2001

A3. Find the value of $\int_0^{\pi/4} 2x\sin 4x\,dx$. **(5)**

A5. (a) Obtain partial fractions for $\frac{x}{x^2-1}, x>1$. **(2)**

(b) Use the result of (a) to find $\int \frac{x^3}{x^2-1}dx, x>1$. **(4)**

2002

A5. Use integration by parts to evaluate $\int_0^1 \ln(1+x)dx$. **(5)**

A6. Use the substitution $x + 2 = 2\tan\theta$ to obtain $\int \frac{1}{x^2+4x+8}dx$ **(5)**

2003

A5. Use the substitution $x = 1+\sin\theta$ to evaluate $\int_0^{\pi/2} \frac{\cos\theta}{(1+\sin\theta)^3}d\theta$. **(3)**

A10. Define $I_n = \int_0^1 x^n e^{-x}dx$ for $n \geq 1$.

a) Use Integration by parts to obtain the value of $I_1 = \int_0^1 xe^{-x}dx$. **(3)**

b) Similarly, show that $I_n = nI_{n-1} - e^{-1}$ for $n \geq 2$. **(4)**

c) Evaluate I_3. **(3)**

2004

5. Express $\frac{1}{x^2-x-6}$ in partial fractions. **(2)**

Evaluate $\int_0^1 \frac{1}{x^2-x-6}dx$. **(4)**

9. Use the substitution $x = (u-1)^2$ to obtain $\int \frac{1}{(1+\sqrt{x})^3}dx$. **(5)**

2005

5. Use the substitution $u = 1 + x$ to evaluate $\int_0^3 \frac{x}{\sqrt{1+x}}dx$. **(5)**

13. Express $\frac{1}{x^3+x}$ in partial fractions. **(4)**

Obtain a formula for $I(k)$ where $I(k) = \int_1^k \frac{1}{x^3+x}dx$, expressing it in the form $\ln\left(\frac{a}{b}\right)$ where a and b depend on k. **(4)**

Write down an expression for $e^{I(k)}$ and obtain the value of $\lim_{k\to\infty} e^{I(k)}$. **(2)**

2005

15. a) Given $f(x)=\sqrt{\sin x}$, where $0<x<\pi$, obtain $f'(x)$. **(2)**

b) If, in general, $f(x)=\sqrt{g(x)},$ where g(x) > 0, show that $f'(x)=\dfrac{g'(x)}{k\sqrt{g(x)}}$, stating the value of *k*. **(3)**

Hence or otherwise, find $\int \frac{x}{\sqrt{1-x^2}}\,dx.$

c) Use integration by parts and the result of (b) to evaluate $\int_0^{1/2} \sin^{-1} x\,dx.$ **(4)**

2006

6. Find $\int \frac{12x^3-6x}{x^4-x^2+1}\,dx.$ **(3)**

17. (a) Show that $\int \sin^2 x\cos^2 x\,dx = \int \cos^2 x - \int \cos^4 x.$ **(1)**

(b) By writing $\cos^4 x = \cos x\cos^3 x$ and using integration by parts, show that

$$\int_0^{\pi/4} \cos^4 x\,dx = \frac{1}{4} + 3\int_0^{\pi/4} \sin^2 x\cos^2 x\,dx.$$ **(3)**

(c) Show that $\int_0^{\pi/4} \cos^2 x\,dx = \frac{\pi+2}{8}$ **(3)**

(d) Hence, using the above results, show that $\int_0^{\pi/4} \cos^4 x\,dx = \frac{3\pi+8}{32}.$ **(3)**

2007

4. Express $\dfrac{2x^2-9x-6}{x(x^2-x-6)}$ in partial fractions. **(3)**

Given that $\int_4^6 \frac{2x^2-9x-6}{x(x^2-x-6)}\,dx = \ln\frac{m}{n}$ determine values for the integers *m* and *n*. **(3)**

10. Use the substitution $u = 1 + x^2$ to obtain $\int_0^1 \frac{x^3}{(1+x^2)^4}\,dx.$ **(5)**

2008

4. Express $\dfrac{12x^2+20}{x(x^2+5)}$ in partial fractions. **(3)**

Hence evaluate $\int_1^2 \frac{12x^2+20}{x(x^2+5)}\,dx$ **(3)**

7. Use integration by parts to obtain $\int 8x^2 \sin 4x\,dx.$ **(5)**

2009

5. Show that $\int_{\ln\frac{3}{2}}^{\ln 2} \frac{e^x + e^{-x}}{e^x - e^{-x}} dx = \ln\frac{9}{5}$. **(6)**

7. Use the substitution $x = 2\sin\theta$ to obtain the exact value of $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} dx$. **(4)**
(Note that $\cos 2A = 1 - 2\sin^2 A$.)

