More Difficult Second Order Differential Equations: Non-homogeneous Linear, Constant Coefficients

1. $y'' - 3y' + 2y = e^{3x}$ 2. $y'' + 6y' + 9y = e^{-x}$ 3. $y'' - 2y' + 5y = e^x$

4. $y'' + y' - 2y = 3e^x$ 5. $y'' - 6y' + 8y = 2e^{2x}$

6. $y'' + 4y' + 13y = 9e^{-2x}$ if $y = 0$ and $y' = 0$ when $x = 0$.

7. $y'' + 4y' + 5y = 1 - 5x$ 8. $4y'' - y = x^2 - 3x$ 9. $y'' - 4y' + 5y = 125x^2$

10. $y'' - y' = 4x + 1$ 11. $y'' + 4y = \sin 3x$ 12. $y'' + 4y' + 5y = 16\sin x$

13. $y'' + 2y' + 2y = \cos x$ if $y = \tfrac{2}{5}$ when $x = 0$, and $y' = -\tfrac{1}{5}$ when $x = \pi/2$.

14. $4y'' + y = 4\sin x$ given that $y = \pi$ and $y' = 0$ when $x = \pi$.

15. $y'' + y = 2\sin x$ 16. $y'' + 4y = \cos 2x$ 17. $y'' + 2y' - 3y = x - \sin x$

18. $y'' + y = x^2 + \sin x$ 19. $y'' + y' - 2y = e^x + e^{-x}$ 20. $y'' - 4y' + 4y = 50e^{2x}$

21. $y'' + 2y' + 2y = e^{-2x} + \cos x$, given that $y = 1$ and $y' = 0$ when $x = 0$.

22. $\dfrac{d^2x}{dt^2} + 2\dfrac{dx}{dt} + 5x = 10 + \sin 3t$, if $x = 0$ and $\dfrac{dx}{dt} = 0$ when $t = 0$.

23. $y'' - 3y' + 2y = e^{2x}$, given that $y = y' = 0$ when $x = 0$.

24. $y'' - 4y' + 4y = e^{2x} + \cos 2x$

Answers are on the next page.

1. $y = Ae^x + Be^{2x} + \frac{1}{2}e^{3x}$

2. $y = (Ax + B)e^{-3x} + \frac{1}{4}e^{-x}$

3. $y = e^x(A\cos 2x + B\sin 2x) + \frac{1}{4}e^x$

4. $y = Ae^{-2x} + Be^x + xe^x$

5. $y = Ae^{2x} + Be^{4x} - xe^{2x}$

6. $y = e^{-2x}(1 - \cos 3x)$

7. $y = e^{-2x}(A\cos x + B\sin x) + 1 - x$

8. $y = Ae^{0.5x} + Be^{-0.5x} - x^2 + 3x - 8$

9. $y = e^{2x}(A\cos x + B\sin x) + 25x^2 + 40x + 22$

10. $y = A + Be^x - 2x^2 - 5x$

11. $y = A\cos 2x + B\sin 2x - \frac{1}{5}\sin 3x$

12. $y = e^{-2x}(A\cos x + B\sin x) + 2(\sin x - \cos x)$

13. $y = \frac{1}{5}e^{-x}(\cos x - \sin x) + \frac{1}{5}\cos x + \frac{2}{5}\sin x$

14. $y = \pi\sin x/2 + 8/3\cos x/2 - 4/3\sin x$

15. $y = A\cos x + B\sin x - x\cos x$

16. $y = A\cos 2x + B\sin 2x + \frac{1}{4}x\sin 2x$

17. $y = Ae^x + Be^{-3x} - \underline{1}x - \underline{2} + \underline{1}(2\sin x + \cos x)$
    $\phantom{y = Ae^x + Be^{-3x} -} 3 \quad 9 \quad 10$

18. $y = A\cos x + B\sin x - \frac{1}{2}x\cos x + x^2 - 2$

19. $y = Ae^x + Be^{-2x} + \frac{1}{3}xe^x - \frac{1}{2}e^{-x}$

20. $y = (Ax + B)e^{2x} + 25x^2e^{2x}$

21. $y = \underline{3}e^{-x}(3\sin x + \cos x) + \underline{1}e^{-2x} + \underline{1}(2\sin x + \cos x)$
    $ 10 \phantom{e^{-x}(3\sin x + \cos x) +} 2 \phantom{e^{-2x} +} 5$

22. $y = -e^{-t}(\underline{49}\cos 2t + \underline{43}\sin 2t) + 2 - \underline{1}\sin 3t - \underline{3}\cos 3t$
    $\phantom{y = -e^{-t}(} 26 52 13 26$

23. $y = e^x - e^{2x} + xe^{2x}$

24. $y = (Ax + B)e^{2x} + \frac{1}{2}x^2e^{2x} - \frac{1}{8}\sin 2x$