2001

A2. Differentiate with respect to x **(4)**

(a) $f(x) = (2+x)\tan^{-1}\sqrt{x-1}, x > 1$ (b) $g(x) = e^{\cot 2x}, 0 < x < \frac{\pi}{2}$. **(2)**

A7. A curve has equation $xy + y^2 = 2$

(a) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y. **(3)**

(b) Hence find an equation of the tangent to the curve at the point (1, 1). **(2)**

2002

A3. A curve is defined by the parametric equations $x = t^2 + t - 1$, $y = 2t^2 - t + 2$ for all t.

Show that the point A (-1, 5) lies on the curve and obtain an equation of the tangent **(6)**

to the curve at the point A.

A4. (a) Given that $f(x) = \sqrt{x}e^{-x}, x \geq 0$, obtain and simplify $f'(x)$. **(4)**

(b) Given $y = (x+1)^2(x+2)^{-4}$ and $x > 0$, use logarithmic differentiation to show that $\frac{dy}{dx}$ can be expressed in the form $\left(\frac{a}{x+1} + \frac{b}{x+2}\right)y$, stating the values of the constants a and b. **(3)**

2003

A1. Given $f(x) = x(1+x)^{10}$, obtain $f'(x)$ and simplify your answer. **(3)**

A1 b) Given $y = 3^x$, use logarithmic differentiation to obtain $\frac{dy}{dx}$ in terms of x. **(3)**

A3. The equation $y^3 + 3xy = 3x^2 - 5$ defines a curve passing through the point A (2,1).
Obtain an equation for the tangent to the curve at A. **(4)**

2004

1. a) Given $f(x) = \cos^2 x e^{\tan x}, -\frac{\pi}{2} < x < \frac{\pi}{2}$, obtain $f'(x)$ and evaluate $f'\left(\frac{\pi}{4}\right)$. **(3,1)**

   b) Differentiate $g(x) = \frac{\tan^{-1} 2x}{1+4x^2}$. **(3)**

3. A curve is defined by the equations $x = 5\cos\theta$, $y = 5\sin\theta$, $(0 \leq \theta \leq 2\pi.)$

   Use parametric differentiation to find $\frac{dy}{dx}$ in terms of θ. **(2)**

   Find the equation of the tangent to the curve at the point where $\theta = \frac{\pi}{4}$. **(3)**

2005

1. a) Given $f(x) = x^3 \tan 2x$, where $0 < x < \frac{\pi}{4}$, obtain $f'(x)$. **(3)**

   b) For $y = \frac{1+x^2}{1+x}$, where $x \neq -1$, determine $\frac{dy}{dx}$ in simplified form. **(3)**

2. Given the equation $2y^2 - 2xy - 4y + x^2 = 0$ of a curve, obtain the x-coordinate of each point at which the curve has a horizontal tangent. **(4)**

15. a) Given $f(x) = \sqrt{\sin x}$, where $0 < x < \pi$, obtain $f'(x)$. **(1)**

    b) If, in general, $f(x) = \sqrt{g(x)}$, where g(x) > 0, show that $f'(x) = \frac{g'(x)}{k\sqrt{g(x)}}$, stating the value of *k*. **(2)**

2006

2. Differentiate, simplifying your answers:

   a) $2\tan^{-1}\sqrt{1+x}$, where $x > -1$; **(3)**

   b) $\frac{1+\ln x}{3x}$, where $x > 0$. **(3)**

4. Given $xy - x = 4$, use implicit differentiation to obtain $\frac{dy}{dx}$ in terms of *x* and *y*. **(2)**

   Hence obtain $\frac{d^2y}{dx^2}$ in terms of *x* and *y*. **(3)**

11. Show that $1 + \cot^2\theta = \operatorname{cosec}^2\theta$, where $0 < \theta < \frac{\pi}{2}$. **(1)**

    By expressing $y = \cot^{-1}x$ as $x = \cot y$, obtain $\frac{dy}{dx}$ in terms of *x*. **(3)**

2007

2. Obtain the derivative of each of the following functions:

   (a) $f(x) = \exp(\sin 2x)$; **(3)**

   (b) $y = 4^{(x^2+1)}$. **(3)**

13. A curve is defined by the parametric equations $x = \cos 2t$, $y = \sin 2t$, $0 < t < \frac{\pi}{2}$.

    (a) Use parametric differentiation to find $\frac{dy}{dx}$ **(5)**

    Hence find the equation of the tangent when $t = \frac{\pi}{8}$.

