**2001**

B4. Find the first four terms in the MacLaurin series for $(2 + x)ln(2 + x)$. — 5 marks

**2002**

A10. Define $S_n(x)$ by $S_n(x) = 1 + 2x + 3x^2 + \ldots + nx^{n-1}$, where $n$ is a positive integer. — 2 marks

Express $S_n(1)$ in terms of $n$. — 4 marks

By considering $(1 - x)S_n(x)$, show that $S_n(x) = \dfrac{1-x^n}{(1-x)^2} - \dfrac{nx^n}{(1-x)}, x \neq 1.$

Obtain the value of $\lim\limits_{n\to\infty}\left\{\dfrac{2}{3} + \dfrac{3}{3^2} + \dfrac{4}{3^3} + \ldots + \dfrac{n}{3^{n-1}} + \dfrac{3}{2}.\dfrac{n}{3^n}\right\}$. — 3 marks

B4. Find the MacLaurin expansion for $f(x) = \ln(\cos x)$, $(0 \leq x \leq \frac{\pi}{2})$, as far as the term $x^4$. — 5 marks

**2003**

A2. Given that $u_k = 11 - 2k$, $(k \geq 1)$, obtain a formula for $S_n = \sum_{k=1}^{n} u_k$. — 3 marks

Find the values of $n$ for which $S_n = 21$. — 2 marks

B3. A recurrence relation is defined by the formula $x_{n+1} = \dfrac{1}{2}\left\{x_n + \dfrac{7}{x_n}\right\}$.
Find the fixed points of this recurrence relation. — 3 marks

B4. Obtain the Maclaurin series for $f(x) = \sin^2 x$ up to the term in $x^4$. — 4 marks
Hence write down a series for $\cos^2 x$ up to the term in $x^4$. — 1 mark

**2004**

7. Obtain the first three non-zero terms in the MacLaurin expansion of $f(x) = e^x \sin x$. — 5 marks

16.a) Obtain the sum of the series 8 + 11 + 14 + …+ 56. — 2 marks

b) A geometric sequence of positive terms has first term 2, and the sum of the first three terms is 266. Calculate the common ratio. — 3 marks

c) An arithmetic sequence, $A$ has first term a and common difference 2, and a geometric sequence, $B$, has first term a and common ratio 2. The first four terms of each sequence have the same sum. Obtain the value of $a$. — 3 marks
Obtain the smallest value of $n$ such that the sum to $n$ terms for sequence $B$ is more than twice the sum to $n$ terms for sequence $A$. — 2 marks

**2005**

3. Write down the Maclaurin expansion of $e^x$ as far as the term in $x^4$. — 2 marks
Deduce the Maclaurin expansion of $e^{x^2}$ as far as the term in $x^4$. — 1 mark
Hence or otherwise, find the Maclaurin expansion of $e^{x+x^2}$ as far as the term in $x^4$.

**2005**

4. The sum $S(n)$, of the first $n$ terms of a sequence, $u_1, u_2, u_3, \ldots$ is given by $S(n) = 8n - n^2$, $n \geq 1$.
Calculate the values of $u_1, u_2, u_3$ and state what type of sequence it is. 3 marks
Obtain a formula for $u_n$ in terms of $n$, simplifying your answer. 2 marks

**2006**

16. The first three terms of a geometric sequence are

$$\frac{x(x+1)}{(x-2)}, \frac{x(x+1)^2}{(x-2)^2} \text{ and } \frac{x(x+1)^3}{(x-2)^3}, \text{ where } x < 2.$$

(a) Obtain expressions for the common ratio and the $n$th term of the sequence. 3 marks
(b) Find an expression for the sum of the first $n$ terms of the sequence. 3 marks
(c) Obtain the range of values of $x$ for which the sequence has a sum to infinity and find an expression for the sum to infinity. 4 marks

**2007**

6. Find the MacLaurin series for $\cos x$ as far as the term $x^4$. 2 marks

Deduce the MacLaurin series for $f(x) = \frac{1}{2}\cos 2x$ as far as the term in $x^4$. 2 marks

Hence write down the first three non-zero terms of the series for $f(3x)$. 1 mark

