2001

B6. Let L_1 and L_2 be the lines $L_1 = x = 8 - 2t, y = -4 + 2t, z = 3 + t$

$$L_2 = \frac{x}{-2} = \frac{y+2}{-1} = \frac{z-9}{2}$$

(a) (i) Show that L_1 and L_2 intersect and find their point of intersection. 4 marks

(ii) Verify that the acute angle between them is $\cos^{-1}\left(\frac{4}{9}\right)$ 2 marks

(b) (i) Obtain the equation of the plane π that is perpendicular to L_2 and passes through the point (1, -4, 2). 3 marks

(ii) Find the coordinates of the point of intersection of the plane π and the line L_1. 2 marks

2002

B1. a) Find an equation of the plane π_1 containing the points A (1, 1, 0), B (3, 1, -1) and C (2, 0, -3). 4 marks

b) Given that π_2 is the plane whose equation is $x + 2y + z = 3$, calculate the size of the acute angle between π_1 and the plane π_2. 3 marks

2003

B1. Find the point of intersection of the line $\frac{x-3}{4} = \frac{y-2}{-1} = \frac{z+1}{2}$

and the plane with equation $2x + y - z = 4$. 4 marks

2004

14. a) Find an equation of the plane π_1 containing the points A (1, 0, 3), B (0, 2, -1) and C (1, 1, 0). 4 marks

Calculate the size of the acute angle between π_1 and the plane π_2 with equation $x + y - z = 0$. 3 marks

b) Find the point of intersection of plane π_2 and the line $\frac{x-11}{4} = \frac{y-15}{5} = \frac{z-12}{2}$. 3 marks

2005

8. The equations of two planes are $x - 4y + 2z = 1$ and $x - y - z = -5$. By letting $z = t$, or otherwise obtain parametric equations for the line of intersection of the planes. 4 marks

Show that this line lies in the plane with equation $x + 2y - 4z = -11$. 1 mark

2006

15. Obtain an equation for the plane passing through the point P (1, 1, 0) which is perpendicular to the line L given by

$$\frac{x+1}{2}=\frac{y-2}{1}=\frac{z}{-1}$$

3 marks

Find the coordinates of the point Q where the plane and L intersect. 4 marks

Hence, or otherwise, obtain the shortest distance from P to L and explain why this is the shortest distance. 2, 1 marks

2007

15. Lines L_1 and L_2 are given by the parametric equations

$$L_1 : x = 2+s, y=-s, z=2-s \qquad L_2 : x=-1-2t, y=t, z=2+3t$$

(a) Show that L_1 and L_2 do not intersect. 3 marks

(b) The line L_3 passes through the point P(1, 1, 3) and its direction is perpendicular to the directions of both L_1 and L_2. Obtain parametric equations for L_3. 3 marks

(c) Find the coordinates of the point Q where L_3 and L_2 intersect and verify the P lies on L_1. 3 marks

(d) PQ is the shortest distance between the lines L_1 and L_2. Calculate PQ. 1 mark

2008

14. (a) Find an equation of the plane π_1 through the points A(1, 1, 1), B(2, -1, 1) and C(0, 3, 3). 3 marks

(b) The plane π_2 has equation $x + 3y - z = 2$.
Given that the point (0, a, b) lies on both the planes π_1 and π_2, find the values of a and b.
Hence find an equation of the line of intersection of the planes π_1 and π_2. 4 marks

(c) Find the size of the acute angle between the planes π_1 and π_2. 3 marks

2009

16. (a) Use Gaussian elimination to solve the following system of equations 5 marks

$$x + y - z = 6$$
$$2x-3y+2z=2$$
$$-5x+2y-4z=1.$$

(b) Show that the line of intersection, L, of the planes $x + y - z = 6$ and $2x - 3y + 2z = 2$ has parametric equations 2 marks

$$x=\lambda$$
$$y=4\lambda-14$$
$$z=5\lambda-20.$$

4 marks

(c) Find the acute angle between line L and the plane $-5x + 2y - 4z = 1$.

2010

6. Given $\mathbf{u} = -2\mathbf{i} + 5\mathbf{k}$, $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{w} = -\mathbf{i} + \mathbf{j} + 4\mathbf{k}$. Calculate $\mathbf{u}.(\mathbf{v}\times\mathbf{w})$. 4 marks

2011

15. The lines L_1 and L_2 are given by the equations $\frac{x-1}{k} = \frac{y}{-1} = \frac{z+3}{1}$ and $\frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$, respectively.