    (b) Obtain an expression for $\frac{d^2y}{dx^2}$ and hence show that $\sin 2t\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = k$, **(5)**

    where *k* is an integer. State the value of *k*.

2008

2. (a) Differentiate $f(x) = \cos^{-1}(3x)$ where $-\frac{1}{3} < x < \frac{1}{3}$. **(2)**

(b) Given $x = 2\sec\theta, y = 3\sin\theta,$ use parametric differentiation to find $\frac{dy}{dx}$ in terms of θ. **(3)**

5. A curve is defined by the equation $xy^2 + 3x^2y = 4$ for $x > 0$ and $y > 0$.

Use implicit differentiation to find $\frac{dy}{dx}$. **(3)**

Hence find an equation of the tangent to the curve where x = 1. **(3)**

9. Write down the derivative of tan x. **(1)**

Show that $1 + \tan^2 x = \sec^2 x$. **(1)**

15. Let $f(x) = \frac{x}{\ln x}$ for $x > 1$.

(a) Derive expressions for $f'(x)$ and $f''(x)$, simplifying your answers. **(2, 2)**

(b) Obtain the coordinates and nature of the stationary point of the curve $y = f(x)$. **(3)**

(c) Obtain the coordinates of the point of infexion. **(2)**

2009

1. (a) Given $f(x) = (x+1)(x-2)^3$ obtain the values of x for which $f'(x) = 0$. **(3)**

(b) Calculate the gradient of the curve defined by $\frac{x^2}{y} + x = y - 5$ at the point (3, -1). **(4)**

11. The curve $y = x^{2x^2+1}$ is defined for $x > 0$. Obtain the values of y and $\frac{dy}{dx}$ at the point where $x = 1$. **(5)**

2010

1. Differentiate the following functions. (a) $f(x) = e^x \sin x^2$. (b) $g(x) = \frac{x^3}{(1+\tan x)}$ **(3)** **(3)**

13. Given $y = t^3 - \frac{5}{2}t^2$ and $x = \sqrt{t}$ for $t > 0$, use parametric differentiation to express $\frac{dy}{dx}$ in terms of t in simplified form. **(4)**

Show that $\frac{d^2y}{dx^2} = at^2 + bt,$ determining the values of the constants a and b. **(3)**

Obtain an equation for the tangent to the curve which passes through the point of inflexion. **(3)**

2011

3. (a) Obtain $\frac{dy}{dx}$ when y is defined as a function of x by the equation $y + e^y = x^2$ **(3)**

(b) Given $f(x) = \sin x \cos^3 x$ obtain $f'(x)$. **(3)**

7. A curve is defined by the equation $y = \frac{e^{\sin x}(2+x)^3}{\sqrt{1-x}}$ for $x < 1$.

Calculate the gradient of the curve when $x = 0$. **(4)**

2012

1. (a) Given $f(x) = \frac{3x+1}{x^2+1}$, obtain $f'(x)$. **(3)**

(b) Let $g(x) = \cos^2 x \exp(\tan x)$. Obtain an expression for $g'(x)$ and simplify your answer. **(4)**

13. A curve is defined parametrically, for all t, by the equations

$$x = 2t + \frac{1}{2}t^2, \qquad y = \frac{1}{3}t^3 - 3t.$$

Obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ as functions of t. **(5)**

Find the values of t at which the curve has stationary points and determine their nature. **(3)**
Show that the curve has exactly two points of inflexion. **(2)**

2013

2. Differentiate $f(x) = e^{\cos x}\sin^2 x$. **(3)**

11. A curve has the equation $x^2 + 4xy + y^2 + 11 = 0$

Find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point (–2, 3). **(6)**

2014

1. (a) Given $f(x) = \frac{x^2-1}{x^2+1}$ obtain $f'(x)$ and simplify your answer. **(3)**

(b) Differentiate $y = \tan^{-1}(3x^2)$. **(3)**

4. Given $x = \ln(1+t^2), y = \ln(1+2t^2)$ use parametric differentiation to find $\frac{dy}{dx}$ in terms of t. **(3)**

6. Given $e^y = x^3\cos^2 x$, $x > 0$, show that $\frac{dy}{dx} = \frac{a}{x} + b\tan x$, for some constants a and b. **(3)**

State the values of a and b.