9. Use integration by parts to obtain the exact value of $\int_0^1 x \tan^{-1} x^2 dx$. **(5)**

2010

3. (b) Integrate $x^2 \ln x$ with respect to x. **(4)**

2011

11. (a) Obtain the exact value of $\int_0^{\pi/4} (\sec x - x)(\sec x + x) dx$ **(3)**

(b) Find $\int \frac{x}{\sqrt{1-49x^4}} dx$. **(4)**

16. Define $I_n = \int_0^1 \frac{1}{(1+x^2)^n} dx$ for $n \geq 1$.

(a) Use integration by parts to show that $I_n = \frac{1}{2^n} + 2n\int_0^1 \frac{x^2}{(1+x^2)^{n+1}} dx$ **(3)**

(b) Find the values of A and B for which $\frac{A}{(1+x^2)^n} + \frac{B}{(1+x^2)^{n+1}} = \frac{x^2}{(1+x^2)^{n+1}}$

and hence show that $I_{n+1} = \frac{1}{n \times 2^{n+1}} + \left(\frac{2n-1}{2n}\right) I_n$. **(5)**

(c) Hence obtain the exact value of $\int_0^1 \frac{1}{(1+x^2)^3} dx$. **(3)**

2012

8. Use the substitution $x = 4\sin\theta$ to evaluate $\int_0^2 \sqrt{16-x^2}\, dx$. **(6)**

11. b) Use integration by parts to obtain $\int \sin^{-1} x . \frac{x}{\sqrt{1-x^2}} dx$. **(4)**

2012

13. A curve is defined parametrically, for all t, by the equations

$$x = 2t + \frac{1}{2}t^2, \qquad y = \frac{1}{3}t^3 - 3t.$$

Obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ as functions of t. **(5)**

Find the values of t at which the curve has stationary points and determine their nature. **(3)**

Show that the curve has exactly two points of inflexion. **(2)**

2013

6. Integrate $\frac{\sec^2 3x}{1+\tan 3x}$ with respect to x. **(4)**

8. Use integration by parts to obtain $\int x^2 \cos 3x\,dx$. **(5)**

2014

12. Use the substitution $x = \tan\theta$ to determine the exact value of $\int_0^1 \frac{dx}{\left(1+x^2\right)^{3/2}}$. **(6)**

15. (a) Use integration by parts to obtain an expression for $\int e^x \cos x\,dx$. **(4)**

(b) Similarly, given $I_n = \int e^x \cos nx\,dx$ where $n \neq 0$, obtain an expression for I_n. **(4)**

(c) Hence evaluate $\int_0^{\frac{\pi}{2}} e^x \cos 8x\,dx$. **(2)**

2015

10. Obtain the exact value of $\int_0^2 x^2 e^{4x}\,dx$. **(5)**

17. Find $\int \frac{2x^3 - x - 1}{(x-3)\left(x^2+1\right)}dx, \quad x > 3$. **(9)**

2016

9. Obtain $\int x^7 (\ln x)^2\,dx$. **6 marks**

13. Express $\frac{3x+32}{(x+4)(6-x)}$ in partial fractions and hence evaluate $\int_3^4 \frac{3x+32}{(x+4)(6-x)}dx$.

Give your answer in the form $\ln\left(\frac{p}{q}\right)$. **9 marks**

2017

6. Use the substitution of $u = 5x^2$ to find the exact value of $\int_0^{\frac{1}{\sqrt{10}}} \frac{x}{\sqrt{1-25x^4}}dx.$ **9 marks**

2018

2. Use partial fractions to find $\int \frac{3x-7}{x^2-2x-15}\,dx.$ 4

8. Using the substitution $u = \sin\theta$, or otherwise, evaluate

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2\sin^4\theta\cos\theta\,d\theta.$$ 4

15. (a) Use integration by parts to find $\int x\sin 3x\,dx.$ 3

(b) Hence find the particular solution of

$$\frac{dy}{dx} - \frac{2}{x}y = x^3\sin 3x, \quad x \neq 0$$

given that $x = \pi$ when $y = 0$.