Find
(a) the value of k for which L_1 and L_2 intersect and the point of intersection; 6 marks
(b) the acute angle between L_1 and L_2. 4 marks

2012

5. Obtain an equation for the plane passing through the points $P(-2,1,-1)$, $Q(1,2,3)$ and $R(3,0,1)$. 5 marks

2013

15. (a) Find an equation of the plane π_1, through the points $A(0,-1,3)$, $B(1,0,3)$ and $C(0,0,5)$. 4 marks

(b) π_2 is the plane through A with normal in the direction $-\mathbf{j} + \mathbf{k}$.
Find an equation of the plane π_2. 2 marks

(c) Determine the acute angle between π_1 and π_2. 3 marks

2014

5. Three vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are given by u, v and w where

$$u = 5i + 13j, \quad v = 2i + j + 3k, \quad w = i + 4j - k$$

Calculate $u.(v \times w)$. **3 marks**

Interpret your result geometrically. **1 mark**

2015

15. A line L_1 passes through the point P (2, 4, 1) and is parallel to $u_1 = i + 2j - k$

and a second line, L_2 passes through Q (–5, 2, 5) and is parallel to $u_2 = -4i + 4j + k$.

a) Write down the vector equations for L_1 and L_2. **2 marks**

b) Show that the lines L_1 and L_2 intersect and find the point of intersection. **4 marks**

c) Determine the equation of the plane containing L_1 and L_2. **4 marks**

2016

14. Two lines L_1 and L_2 are given by the equations:

$$L_1: \quad x = 4 + 3\lambda, \quad y = 2 + 4\lambda, \quad z = -7\lambda$$

$$L_2: \quad \frac{x-3}{-2} = \frac{y-8}{1} = \frac{z+1}{3}$$

a) Show that the lines L_1 and L_2 intersect and find the point of intersection. **4 marks**

b) Calculate the obtuse angle between the lines L_1 and L_2 . **5 marks**

2017

15. (a) A beam of light passes through the points B(7, 8, 1) and T(–3, –22, 6).
Obtain parametric equations of the line representing the beam of light. **2 marks**

(b) A sheet of metal is represented by a plane containing the points P(2, 1, 9), Q(1, 2, 7) and R(–3, 7, 1).
Find the Cartesian equation of the plane. **4 marks**

(c) The beam of light passes through a hole in the metal at point H. Find the coordinates of H. **3 marks**

2018

16. Planes π_1 , π_2 and π_3 have equations:

$$\pi_1: \quad x - 2y + z = -4$$
$$\pi_2: \quad 3x - 5y - 2z = 1$$
$$\pi_3: \quad -7x + 11y + az = -11$$

where $a \in \mathbb{R}$.

(a) Use Gaussian elimination to find the value of a such that the intersection of the planes π_1 , π_2 and π_3 is a line. 4

(b) Find the equation of the line of intersection of the planes when a takes this value. 2

The plane π_4 has equation $-9x + 15y + 6z = 20$.

(c) Find the acute angle between π_1 and π_4. 3

(d) Describe the geometrical relationship between π_2 and π_4.
Justify your answer. 1

2019

15. The equations of two planes are given below.

$$\pi_1: \; 2x - 3y - z = 9$$
$$\pi_2: \; x + y - 3z = 2$$

(a) Verify that the line of intersection, L_1, of these two planes has parametric equations

$$x = 2\lambda + 3$$
$$y = \lambda - 1$$
$$z = \lambda$$

2

(b) Let π_3 be the plane with equation $-2x + 4y + 3z = 4$.

Calculate the acute angle between the line L_1 and the plane π_3. **3**

(c) L_2 is the line perpendicular to π_3 passing through P$(1, 3, -2)$.

Determine whether or not L_1 and L_2 intersect. **4**

Answers

2001 B6. (a) (i) proof **(4)** (ii) proof **(2)** (b) (i) $-2x - y + 2z = 6$ **(3)** (ii) (2, 2, 6) **(2)**

2002 B1.a) $-x + 5y - 2z = 4$ **(4)** b) $\theta = 58.6^o$ **(3)**

2003 $(-1,3,-3)$ **(4)**

2004 14.a) $2x + 3y + z = 5$ **(4)** $\theta = 51.9^o$ **(3)** b) $x = 3$ $y = 5$ $z = 8$ **(3)**

