



7. (a) Use the substitution $u = y^2 + 1$, or otherwise, to find the exact value of

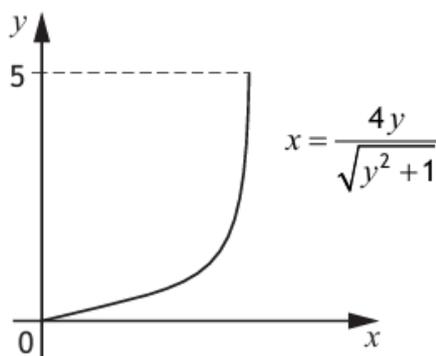
$$\int_0^5 \frac{4y}{\sqrt{y^2 + 1}} dy.$$

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Student engineers are using a 3D printer to make a model.

Relative to a suitable set of axes, the cross-section of the model is **symmetrical about the y -axis** and is represented in the **first quadrant** by the curve with equation

$$x = \frac{4y}{\sqrt{y^2 + 1}}, \quad 0 \leq y \leq 5, \text{ as shown in the diagram.}$$



- (b) State the area of the cross-section.

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- (c) Express $\frac{y^2}{y^2 + 1}$ in the form $a + \frac{b}{y^2 + 1}$ where a and b are real numbers.

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The curve $x = \frac{4y}{\sqrt{y^2 + 1}}$, $0 \leq y \leq 5$, will be rotated through 2π radians about the y -axis to make the model.

- (d) Find the volume of the model.

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Answers:

(a) $4(\sqrt{26} - 1)$

(b) $8(\sqrt{26} - 1)$

(c) $1 - \frac{1}{y^2 + 1}$

(d) $16\pi(5 - \tan^{-1}5)$