix}$

(e) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ **OR** $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ **(f)** $\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$ **OR** $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$

(g) $\begin{pmatrix} 3 \\ -1 \\ 6 \end{pmatrix} + t\begin{pmatrix} -3 \\ -2 \\ -7 \end{pmatrix}$ **OR** $\begin{pmatrix} 0 \\ -3 \\ -1 \end{pmatrix} + t\begin{pmatrix} -3 \\ -2 \\ -7 \end{pmatrix}$ **(h)** $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + t\begin{pmatrix} -2 \\ -2 \\ 2 \end{pmatrix}$ **OR** $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + t\begin{pmatrix} -2 \\ -2 \\ 2 \end{pmatrix}$

3 Equation of each line in <u>Parametric Form</u>:

(a) $x = 2t + 1, \quad y = t - 2, \quad z = -t + 3$ **(b)** $x = t - 1, \quad y = -t + 2, \quad z = t - 2$

(c) $x = 3t + 4, \quad y = t + 2, \quad z = 3t - 1$ **(d)** $x = 2t + 1, \quad y = 2t - 1, \quad z = t + 1$

(e) $x = 2t + 3, \quad y = t + 4, \quad z = -3t + 5$ **(f)** $x = -2t - 1, \quad y = -t + 3, \quad z = 4t$

3 Equation of each line in <u>Symmetric Form</u>:

(a) $\frac{x-1}{2} = \frac{y+2}{1} = \frac{z-3}{-1}$ **(b)** $\frac{x+1}{1} = \frac{y-2}{-1} = \frac{z+2}{1}$ **(c)** $\frac{x-4}{3} = \frac{y-2}{1} = \frac{z+1}{3}$

(d) $\frac{x-1}{2} = \frac{y+1}{2} = \frac{z-1}{1}$ **(e)** $\frac{x-3}{2} = \frac{y-4}{1} = \frac{z-5}{-3}$ **(f)** $\frac{x+1}{-2} = \frac{y-3}{-1} = \frac{z}{4}$

3 Equation of each line in <u>Vector Form</u>:

(a) $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ **(b)** $\begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ **(c)** $\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + t\begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}$

(d) $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + t\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ **(e)** $\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ **(f)** $\begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} + t\begin{pmatrix} -2 \\ -1 \\ 4 \end{pmatrix}$

4 **(a)** **Cartesian =** $2x+3y+z=12$

Parametric = $x=t,\ y=\dfrac{2+3t}{3},\ z=6+t$

Vector = $\begin{pmatrix}0\\2\\6\end{pmatrix}+\begin{pmatrix}2\\3\\1\end{pmatrix}t$

(b) **Cartesian =** $5x+4y-3z=17$

Parametric = $x=\dfrac{2+5t}{5},\ y=\dfrac{1+4t}{4},\ z=\dfrac{1+3t}{3}$

Vector = $\begin{pmatrix}2\\1\\-1\end{pmatrix}+\begin{pmatrix}5\\4\\-3\end{pmatrix}t$

(c) **Cartesian =** $2x-3y+z=-1$

Parametric = $x=\dfrac{5+2t}{2},\ y=\dfrac{2-3t}{-3},\ z=6+t$

Vector = $\begin{pmatrix}5\\3\\-2\end{pmatrix}+\begin{pmatrix}2\\-3\\1\end{pmatrix}t$

(d) **Cartesian =** $-4x+6y+7z=101$

Parametric = $x=1+t,\ y=1+t,\ z=1+t$

Vector = $\begin{pmatrix}-4\\6\\7\end{pmatrix}+\begin{pmatrix}-4\\6\\7\end{pmatrix}t$

(e) **Cartesian =** $x-3y+2z=-5$

Parametric = $x=-1+t,\ y=\dfrac{2-3t}{-3},\ z=\dfrac{1+2t}{2}$

Vector = $\begin{pmatrix}-1\\2\\1\end{pmatrix}+\begin{pmatrix}1\\-3\\2\end{pmatrix}t$

(f) **Cartesian =** $x+2y-2z=-7$

Parametric = $x=1+t,\ y=\dfrac{-3+2t}{2},\ z=\dfrac{1-2t}{-2}$

Vector = $\begin{pmatrix}1\\-3\\1\end{pmatrix}+\begin{pmatrix}1\\2\\-2\end{pmatrix}t$

(g) **Cartesian =** $-4x+2y-8z=-2$ **OR** $2x-y+4z=1$

Parametric = $x=1+3s+4t,\ y=1+2s,\ z=-s-2t$

Vector = $\begin{pmatrix}1\\1\\0\end{pmatrix}+\begin{pmatrix}3\\2\\-1\end{pmatrix}s+\begin{pmatrix}4\\0\\-2\end{pmatrix}t$

(h) **Cartesian =** $-5x+y+2z=-5$

Parametric = $x=1+s,\ y=2+s+2t,\ z=-1+2s-t$

Vector = $\begin{pmatrix}1\\2\\-1\end{pmatrix}+\begin{pmatrix}1\\1\\2\end{pmatrix}s+\begin{pmatrix}0\\2\\-1\end{pmatrix}t$

(i) **Cartesian =** $7x-6y-z=0$

Parametric = $x=s-2t,\ y=s-3t,\ z=s+4t$

Vector = $\begin{pmatrix}1\\1\\1\end{pmatrix}s+\begin{pmatrix}-2\\-3\\4\end{pmatrix}t$