**2008**

1. The first term of an arithmetic sequence is 2 and the 20th term is 97.
Obtain the sum of the first 50 terms. 4 marks

12. Obtain the first three non-zero terms in the Maclaurin expansion of $x\ln(2 + x)$. 3 marks
Hence or otherwise, deduce the first three non-zero terms in the Maclaurin expansion of $x\ln(2 - x)$. 2 marks
Hence obtain the first two non-zero terms in the Maclaurin expansion of $x\ln(4 - x^2)$.
[*Throughout this question, it can be assumed that -2 < x < 2.*] 2 marks

**2009**

12. The first two terms of a geometric sequence are $a_1 = p$ and $a_2 = p^2$.

Obtain expressions for $S_n$ and $S_{2n}$, in terms of $p$, where $S_k = \sum_{j=1}^{k} a_j$. 1, 1 marks

Given that $S_{2n} = 65S_n$ show that $p^n = 64$. 2 marks

Given also that $a_3 = 2p$ and that $p > 0$, obtain the exact value of $p$ and hence the value of $n$. 1, 1 marks

**2010**

2. The second and third terms of a geometric series are -6 and 3 respectively.
   Explain why the series has a sum to infinity, and obtain this sum. 5 marks

9. Obtain the first three non-zero terms in the Maclaurin expansion of $(1 + \sin^2 x)$. 4 marks

**2011**

5. Obtain the first four terms in the Maclaurin series of $\sqrt{1+x}$ and hence write down the first four terms in the Maclaurin series of $\sqrt{1+x^2}$ 3 marks

   Hence obtain the first four terms in the Maclaurin series of $\sqrt{(1+x)(1+x^2)}$ 2 marks

**2012**

2. The first and fourth terms of a geometric series are 2048 and 256 respectively.
   Calculate the value of the common ratio. 2 marks

   Given that the sum of the first $n$ terms is 4088, find the value of $n$. 3 marks

6. Write down the Maclaurin expansion of $e^x$ as far as the term in $x^3$.
   Hence or otherwise, obtain the Maclaurin expansion of $(1 + e^x)^2$ as far as the term in $x^3$. 4 marks

**2013**

17. Write down the sums to infinity of the geometric series $1+x+x^2+x^3+\ldots\ldots$

    and $1-x+x^2-x^3+\ldots\ldots$

    valid for $|x|<1$.

    Assuming that it is permitted to integrate an inifinite series term by term, show that, for $|x|<1$,

$$\ln\left(\frac{1+x}{1-x}\right)=2\left(x+\frac{x^3}{3}+\frac{x^5}{5}+\ldots\ldots\right).$$

7 marks

    Show how this series can be used to evaluate ln2.

    Hence determine the value of ln2 correct to 3 decimal places. 3 marks

**2014**

9. Give the first three no zero terms of the Maclaurin series for Cos $3x$. 2 marks
Write down the first four terms of the Maclaurin series for $e^{2x}$. 1 mark
Hence, or otherwise, determine the Maclaurin series for $e^{2x}\cos 3x$ up to, and including the term $x^3$. 3 marks

14. (a) Given the series $1 + r + r^2 + r^3 + \ldots,$ write down the sum to infinity when $|r| < 1$.

Hence obtain an infinite geometric series for $\frac{1}{2-3r}$.
For what values of $r$ is this series valid? 4 marks

(b) Express $\frac{1}{3r^2 - 5r + 2}$ in partial fractions.

Hence, or otherwise, determine the first three terms of an infinite series for
For what values of $r$ does the series converge? 6 marks

**2015**

3. The sum of the first twenty terms of an arithmetic sequence is 320.
The twenty first term is 37.
What is the sum of the first ten terms. 5 marks

**2016**

2. A geometric sequence has second and fifth terms 108 and 4 respectively.

a) Calculate the value of the common ratio. 3 marks
b) State why the associated geometric series has a sum to infinity. 1 marks
c) Find the value of this sum to infinity. 2 marks

6. Find Maclaurin expansion for $\sin 3x$ and $e^{4x}$ up to and including the term in $x^3$.

Hence obtain an expansion for $e^{4x}\sin 3x$ up to and including the term in $x^3$. 6 marks

**2017**

4. The fifth term on an arithmetic sequence is –6 and the twelfth term is –34.

(a) Determine the values of the first term and the common difference. 2 marks
(b) Obtain algebraically the value of $n$ for which $S_n = -144$. 3 marks

10. $S_n$ is defined by $\sum_{r=1}^{n}\left(r^2 + \frac{1}{3}r\right)$.

a) Find an expression for $S_n$, fully factorising your answer. 2 marks
b) Hence find an expression for $\sum_{r=10}^{2p}\left(r^2 + \frac{1}{3}r\right)$ where $p > 5$. 2 marks

**2018**

**14.** A geometric sequence has first term 80 and common ratio $\frac{1}{3}$.

(a) For this sequence, calculate:

(i) the 7th term; **2**

(ii) the sum to infinity of the associated geometric series. **2**

The first term of this geometric sequence is equal to the first term of an arithmetic sequence.

