Prelim Revision **Differentiation 2**

Differentiate the following

1a) $x = (t + 1)^2$, $y = t^2 - 1$ b) $x = \frac{1}{1-t}$, $y = \frac{t^2}{t+3}$

c) $x = \cos 2\theta$, $y = 4\sin\theta$ d) $x = a\cos^2\theta$, $y = a\sin^3\theta$

e) $x = e^t \cos t$, $y = e^t \sin t$ f) $x = a(t - \cos t)$, $y = a(1 + \sin t)$

2) a)Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the curve defined by $x = t^2 - \frac{1}{t^2}$, $y = t^2 + \frac{1}{t^2}$.

b) Determine the coordinates of the turning point on the curve.

c) Determine the nature of this turning point.

3) Find the equations of the tangents at the given points on the following curves.

a) $x^3 - 2xy^3 = 3xy$ at (2, 1) b) $x^2y^2 = x^2 + 5y^2$ at (3, $\frac{3}{2}$)

4) For the curve, $4x^2 + y^3 = 2x + 7y$, find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at (-1, 2)

5) Differentiate logarithmically

a) $y = \frac{x^3}{\sqrt{x+4}}$ b) $y = \frac{x^3(2x-1)^5}{(x+1)^2}$ c) $y = \frac{x^3\sqrt{x+2}}{x+3}$

6) Differentiate, simplifying your answer as far as possible.

a) $y = \cos^{-1}(x^3)$ b) $y = \sin^{-1}(2x-1)$ c) $y = \tan^{-1}(2x+3)$

d) $y = \tan^{-1}\left(\sqrt{x}\right)$ e) $y = \tan^{-1}\left(\sqrt{2x-1}\right)$ f) $y = \sin^{-1}(x^2-1)$

g) $y = \tan^{-1}\left(\frac{x-1}{x+1}\right)$ h) $y = \tan^{-1}\left(\frac{2x}{x+1}\right)$ i) $y = \tan^{-1}\left(\frac{x+1}{x-3}\right)$

Answers

1a) $\frac{t}{t+1}$ b) $\frac{(t+6)(1-t)^2}{(t+3)^2}$ c) – cosec θ d) $\frac{-3}{2}\sin\theta$

e) $\frac{\cos t+\sin t}{\cos t-\sin t}$ f) $\frac{\cos t}{1+\sin t}$ 2a) $\frac{t^4-1}{t^4+1}, \frac{4t^6}{\left(t^4+1\right)^3}$ b)(0,2) c) minimum

3a) $3x - 4y = 2$ b) $5x + 8y = 27$ 4) 2, $\frac{-56}{5}$

5a) $\frac{x^2(5x+24)}{2(x+4)^{\frac{3}{2}}}$ b) $\frac{3x^2(2x-4)^4\left(4x^2+5x-1\right)}{(x+1)^3}$ c) $\frac{x^2\left(5x^2+29x+36\right)}{2(x+2)^{\frac{1}{2}}(x+3)^2}$

6a) $-\frac{3x^2}{\sqrt{1-x^6}}$ b) $\frac{1}{\sqrt{x-x^2}}$ c) $\frac{1}{2x^2+6x+5}$

d) $\frac{1}{2\sqrt{x}(1+x)}$ e) $\frac{1}{2x\sqrt{2x-1}}$ f) $\frac{2}{\sqrt{2-x^2}}$

g) $\frac{1}{x^2+1}$ h) $\frac{2}{3x^2+2x+1}$ i) $\frac{-1}{x^2-2x+5}$