

TLMaths BUMPER Worksheet of Differential Equations (Separation of Variables)

Prerequisites: Reciprocal Trigonometric Functions, Double Angle Formulae, Integrals of e^{kx} , $\sin kx$, $\cos kx$, $\sec^2 kx$, $\sin^2 kx$, $\cos^2 kx$, Integration by Substitution, Integration by Parts, Partial Fractions.

- Solutions show the separation of variables and then the general solution.
- Care has been taken to avoid \pm in general solutions.
- Not all general solutions have been rearranged to make y the subject.
- c_1 is used to represent the constant of integration in each case.
- Nested fractions have been avoided in the general solution.

01	$\frac{dy}{dx} = \frac{x}{y}$	02	$\frac{dy}{dx} = \frac{x^2}{y}$
03	$\frac{dy}{dx} = \frac{x^3}{y^2}$	04	$\frac{dy}{dx} = \frac{x^4}{y^3}$
05	$\frac{dy}{dx} = xy$	06	$\frac{dy}{dx} = x^2y$
07	$\frac{dy}{dx} = x^3y^2$	08	$\frac{1}{x^4} \frac{dy}{dx} = y^3$
09	$\frac{dy}{dx} = \frac{y}{x}$	10	$x \frac{dy}{dx} = y^2$
11	$\frac{dy}{dx} = \frac{y^3}{x^2}$	12	$x^3 \frac{dy}{dx} = y^4$
13	$\frac{dy}{dx} = \frac{x}{y+1}$	14	$\frac{1}{x+1} \frac{dy}{dx} = \frac{1}{y}$
15	$\frac{dy}{dx} = x(y+1)$	16	$\frac{dy}{dx} = (x+1)y$
17	$\frac{dy}{dx} = \frac{y}{x+1}$	18	$x \frac{dy}{dx} = y+1$
19	$x \frac{dy}{dx} = (x+1)y$	20	$\frac{dy}{dx} = \frac{xy}{y+1}$
21	$\frac{dy}{dx} = \frac{(x^2-3)y^2}{x}$	22	$\frac{1}{5x} \frac{dy}{dx} = \frac{y^3}{y^2-1}$
23	$\frac{dy}{dx} = \frac{(x^3+3x+9)y^3}{x^2}$	24	$x^5 \frac{dy}{dx} = 2(x^6-x^3+8)y^4$

25	$\frac{dy}{dx} = \frac{(x+2)^2 y^4}{x}$	26	$\frac{1}{(2x-3)^2} \frac{dy}{dx} = \frac{y^5}{x}$
27	$\frac{dy}{dx} = \frac{(5x-4)^2 y^3}{x^2}$	28	$x^3 \frac{dy}{dx} = (9x+2)^2 y^6$
29	$\frac{dy}{dx} = \frac{(3x+2)^2 y^4}{x^3(3y-2)^2}$	30	$x^2(5y-4)^2 \frac{dy}{dx} = (4x-5)^2 y^5$
31	$\frac{dy}{dx} = \frac{\sqrt{x}}{y^8}$	32	$x^8 \frac{dy}{dx} = \sqrt{y}$
33	$\frac{dy}{dx} = \sqrt{\frac{x}{y}}$	34	$\frac{dy}{dx} = \sqrt{\frac{y}{x}}$
35	$\frac{dy}{dx} = \sqrt[3]{\frac{x}{y}}$	36	$\frac{dy}{dx} = \sqrt[3]{\frac{y}{x}}$
37	$\frac{dy}{dx} = \frac{e^x}{y}$	38	$\frac{dy}{dx} = \frac{e^y}{x}$
39	$\frac{1}{e^x} \frac{dy}{dx} = \frac{1}{e^y}$	40	$\frac{dy}{dx} = e^{2x-y}$
41	$\frac{dy}{dx} = \frac{e^y}{e^x}$	42	$\frac{dy}{dx} = e^{2y-x}$
43	$\frac{dy}{dx} = e^x e^y$	44	$\frac{dy}{dx} = e^{2x+y}$
45	$\frac{dy}{dx} = \frac{e^{-3x}}{e^{-4y}}$	46	$\frac{dy}{dx} = \frac{e^{-4y}}{e^{-2x}}$
47	$\frac{dy}{dx} = \frac{\sin x}{y}$	48	$\frac{dy}{dx} = \frac{\cos x}{y}$
49	$\frac{dy}{dx} = \frac{\sin 2x}{2y}$	50	$\sec 3x \frac{dy}{dx} = \frac{1}{3y}$
51	$\frac{dy}{dx} = \frac{\cos x}{\sin y}$	52	$\frac{dy}{dx} = \frac{\sin x}{\cos y}$