2015

2. a) For $y = \frac{5x+1}{x^2+2}$, find $\frac{dy}{dx}$. Express your answer as a single, simplified fraction. **(3)**

b) Given $f(x) = e^{2x}\sin^2 3x$, obtain $f'(x)$. **(3)**

4. The equation $x^4 + y^4 + 9x - 6y = 14$ defines a curve passing through the point A (1, 2).

Obtain the equation of the tangent to the curve at A. **(4)**

6. For $y = 3^x$, obtain $\frac{dy}{dx}$. **(3)**

8. Given $x = \sqrt{t+1}$ and $y = \cot t$, $0 < t < \pi$, obtain $\frac{dy}{dx}$ in terms of y. **(3)**

2016

1a. Differentiate $y = x\tan^{-1} 2x$. **3 marks**

1b. Given $f(x) = \frac{1-x^2}{1+4x^2}$, find $f'(x)$, simplifying your answer. **3 marks**

1c. A curve is given by the parametric equations $x = 6t$ and $y = 1 - \cos t$. **2 marks**

Find $\frac{dy}{dx}$ in terms of t.

2017

3. On a suitable domain, a function is defined by $f(x) = \frac{e^{x^2-1}}{x^2-1}$
Find $f'(x)$, simplifying your answer. **3 marks**

11. Given $y = x^{2x^3+1}$, use logarithmic differentiation to find $\frac{dy}{dx}$.

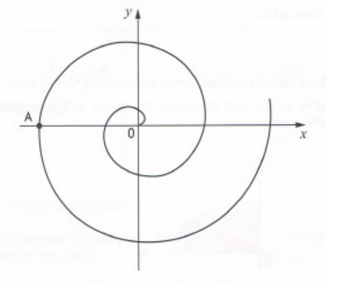
Write your answer in terms of x. **5 marks**

18. The position of a particle at time t is given by the parametric equations

$$x = t\cos t, \qquad y = t\sin t, \quad t \geq 0.$$

a) Find an expression for the instantaneous speed of the particle. **5 marks**

The diagram shows the path that the particle takes.



b) Calculate the instantaneous speed of the particle at point A. **2 marks**

2018

1. (a) Given $f(x)=\sin^{-1}3x$, find $f'(x)$. **2**

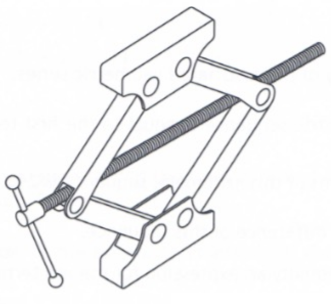
(b) Differentiate $y=\dfrac{e^{5x}}{7x+1}$. **2**

(c) For $y\cos x+y^2=6x$, use implicit differentiation to find $\dfrac{dy}{dx}$. **4**

6. On a suitable domain, a curve is defined parametrically by $x=t^2+1$ and $y=\ln(3t+2)$.

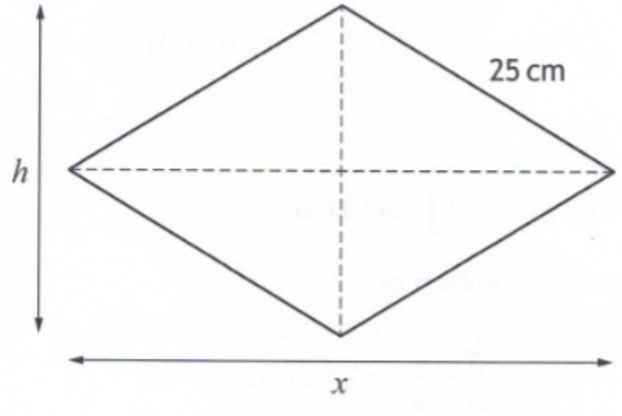
Find the equation of the tangent to the curve where $t=-\dfrac{1}{3}$. **5**

13. An engineer has designed a lifting device. The handle turns a screw which shortens the horizontal length and increases the vertical height.



The device is modelled by a rhombus, with each side 25 cm.

The horizontal length is x cm, and the vertical height is h cm as shown.



(a) Show that $h=\sqrt{2500-x^2}$. **1**

(b) The horizontal length decreases at a rate of 0·3 cm per second as the handle is turned.

