

Advanced Higher Maths  
SQA 2016 Exemplar  
Question 18



Vegetation can be irrigated by putting a small hole in the bottom of a cylindrical tank, so that the water leaks out slowly.

Torricelli's law states that the rate of change of volume,  $V$ , of water in the tank is proportional to the square root of the height,  $h$ , of the water above the hole.

This is given by the differential equation:

$$\frac{dV}{dt} = -k\sqrt{h}, k > 0.$$

- (a) For a cylindrical tank with constant cross-sectional area,  $A$ , show that the rate of change of the height of the water in the tank is given by

$$\frac{dh}{dt} = \frac{-k}{A}\sqrt{h}. \quad 2$$

- (b) Initially, when the height of the water is 144 cm, the rate at which the height is changing is  $-0.3$  cm/hr.

By solving the differential equation in part (a), show that

$$h = \left(12 - \frac{1}{80}t\right)^2. \quad 4$$

- (c) How many days will it take for the tank to empty? 2
- (d) Given that the tank has radius of 20 cm, find the rate, in  $\text{cm}^3/\text{hr}$ , at which the vegetation was receiving water from the tank at the end of the fourth day. 3

Answers/methods:

- (a) Proof, using related rates of change. See marking instructions.
- (b) Proof, by substituting values, separating variables, integrating and evaluating the constant. See marking instructions.
- (c) 40 days
- (d)  $108\pi(\text{cm}^3/\text{hr})$