53	$\frac{dy}{dx} = \frac{\cos 4x}{\sin 6y}$	54	$\operatorname{cosec} 5x \frac{dy}{dx} = \frac{2}{\cos 10y}$
55	$\frac{dy}{dx} = \frac{e^{3x}}{\sin 3y}$	56	$e^{-4x} \frac{dy}{dx} = \frac{1}{\cos 8y}$
57	$\frac{dy}{dx} = \frac{\sin^2 x}{y}$	58	$\frac{dy}{dx} = \frac{\cos^2 x}{y}$
59	$\frac{dy}{dx} = \frac{\sin^2 x}{\cos^2 y}$	60	$\sec^2 x \frac{dy}{dx} = \operatorname{cosec}^2 y$
61	$\frac{dy}{dx} = \frac{4 \sin^2 2x}{y^2}$	62	$\frac{dy}{dx} = \frac{6 \cos^2 3x}{y^3}$
63	$\frac{dy}{dx} = \frac{\sec^2 x}{\sec^2 y}$	64	$\frac{dy}{dx} = \frac{\cos^2 2y}{\cos^2 2x}$
65	$\frac{dy}{dx} = \frac{\ln x}{y}$	66	$\frac{1}{\ln x} \frac{dy}{dx} = \frac{x}{y}$
67	$\frac{dy}{dx} = \frac{x}{\ln y}$	68	$\frac{1}{x^2} \frac{dy}{dx} = \frac{10}{y^2 \ln y}$
69	$\frac{dy}{dx} = \frac{x^5 \ln x}{y^3 \ln y}$	70	$\frac{x^5}{\ln x} \frac{dy}{dx} = \frac{y^3}{\ln y}$
71	$\frac{dy}{dx} = \frac{x \sin x}{y \sin y}$	72	$\frac{dy}{dx} = \frac{x \cos x}{y \cos y}$
73	$\frac{dy}{dx} = \frac{(x+1)^3}{y^2}$	74	$\frac{dy}{dx} = \frac{(x+2)^5}{y^4}$
75	$\frac{dy}{dx} = \frac{(2x-3)^4}{(3y+4)^3}$	76	$(3x+4)^3 \frac{dy}{dx} = (2y-3)^4$
77	$\frac{dy}{dx} = \frac{(4x+5)^6}{(5y-3)^4}$	78	$(5x-3)^4 \frac{dy}{dx} = (4y+5)^6$
79	$\frac{dy}{dx} = \frac{\sqrt{x-4}}{y^3}$	80	$\frac{dy}{dx} = \frac{\sqrt{y-4}}{x^3}$

81	$\frac{dy}{dx} = \frac{\sqrt{2x-5}}{y^4}$	82	$x^4 \frac{dy}{dx} = \sqrt{2y-5}$
83	$\frac{dy}{dx} = \frac{y}{(x+1)(x+2)}$	84	$x \frac{dy}{dx} = (y+1)(y+2)$
85	$(x+2)(x+3) \frac{dy}{dx} = y^2$	86	$x^2 \frac{dy}{dx} = (y+2)(y+3)$
87	$\frac{dy}{dx} = \frac{12y^3}{(x+1)(x+4)}$	88	$\frac{dy}{dx} = \frac{(y+1)(y+4)}{18x^4}$
89	$\frac{dy}{dx} = \frac{x-17}{(x-3)(x+4)y}$	90	$x(y-17) \frac{dy}{dx} = (y-3)(y+4)$
91	$\frac{dy}{dx} = \frac{(8x+1)e^{2y}}{x(2x-1)}$	92	$e^{3x} \frac{dy}{dx} = \frac{y(4y-1)}{32y-6}$
93	$\frac{dy}{dx} = \frac{8x^2 + 25x + 27}{x(x+3)^2 e^{4y}}$	94	$\frac{dy}{dx} = \frac{x^2 y^2 (y+1)}{5y^2 - 6y - 9}$
95	$(x-3)(x+2)(x+5) \frac{dy}{dx} = 24(x+7) \sec y$	96	$\frac{dy}{dx} = \frac{(y-6)(y+1)(y+2)}{(79y+198) \operatorname{cosec} x}$
97	$\frac{1}{13x+46} \frac{dy}{dx} = \frac{\sqrt{4y-5}}{(x+7)(2x+5)}$	98	$(8x-5)^{\frac{3}{5}} \frac{dy}{dx} = \frac{(y-3)(3y-4)}{15y-5}$
99	$\frac{dy}{dx} = \frac{(90x+140)e^y}{y(20+9x-18x^2)}$	100	$\frac{1}{x^2} \frac{dy}{dx} = \frac{(4-13y-35y^2) \sin 8x}{40-11y}$