Find the rate of change of the vertical height when $x=30$. **5**

2019

1. (a) Differentiate $f(x) = x^6 \cot 5x$. **2**

(b) Given $y = \dfrac{2x^3+1}{x^3-4}$, find $\dfrac{dy}{dx}$. Simplify your answer. **3**

(c) For $f(x) = \cos^{-1} 2x$ evaluate $f'\left(\dfrac{\sqrt{3}}{4}\right)$. **3**

5. For $x = \ln(2t+7)$ and $y = t^2$, $t > 0$, find

(a) $\dfrac{dy}{dx}$ **2**

(b) $\dfrac{d^2y}{dx^2}$. **2**

6. A spherical balloon of radius r cm, $r > 0$, deflates at a constant rate of $60\ \text{cm}^3\,\text{s}^{-1}$.

Calculate the rate of change of the radius with respect to time when $r = 3$. **3**

[The volume of a sphere is given by $V = \dfrac{4}{3}\pi r^3$.]

10. A curve is defined implicitly by the equation $x^2 + y^2 = xy + 12$.

(a) Find an expression for $\dfrac{dy}{dx}$ in terms of x and y. **3**

(b) There are two points where the tangent to the curve has equation $x = k$, $k \in \mathbb{R}$. Find the values of k. **2**

Answers

2001

A2 a) $\tan^{-1}\sqrt{x+1}+\dfrac{2+x}{2x\sqrt{x-1}}$ A2 b) $g'(x)=-2e^{\cot 2x}\text{cosec}^2 2x$

A7 a) $\dfrac{dy}{dx}=\dfrac{-y}{x+2y}$ A7 b) $3y=4-x$

2002

A3 $f'(x)=\dfrac{1}{2\sqrt{x}}e^{-x}(1-2x)$ A4. a) $f'(x)=\dfrac{1}{2\sqrt{x}}e^{-x}(1-2x)$ b) $a=2,\quad b=-4$

2003

A1 a) $f'(x)=(1+11x)(1+x)^9$ A1 b) $\dfrac{dy}{dx}=\ln 3y=\ln 3.3^x$ A3 $y=x-1$

2004

1 a) $f'(x)=(1-\sin 2x)e^{\tan x};\ f'\left(\dfrac{\pi}{4}\right)=0$ 1 b) $g'(x)=\dfrac{2-8x\tan^{-1}2x}{\left(1+4x^2\right)^2}$

3a) $\dfrac{dy}{dx}=\dfrac{5\cos\theta}{-5\sin\theta}$ 3b) $x+y=5\sqrt{2}$

2005

1 a) $f'(x)=3x^2\tan 2x+x^3(2\sec^2 2x)$

1 b) $\dfrac{dy}{dx}=\dfrac{x^2+2x-1}{(1+x)^2}$ or $1-\dfrac{2}{(1+x)^2}$ or $\dfrac{2x}{(1+x)}-\dfrac{1+x^2}{(1+x)^2}$

2) $x=0$ or $x=4$

15) $f'(x)=\dfrac{1}{2}\dfrac{\cos x}{(\sin x)^{1/2}}$ $k=2;$

2006

2a) $f'(x)=\dfrac{1}{(2+x)\sqrt{1+x}}$ 2b) $f'(x)=\dfrac{-\ln x}{3x^2}$

4a) $\dfrac{dy}{dx}=\dfrac{1-y}{x}$ 4b) $\dfrac{d^2y}{dx^2}=\dfrac{2(y-1)}{x^2}$

11) proof $\dfrac{dy}{dx}=-\dfrac{1}{1+x^2}$

2007

2a) $f'(x)=2\cos 2x\exp(\sin 2x)$ 2b) $\dfrac{dy}{dx}=2x\ln 4.4^{(x^2+1)}$ or $\ln 4.2xe^{\ln 4(x^2+1)}$

13a) $\dfrac{dy}{dx}=-\cot 2t.\quad x+y=\sqrt{2}$ 13b) $k=-1$

2008

2a) $f'(x)=\dfrac{-3}{\sqrt{1-9x^2}}$ 2b) $f'(x)=\dfrac{3\cos^3\theta}{2\sin\theta}$

5a) $\dfrac{dy}{dx}=\dfrac{-y^2-6xy}{2xy+3x^2}$ 5b) $5y+7x=12$

9a) $\sec^2 x$ b) Proof c) $\tan x-x+c$

15a) $\dfrac{2-\ln x}{x(\ln x)^3}$ b) (e, e) Min c) $x=e^2\Rightarrow y=\dfrac{1}{2}e^2$