The sum of the first five terms of this arithmetic sequence is 240.

(b) (i) Find the common difference of this sequence. **2**

(ii) Write down and simplify an expression for the $n$th term. **1**

Let $S_n$ represent the sum of the first $n$ terms of this arithmetic sequence.

(c) Find the values of $n$ for which $S_n = 144$. **3**

**17.** (a) Given $f(x) = e^{2x}$, obtain the Maclaurin expansion for $f(x)$ up to, and including, the term in $x^3$. **2**

(b) On a suitable domain, let $g(x) = \tan x$.

(i) Show that the third derivative of $g(x)$ is given by

$g'''(x) = 2\sec^4 x + 4\tan^2 x \sec^2 x$. **3**

(ii) Hence obtain the Maclaurin expansion for $g(x)$ up to and including the term in $x^3$. **2**

(c) Hence, or otherwise, obtain the Maclaurin expansion for $e^{2x}\tan x$ up to, and including, the term in $x^3$. **2**

(d) Write down the first three non-zero terms in the Maclaurin expansion for $2e^{2x}\tan x + e^{2x}\sec^2 x$. **1**

**2019**

**7.** (a) Find an expression for $\sum_{r=1}^{n}(6r+13)$ in terms of $n$. **1**

(b) Hence, or otherwise, find $\sum_{r=p+1}^{20}(6r+13)$. **2**

**17.** The first three terms of a sequence are given by

$$5x+8,\ -2x+1,\ x-4$$

(a) When $x=11$, show that the first three terms form the start of a geometric sequence, and state the value of the common ratio. **2**

(b) Given that the entire sequence is geometric for $x=11$

(i) state why the associated series has a sum to infinity **1**

(ii) calculate this sum to infinity. **2**

(c) There is a second value for $x$ that also gives a geometric sequence.

For this second sequence

(i) show that $x^2-8x-33=0$ **2**

(ii) find the first three terms **2**

(iii) state the value of $S_{2n}$ and justify your answer. **1**

## Answers

**2001** B4 $f(x) = 2\ln 2 + (1+\ln 2)x + \frac{x^2}{4} - \frac{x^3}{24} + ..$**(5)**

**2002** A10. $S_n(1) = \frac{1}{2}n(n+1)$ $\lim = \frac{4}{5}$ B4 $\ln(\cos x) = -\frac{x^2}{2} - \frac{x^4}{12} + ...$ **(5)**

**2003** A2 a) $S_n = -n^2 + 10n$ or $S_n = \frac{n}{2}[18 + (n-1)(-2)]$ **(3)** b) $n = 3$ and 7. **(2)**

B3. Fixed Points $\sqrt{7}$ and $-\sqrt{7}$ **(3)** B4. $\sin^2 x = x^2 - \frac{1}{3}x^4$ therefore $\cos^2 x = 1 - x^2 + \frac{1}{3}x^4$ **(5)**

**2004** 7) $x + x^2 + \frac{x^3}{3} - ..$**(5)** 16. a) 544 b) $r = 11$ c) smallest $n$ is 7

**2005**

3) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ...$**(2)** $e^{x^2} = 1 + x^2 + \frac{x^4}{2!} + ...$**(1)** $e^{x+x^2} = 1 + x + \frac{3}{2}x^2 + \frac{7}{6}x^3 + \frac{25}{24}x^4 + ...$ **(3)**

4) a) $u_1 = 7;\ u_2 = 5;\ u_3 = 3.$ Arithmetic series. **(3)** b) $9 - 2n$ **(2)**

**2006** 16) $\frac{x(x+1)^n}{(x-2)^n}$**(3)** $S_n = \frac{1}{3}x(x+1)\left(\frac{(x+1)^n}{(x-2)^n} - 1\right)$ **(3)** $-1 < r < 1$ i.e. $r^2 < 1$ $\frac{-x(x+1)}{3}$ **(4)**

