

2004

11. A solid is formed by rotating the curve $y = e^{-2x}$ between $x = 0$ and $x = 1$ through 360° about the x -axis.

Calculate the volume of the solid that is formed.

5 marks

2006

10. The amount x micrograms of an impurity removed per kg of a substance by a chemical process depends on the temperature $T^\circ\text{C}$ as follows:

$$x = T^3 - 90T^2 + 2400T. \quad 10 \leq T \leq 60.$$

At what temperature in the given range should the process be carried out to remove as much impurity per kg as possible?

4 marks

2008

10. A body moves along a straight line with velocity $v = t^3 - 12t^2 + 32t$ and time t .

(a) Obtain the value of its acceleration when $t = 0$.

1 mark

(b) At time $t = 0$, the body is at the origin O . Obtain a formula for the displacement of the body at time t .

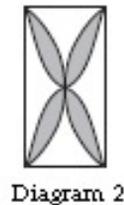
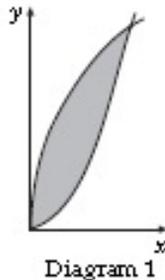
2 marks

Show that the body returns to O , and obtain the time, T , when this happens.

2 marks

2010

15. A new board game has been invented and the symmetrical design on the board is made from four identical “petal” shapes. One of these petals is the region enclosed between the curves $y = x^2$ and $y^2 = 8x$ as shown shaded in diagram 1 below. Calculate the area of the complete design, as shown in diagram 2.



The counter used in the game is formed by rotating the shaded area shown in diagram 1 above, through 360° about the y -axis. Find the volume of plastic required to make one counter.

5 marks

2012

12. The radius of a cylindrical column of liquid is decreasing at the rate of 0.02 m s^{-1} , while the height is increasing at the rate of 0.01 m s^{-1} . Find the rate of change of the volume when the radius is 0.6 metres and the height is 2 metres.

[Recall that the volume of a cylinder is given by $V = \pi r^2 h$].

5 marks

2013

4. The velocity, v , of a particle P at time t is given by $v = e^{3t} + 2e^t$.

(a) Find the acceleration of P at time t .

2 marks

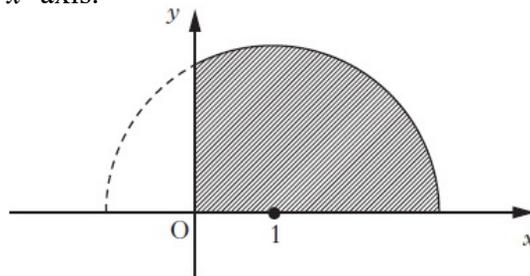
(a) Find the distance covered by P between $t = 0$ and $t = \ln 3$.

3 marks

2014

10. A semi-circle with centre $(1, 0)$ and radius 2 , lies on the x -axis as shown.

Find the volume of the solid of revolution formed when the shaded region is rotated completely about the x -axis.



5 marks

13. The fuel efficiency, F , in km per litre of a vehicle varies with its speed, s km per hour and for a particular vehicle the relationship is thought to be

$$F = 15 + e^x (\sin x - \cos x - \sqrt{2}), \text{ where } x = \frac{\pi(s-40)}{80},$$

for speeds in the range $40 \leq s \leq 120$ km per hour.

What is the greatest and least efficiency over the range and at what speeds do they occur?

10 marks

2016

11. The height of a cube is increasing at the rate of 5 cms^{-1} .
Find the rate of increase of the volume when the height of the cube is 3cm. **4 marks**

16. A beaker of liquid was placed in a fridge.
The rate of cooling is given by $\frac{dT}{dt} = -k(T - T_F)$, $k > 0$,

where T_F is the constant temperature in the fridge and T is the temperature of the liquid at time t .

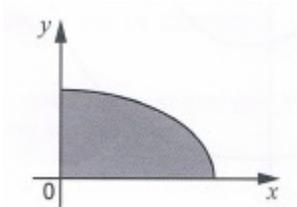
- The constant temperature in the fridge is 4°C .
- When first placed in the fridge, the temperature of the liquid was 25°C .
- At 12 noon, the temperature of the liquid was 9.8°C .
- At 12:15pm, the temperature of the liquid had dropped to 6.5°C .

At what time, to the nearest minute, was the liquid placed in the fridge? **9 marks**

2017

16. On a suitable domain, a curve is defined by the equation $4x^2 + 9y^2 = 36$.

A section of the curve in the first quadrant, illustrated in the diagram below, is rotated 360° about the **y-axis**.



Calculate the exact value of the volume generated. **5 marks**

2019

16. (a) Use integration by parts to find the exact value of $\int_0^1 (x^2 - 2x + 1)e^{4x} dx$. **5**

- (b) A solid is formed by rotating the curve with equation $y = 4(x-1)e^{2x}$ between $x = 0$ and $x = 1$ through 2π radians about the x -axis.

Find the exact value of the volume of this solid. **3**