2009 1a) $x = 2$ and $x = -\frac{1}{4}$ 1b) $\frac{dy}{dx} = -\frac{1}{2}$ 11) $x = 1, \quad y = 1$ and $\frac{dy}{dx} = 3$

2010 1 a) $f'(x) = e^x \sin x^2 + e^x\left(2x\cos x^2\right)$ b) $g'(x) = \frac{3x^2(1+\tan x) - x^3\sec^2 x}{(1+\tan x)^2}$

13) $\frac{dy}{dx} = 6t^{\frac{5}{2}} - 10t^{\frac{3}{2}}$ $a = 30, \quad b = -30$ $2y + 8x = 5$

2011 3a) $\frac{dy}{dx} = \frac{2}{1+e^y}$ 3b) $\frac{dy}{dx} = \cos^4 x - 3\sin^2 x\cos^2 x$ 7) $\frac{dy}{dx} = 24$

2012 1a) $f'(x) = \frac{-3x^2 - 2x + 3}{(x^2+1)^2}$

1b) $g'(x) = (1 - \sin 2x)\tan x$

13. $\frac{dy}{dx} = \frac{t^2 - 3}{2+t}$ $\frac{d^2y}{dx^2} = \frac{t^2 + 4t + 3}{(2+t)^3}$

When $t = \sqrt{3}, \frac{d^2y}{dx^2} = \frac{3 + 4\sqrt{3} + 3}{\left(2+\sqrt{3}\right)^3} > 0$

which gives a minimum.

When $t = -\sqrt{3}, \frac{d^2y}{dx^2} = \frac{3 - 4\sqrt{3} + 3}{\left(2-\sqrt{3}\right)^3} < 0$

which gives a maximum.

At a point of infexion $\frac{d^2y}{dx^2} = 0.$

In this case, that means

$t^2 + 4t + 3 = (t+1)(t+3) = 0$

and this has exactly two roots.

2013 2) $e^{\cos x}\sin x\left(2\cos x - \sin^2 x\right)$

11) $\frac{dy}{dx} = 4$

$\frac{d^2y}{dx^2} = 33$

2014 1) $a)\frac{4x}{\left(x^2+1\right)^2}$ $b)\frac{6x}{1+9x^4}$

4) $\frac{dy}{dx} = \frac{2\left(1+t^2\right)}{\left(1+2t^2\right)}$ or $\frac{2+2t^2}{1+2t^2}$ 6) $a = 3, b = -2$

2015 2.a) $\frac{-5x^2 - 2x + 10}{(x^2+2)^2}$ b) $2e^{2x}\sin 3x(\sin 3x + 3\cos 3x)$ 4. $2y + x = 5$

6. $\frac{dy}{dx} = 2x\ln 3.3^{x^2}$ 8. $-2\sqrt{t+1}.\cos ec^2 t$

2016 1a. $\tan^{-1} 2x + \frac{2x}{1+4x^2}$ 1b. $\frac{-10x}{\left(1+4x^2\right)^2}$ 1c. $\frac{1}{6}\sin t$

2017 3. $\frac{2xe^{x^2-1}\left(x^2-2\right)}{\left(x^2-1\right)^2}$ **(3)** 11. $\frac{dy}{dx} = x^{2x^3+1}\left(6x^2\ln x + \frac{2x^3+1}{x}\right)$ **(5)**

18. $\sqrt{\left(\cos t - t\sin t\right)^2 + \left(\sin t + t\cos t\right)^2}$ $t = 3\pi$ speed = $\sqrt{1+9\pi^2}$ **(7)**

$= \sqrt{1+t^2}$

2018

1.a) $f'(x) = \dfrac{3}{\sqrt{1-9x^2}}$ b) $\dfrac{dy}{dx} = \dfrac{e^{5x}(35x-2)}{(7x+1)^2}$ c) $\dfrac{dy}{dx} = \dfrac{6+y\sin x}{\cos x + 2y}$

6. $2y = -9x + 10$

13.a) proof b) 0·225cm/s

2019

1.a) $6x^5\cot 5x - 5x^6\operatorname{cosec}^2 5x$ b) $\dfrac{-27x^2}{(x^3-4)^2}$ c) -4

5.a) $2t^2 + 7t$ b) $\dfrac{1}{2}(2t+7)(4t+7)$

6. $-\dfrac{5}{3\pi}\text{cms}^{-1}$

10. a) $\dfrac{dy}{dx} = \dfrac{y-2x}{2y-x}$ b) $k = \pm 4$