Express your answer in the form $y = f(x)$. 7

Answers

2001 A3 $\frac{\pi}{8}$ A5 $\frac{x}{x^2-1}=\frac{1}{2(x+1)}+\frac{1}{2(x-1)}$ $\frac{1}{2}\left[x^2+\ln\left(x^2-1\right)\right]+c$

2002 A5. 2ln2 – 1 = 0.3863 A6. $\frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right)+c$

2003 *A*5. $\frac{-1}{8}-\frac{-1}{2}=\frac{3}{8}$ A10 a) $I_1=1-\frac{2}{e}=0.264$ b) Proof c) $6-16e^{-1}=0.1139$

2004 5. $\frac{1}{x^2-x-6}=\frac{1}{5(x-3)}-\frac{1}{5(x+2)}$ $\frac{1}{5}\ln\frac{4}{9}=-0.162$ 9. $\left(\frac{1}{\left(1+\sqrt{x}\right)^2}-\frac{2}{\left(1+\sqrt{x}\right)}\right)+c$

2005 5. $2\frac{2}{3}$ 13. $\frac{1}{x^3+x}=\frac{1}{x}-\frac{x}{x^2+1}$ $\ln\frac{k\sqrt{2}}{\sqrt{k^2+1}}$ $\sqrt{2}$

15 $f'(x)=\frac{1}{2}\frac{\cos x}{(\sin x)^{1/2}}$ $k=2;$ $-\sqrt{1-x^2}+c$ $\frac{\pi}{12}+\frac{\sqrt{3}}{2}-1$

2006 6. $3\ln\left(x^4-x^2+1\right)+c$ 17. Proofs

2007 4. $\frac{1}{x}+\frac{2}{x+2}-\frac{1}{x-3}$ $\ln\frac{8}{9}$ 10. $\frac{1}{24}$

2008 4. $\frac{4}{x}+\frac{8x}{x^2+5}$ 4 ln 3 = 4.39 7. $-2x^2\cos 4x+x\sin 4x+\frac{1}{4}\cos 4x+c$

2009 3. $y=\ln\left(2-\frac{1}{x}\right)$ 5. Proof 7. $\frac{\pi}{2}-1$ 9. $\frac{\pi}{8}-\frac{1}{4}\ln 2$

2010 3. $=\frac{1}{4}\tan^{-1}x^4+c$ $=\frac{1}{3}x^3\ln x-\frac{1}{9}x^3+c$

2011 11. $1-\frac{\pi^3}{192}$ $\frac{1}{14}\sin^{-1}\left(7x^2\right)+C$ 16.a) Proof b) A = 1, B = -1 c) $\frac{1}{4}+\frac{3\pi}{32}$

2012

8. $\int_0^2 \sqrt{16-x^2}\,dx$

$= \frac{4\pi}{3} + 2\sqrt{3}\ (\approx 7.65)$

11. $\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$

$\int \sin^{-1} x.\frac{x}{\sqrt{1-x^2}}\,dx = x - \sin^{-1} x.\sqrt{1-x^2} + c$

13. $\frac{dy}{dx} = \frac{t^2-3}{2+t} \quad \frac{d^2y}{dx^2} = \frac{t^2+4t+3}{(2+t)^2}$

When $t = \sqrt{3}, \frac{d^2y}{dx^2} = \frac{3+4\sqrt{3}+3}{\left(2+\sqrt{3}\right)^3} > 0$

which gives a minimum.

When $t = -\sqrt{3}, \frac{d^2y}{dx^2} = \frac{3-4\sqrt{3}+3}{\left(2-\sqrt{3}\right)^3} < 0$

which gives a maximum.

At a point of infexion $\frac{d^2y}{dx^2} = 0.$

In this case, that means

$t^2 + 4t + 3 = (t+1)(t+3) = 0$

and this has exactly two roots.

2013

6. $\frac{1}{3}\ln\left|1+\tan 3x\right| + c$

8. $\frac{1}{3}x^2 \sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x + c$

2014

12. $\left[\sin\theta\right]_0^{\frac{\pi}{4}} = \frac{1}{\sqrt{2}}$

15. $a)\int e^x \cos x\,dx = \frac{1}{2}e^x\left(\sin x + \cos x\right) + c$

$b) I_n = \left(\frac{e^x}{1+n^2}\right)\left(n\sin nx + \cos nx\right) + c \qquad c)\frac{1}{65}\left(e^{\frac{\pi}{2}} - 1\right)$

2015

10. $\frac{25}{32}e^8 - \frac{1}{32}$

17. $2x + 5\ln\left|x-3\right| + \frac{1}{2}\ln\left(x^2+1\right) + k$

2016

9. $\frac{1}{8}x^8(\ln x)^2 - \frac{1}{32}x^8(\ln x) + \frac{1}{256}x^8 + c$

13. $\int_3^4 \left(\frac{2}{x+4} + \frac{5}{6-x}\right)dx$

$= \ln\left|\frac{486}{49}\right|$

2017

6. $\frac{\pi}{60}$ **(6)**

2018

2. $\ln\left|x-5\right| + 2\ln\left|x+3\right| + c$

8. $\frac{31}{80}$

15. $-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + C$

$y = -\frac{1}{3}x^3\cos 3x + \frac{1}{9}x^2\sin 3x - \frac{1}{3}\pi x^2$