2005 8. a) $x = 2t - 7,\ y = t - 2,\ z = t$ **(4)** b) Proof **(1)**

2006 a) $2x + y - z = 3$ **(3)** b) $Q = \left(0,\ 2\frac{1}{2},\ -\frac{1}{2}\right)$ **(4)** c) Perpendicular to L. $\sqrt{\frac{7}{2}} = 1.87$ **(3)**

2007 a) Z coordinates differ therefore does not intersect. **(3)** b) $x = 1 - 2u;\ y = 1;\ z = 3 - u$ **(3)**

c) Q (-1, 0, 2) **(3)** d) $PQ = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{6}$ **(1)**

2008 a) $2x + y = 3$ **(3)** b) $a = 3$ $b = 7$ $x = 0 + 2t;\ y = 3 - 4t;\ z = 7 - 10t$ **(4)** c) 47.6^o **(3)**

2009 16.a) $x = 3,\ y = -2,\ z = -5$ **(5)** b) Proof **(2)** c) 23^o **(4)**

2010 6) 7

2011 15. $k = 2$ $(5,-2,-1)$ $\theta = 60^\circ$

2012 $3x + 7y - 4z = 5$

2013 15. $a)\ r = \begin{pmatrix}0\\-1\\3\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\0\end{pmatrix} + \mu\begin{pmatrix}0\\1\\2\end{pmatrix}$ or equivalent

$b)\ -y + z = 4$

$c)\ 45^o$ or $\frac{\pi}{4}$

2014 5. $u.(v \times w) = \begin{pmatrix}5\\13\\0\end{pmatrix}.\begin{pmatrix}-13\\5\\7\end{pmatrix} = 0$

u lies in the same plane as the one containing both ***v*** and ***w***.
OR ***u*** is parallel to the plane containing ***v*** and ***w***.
OR ***u*** is perpendicular to the normal of ***v*** and ***w***.
OR All **4** points lie in the same plane.
OR ***u*** is perpendicular to ***v*** × ***w***.
OR Volume of parallelepiped is zero.
OR ***u***, ***v*** and ***w*** are coplanar/linearly dependent.

2015

15.a) $v_1 = \begin{pmatrix}2\\4\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\-1\end{pmatrix},\quad v_2 = \begin{pmatrix}-5\\2\\5\end{pmatrix} + \mu\begin{pmatrix}-4\\4\\1\end{pmatrix}$

15.b) $(-1,-2,4)$

15.c) $6x + 3y + 12z = 36$

2016

14. $(7,6,-7)$

$\cos^{-1}\left(\frac{-23}{\sqrt{74}\sqrt{14}}\right) \approx 135.6^\circ$

2017

15. $x = 2\lambda + 7$, $y = 6\lambda + 8$, $z = -\lambda + 1$

$x = 2\lambda - 3$, $y = 6\lambda - 22$, $z = -\lambda + 6$

$4x + 2y - z = 1$

$H(3,-4,3)$ **(9)**

2018

16.a) $a = 8$ (last row redundant)

b) $\lambda = \frac{x-22}{9} = \frac{y-13}{5} = z$

c) $\theta = 43\cdot24^{\circ}$

d) $n_4 = -3n_2$ therefore parallel

2019

a) •[1] eg $2(2\lambda+3)-3(\lambda-1)-\lambda=9$

•[2] eg $2\lambda+3+\lambda-1-3\lambda=2$;
therefore the line lies on both planes

OR

•[1] eg $2x-3y-\lambda=9$
$x+y-3\lambda=2$

•[2] $x=2\lambda+3; y=\lambda-1; z=\lambda$

OR

•[1] eg $10\mathbf{i}+5\mathbf{j}+5\mathbf{k}$
OR $(3, -1, 0)$

•[2] $(3, -1, 0)$
OR
$10\mathbf{i}+5\mathbf{j}+5\mathbf{k}$
AND
$x=2\lambda+3; y=\lambda-1; z=\lambda$

b) •[3] $\begin{pmatrix}2\\1\\1\end{pmatrix}, \begin{pmatrix}-2\\4\\3\end{pmatrix}$

•[4] $\cos\theta = \left(\frac{3}{\sqrt{6}\sqrt{29}}\right)$

•[5] any answer which rounds to 0·229 or 13°

c) •[6] $x=-2\mu+1; y=4\mu+3;$
$z=3\mu-2$

•[7] any two from
$2\lambda+3=-2\mu+1;$
$\lambda-1=4\mu+3; \lambda=3\mu-2$

•[8] eg $\mu=-1; \lambda=0$

•[9] eg LHS $=0$, RHS $=-5$
so lines do not intersect.