**2007** 6) $\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24}$ **(2)** $\frac{1}{2} - x^2 + \frac{x^4}{3} - ...$ **(2)** $\frac{1}{2} - 9x^2 + 27x^4 - ...$ **(1)**

**2008** 1) 6225 12) $f(x) = (\ln 2)x + \frac{x^2}{2} - \frac{x^3}{8} + ...$ **(3)** $(\ln 2)x - \frac{x^2}{2} - \frac{x^3}{8} + ...$ **(2)** $2(\ln 2)x - \frac{x^3}{4} + ...$**(2)**

**2009** 12.a) $S_n = \frac{p(p^n - 1)}{p-1};\ S_{2n} = \frac{p(p^{2n} - 1)}{p-1}$ **(2)** b) Proof **(2)** c) $p = \sqrt{2}$ $n = 12$ **(2)**

**2010** 2. Sum to infinity as $r = -\frac{1}{2}$ Sum to infinity = 8. 9. $f(x) = 1 + x^2 - \frac{1}{3}x^4 + ...$

**2011** 5. $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - ...$ $1 + \frac{1}{2}x^2 - \frac{1}{8}x^4 + \frac{1}{16}x^6 - ...$ $1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3$

**2012** 2. $2^n = 512$

$n = 9$ **(5)**

6. $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + ...$

$4 + 4x + 3x^2 + \frac{5}{3}x^3 ...$ **(5)**

**2013**

17. $1 + x + x^2 + x^3 + ... = \frac{1}{1-x}$

$1 - x + x^2 - x^3 + ... = \frac{1}{1+x}$ **(4, 3, 3)**

$\frac{1+x}{1-x} = 2$

$1 + x = 2 - 2x$, so $x = \frac{1}{3}$

So $\ln 2 = 2\left(\frac{1}{3} + \frac{1}{81} + \frac{1}{1215} + \frac{1}{15309} + ...\right)$

$= 0.693$ to 3 d.p. **(7, 3)**

**2014**

9. $1-\frac{9x^2}{2}+\frac{27x^4}{8}...$

$f(x)=\cos 3x$ $f(0)=1$
$f'(x)=-3\sin 3x$ $f'(0)=0$
$f''(x)=-9\cos 3x$ $f''(0)=-9$
$f'''(x)=27\sin 3x$ $f'''(0)=0$
$f''''(x)=81\cos 3x$ $f''''(0)=81$

$1+2x-\frac{5x^2}{2}-\frac{23x^3}{3}$ **(2, 1, 3)**

14. $a) 1+r+r^2+r^3+...=\frac{1}{1-r}, \quad |r|<\frac{2}{3}$

$b) f(x)=\frac{1}{2}+\frac{5r}{4}+\frac{19r^2}{8}...$ **(4, 6)**

**2015**

3. $S_{10}=60$

**2016**

2. $r=\frac{1}{3}, \quad -1<\frac{1}{3}<1 \quad 486$

6. $f(x)=3x-\frac{9}{2}x^3...$

$e^{4x}\sin 3x=3x+12x^2+\frac{39}{2}x^3...$

**2017**

4. $a=10, d=-4$

$n>0 \therefore n=12$ **(5)**

10. $=\frac{n(n+1)^2}{3}$ $\quad =\frac{2p(2p+1)^2}{3}-300$ **(4)**

**2018**

14. a) $u_7=\frac{80}{729}, \quad S_\infty=120$

b) $d=-16 \quad u_n=96-16n$

c) $n=2 \quad n=9$

17. a) $f(x)=1+2x+2x^2+\frac{4}{3}x^3+...$

b) proof, $g(x)=x+\frac{1}{3}x^3+...$

c) $x+2x^2+\frac{7}{3}x^3+...$

d) $\frac{dy}{dx}=1+4x+7x^2+...$

**2019**

7. a) $3n^2+16n$ b) $1520-3p^2-16p$

17. a) $r=-\frac{1}{3}$ b) i) $\left|-\frac{1}{3}\right|<1$ ii) $\frac{189}{4}$ or $47\cdot 25$

c) i) $x^2-8x-33=0$ ii) $-7, 7, -7$ iii) 0 since eg $2n$ is even and so pairs of terms cancel each other out