1) $A = \begin{pmatrix} 2 & -3 \\ 4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -4 & 1 \\ 3 & -2 \end{pmatrix}$.

Find:

(a) $2A$ (b) $2B$ (c) $3A$
(d) $5A$ (e) $2(A+B)$ (f) $A-B$
(g) $4A+3B$ (h) $5A-2B$

2) $A = \begin{pmatrix} 0 & 4 & 2 \\ -1 & 1 & 3 \\ 2 & 0 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -3 & 5 \\ 2 & 0 & -4 \\ 3 & 2 & 0 \end{pmatrix}$.

Find:

(a) AB (b) BA (c) A^2 (d) B^2
(e) A^3 (f) B^4

3) Find the value or values of x for which:

(a) $\det \begin{pmatrix} 1 & 2 & 1 \\ 3 & x & 2 \\ 1 & 5 & 2 \end{pmatrix} = 1$ (b) $\det \begin{pmatrix} 2 & x & 1 \\ 3 & x+4 & 0 \\ 1 & 4 & x \end{pmatrix} = -10$

(c) $\det \begin{pmatrix} x & 2 & -1 \\ -2 & x & 1 \\ 4 & 1 & 2 \end{pmatrix} = 45$ (d) $\det \begin{pmatrix} -2 & 3 & 1 \\ 1 & x & 2 \\ 0 & 4 & x \end{pmatrix} = 21$

(e) $\det \begin{pmatrix} 1 & x & 2 \\ 2 & 1 & 3 \\ -1 & x+1 & 4 \end{pmatrix} = 37$ (f) $\det \begin{pmatrix} x & 3 & -1 \\ 0 & 4 & x \\ -2 & 6 & 8 \end{pmatrix} = 20$

(g) $\det \begin{pmatrix} 2 & 3 & -2 \\ x+1 & x & 4 \\ 1 & 2 & x-2 \end{pmatrix} = 0$ (h) $\det \begin{pmatrix} x & 2 & 0 \\ 1 & 3 & -1 \\ 2 & -2 & x-2 \end{pmatrix} = 8$

4) Find the inverses of the following matrix if they exist.

a) $D = \begin{pmatrix} 5 & 7 \\ 6 & 9 \end{pmatrix}$ b) $E = \begin{pmatrix} -2 & 4 \\ 1 & -1 \end{pmatrix}$ c) $F = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ d) $\begin{pmatrix} 3 & 4 & 5 \\ 4 & 3 & 11 \\ 1 & 0 & 3 \end{pmatrix}$

Answers

1) (a) $\begin{pmatrix} 4 & -6 \\ 8 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} -8 & 2 \\ 6 & -4 \end{pmatrix}$ (c) $\begin{pmatrix} 6 & -9 \\ 12 & 3 \end{pmatrix}$

   (d) $\begin{pmatrix} 10 & -15 \\ 20 & 5 \end{pmatrix}$ (e) $\begin{pmatrix} -4 & -4 \\ 14 & -2 \end{pmatrix}$ (f) $\begin{pmatrix} 6 & -4 \\ 1 & 3 \end{pmatrix}$

   (g) $\begin{pmatrix} -4 & -9 \\ 25 & -2 \end{pmatrix}$ (h) $\begin{pmatrix} 18 & -17 \\ 14 & 9 \end{pmatrix}$

2)(a) $\begin{pmatrix} 14 & 4 & -16 \\ 10 & 9 & -9 \\ -4 & -10 & 10 \end{pmatrix}$ (b) $\begin{pmatrix} 13 & 1 & -17 \\ -8 & 8 & 12 \\ -2 & 14 & 12 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 4 & 8 \\ 5 & -3 & -5 \\ -4 & 8 & 8 \end{pmatrix}$

   (d) $\begin{pmatrix} 10 & 7 & 17 \\ -10 & -14 & 10 \\ 7 & -9 & 7 \end{pmatrix}$ (e) $\begin{pmatrix} 12 & 4 & -4 \\ -7 & 17 & 11 \\ 8 & -8 & 0 \end{pmatrix}$ (f) $\begin{pmatrix} 149 & -181 & 359 \\ 110 & 36 & -240 \\ 209 & 112 & 78 \end{pmatrix}$

3) (a) $x = 4$ (b) $x = -9$ or $x = 2$

   (c) $x = -\dfrac{9}{2}$ or $x = 3$ (d) $x = -\dfrac{1}{2}$ or $x = 1$

   (e) $x = -3$ (f) $x = 2$ or $x = \dfrac{7}{3}$

   (g) $x = -2$ or $x = -1$ (h) $x = -\dfrac{2}{3}$ or $x = 4$

4)a) $\begin{pmatrix} 3 & -\frac{7}{3} \\ -2 & \frac{5}{3} \end{pmatrix}$ b) $\begin{pmatrix} \frac{1}{2} & 2 \\ \frac{1}{2} & 1 \end{pmatrix}$ c) $\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$ d) $\dfrac{1}{8}\begin{pmatrix} 9 & -12 & 29 \\ -1 & 4 & -13 \\ -3 & 4 & -7 \end{pmatrix}$