Prelim Revision   **Partial Fractions**

Express in partial fractions:

1. $\dfrac{x+7}{(x+1)(x-2)}$
2. $\dfrac{3(x+5)}{x(x+3)}$
3. $\dfrac{x^2-8x+57}{(x-1)(x-6)^2}$

4. $\dfrac{4x+13}{(x-2)(x+5)}$
5. $\dfrac{7x^2+5x+18}{(x+2)(x^2+5)}$
6. $\dfrac{x-3}{x(x+6)}$

7. $\dfrac{x^2+12x-17}{(x+4)(x-3)^2}$
8. $\dfrac{10(x+1)}{(x-2)(2x^2-x+4)}$
9. $\dfrac{x-19}{x^2+2x-15}$

10. $\dfrac{3x^2+5x+3}{x^2-3x}$
11. $\dfrac{11x-6}{(2x-1)(3x-2)}$
12. $\dfrac{3x+13}{(2x+5)(x-3)}$

13. $\dfrac{x^3+9x^2+15x-39}{x^2+5x-6}$

14. The cubic polynomial $c(x)$ is defined by $c(x)=x^3-x^2-x-15$.

   (a) Factorise $c(x)$ into the product of a linear factor $l(x)$ and a quadratic factor $q(x)$, and prove that $q(x)$ cannot be further factorised into the product of linear factors with real coefficients.

   (b) Hence express $\dfrac{13x+1}{x^3-x^2-x-15}$ in partial fractions.

ANSWERS

1. $-\dfrac{2}{x+1} + \dfrac{3}{x-2}$

2. $\dfrac{5}{x} - \dfrac{2}{x+3}$

3. $\dfrac{2}{x-1} - \dfrac{1}{x-6} + \dfrac{9}{(x-6)^2}$

4. $\dfrac{3}{x-2} + \dfrac{1}{x+5}$

5. $\dfrac{4}{x+2} + \dfrac{3x-1}{x^2+5}$

6. $-\dfrac{1/2}{x} + \dfrac{3/2}{x+6} \quad \left[= -\dfrac{1}{2x} + \dfrac{3}{2(x+6)} \right]$

7. $-\dfrac{1}{x+4} + \dfrac{2}{x-3} + \dfrac{4}{(x-3)^2}$

8. $\dfrac{3}{x-2} + \dfrac{-6x+1}{2x^2-x+4}$

9. $\dfrac{3}{x+5} - \dfrac{2}{x-3}$

10. $3 - \dfrac{1}{x} + \dfrac{15}{x-3}$

11. $\dfrac{1}{2x-1} + \dfrac{4}{3x-2}$

12. $-\dfrac{1}{2x+5} + \dfrac{2}{x-3}$

13. $x+4 + \dfrac{3}{x+6} - \dfrac{2}{x-1}$

14. (a) $c(x) = (x-3)(x^2+2x+5)$;

$q(x) = x^2 + 2x + 5$ and the discriminant of $q(x)$ is -16, which is < 0, hence $q(x)$ has no real roots and thus does not factorise into the product of linear factors with real coefficients.

(b) $\dfrac{13x+1}{x^3 - x^2 - x - 15} = \dfrac{2}{x-3} + \dfrac{-2x+3}{x^2+2x+5}$