01

$$\frac{dy}{dx} = \frac{x}{y}$$

$$\int y \, dy = \int x \, dx$$

$$y^2 = c_1 + x^2$$

02

$$\frac{dy}{dx} = \frac{x^2}{y}$$

$$\int y \, dy = \int x^2 \, dx$$

$$y^2 = c_1 + \frac{2}{3}x^3$$

03

$$\frac{dy}{dx} = \frac{x^3}{y^2}$$

$$\int y^2 \, dy = \int x^3 \, dx$$

$$y = \sqrt[3]{c_1 + \frac{3}{4}x^4}$$

04

$$\frac{dy}{dx} = \frac{x^4}{y^3}$$

$$\int y^3 \, dy = \int x^4 \, dx$$

$$y^4 = c_1 + \frac{4}{5}x^5$$

05

$$\frac{dy}{dx} = xy$$

$$\int \frac{1}{y} \, dy = \int x \, dx$$

$$y = c_1 e^{\frac{1}{2}x^2}$$

06

$$\frac{dy}{dx} = x^2 y$$

$$\int \frac{1}{y} \, dy = \int x^2 \, dx$$

$$y = c_1 e^{\frac{1}{3}x^3}$$

07

$$\frac{dy}{dx} = x^3 y^2$$

$$\int \frac{1}{y^2} \, dy = \int x^3 \, dx$$

$$y = \frac{4}{c_1 - x^4}$$

08

$$\frac{1}{x^4} \frac{dy}{dx} = y^3$$

$$\int \frac{1}{y^3} \, dy = \int x^4 \, dx$$

$$y^2 = \frac{5}{c_1 - 2x^5}$$

09

$$\frac{dy}{dx} = \frac{y}{x}$$

$$\int \frac{1}{y} \, dy = \int \frac{1}{x} \, dx$$

$$y = c_1 x$$

10

$$x \frac{dy}{dx} = y^2$$

$$\int \frac{1}{y^2} \, dy = \int \frac{1}{x} \, dx$$

$$y = \frac{1}{c_1 - \ln x}$$

<p>11</p> $\frac{dy}{dx} = \frac{y^3}{x^2}$ $\int \frac{1}{y^3} dy = \int \frac{1}{x^2} dx$ $y^2 = \frac{x}{2 + c_1 x}$	<p>12</p> $x^3 \frac{dy}{dx} = y^4$ $\int \frac{1}{y^4} dy = \int \frac{1}{x^3} dx$ $y = \sqrt[3]{\frac{2x^2}{3 + c_1 x^2}}$
<p>13</p> $\frac{dy}{dx} = \frac{x}{y + 1}$ $\int (y + 1) dy = \int x dx$ $y^2 + 2y = c_1 + x^2$	<p>14</p> $\frac{1}{x + 1} \frac{dy}{dx} = \frac{1}{y}$ $\int y dy = \int (x + 1) dx$ $y^2 = c_1 + x^2 + 2x$
<p>15</p> $\frac{dy}{dx} = x(y + 1)$ $\int \frac{1}{y + 1} dy = \int x dx$ $y = c_1 e^{\frac{1}{2}x^2} - 1$	<p>16</p> $\frac{dy}{dx} = (x + 1)y$ $\int \frac{1}{y} dy = \int (x + 1) dx$ $y = c_1 e^{\frac{1}{2}x^2 + x}$
<p>17</p> $\frac{dy}{dx} = \frac{y}{x + 1}$ $\int \frac{1}{y} dy = \int \frac{1}{x + 1} dx$ $y = c_1(x + 1)$	<p>18</p> $x \frac{dy}{dx} = y + 1$ $\int \frac{1}{y + 1} dy = \int \frac{1}{x} dx$ $y = c_1 x - 1$
<p>19</p> $x \frac{dy}{dx} = (x + 1)y$ $\int \frac{1}{y} dy = \int \left(1 + \frac{1}{x}\right) dx$ $y = c_1 x e^x$	<p>20</p> $\frac{dy}{dx} = \frac{xy}{y + 1}$ $\int \left(1 + \frac{1}{y}\right) dy = \int x dx$ $y + \ln y = c_1 + \frac{1}{2}x^2$

21

$$\frac{dy}{dx} = \frac{(x^2 - 3)y^2}{x}$$

$$\int \frac{1}{y^2} dy = \int \left(x - \frac{3}{x}\right) dx$$

$$y = \frac{2}{c_1 + 6 \ln x - x^2}$$

22

$$\frac{1}{5x} \frac{dy}{dx} = \frac{y^3}{y^2 - 1}$$

$$\int \left(\frac{1}{y} - \frac{1}{y^3}\right) dy = \int 5x dx$$

$$\ln y - \frac{1}{2y^2} = c_1 + \frac{5}{2}x^2$$

23

$$\frac{dy}{dx} = \frac{(x^3 + 3x + 9)y^3}{x^2}$$

$$\int \frac{1}{y^3} dy = \int \left(x + \frac{3}{x} + \frac{9}{x^2}\right) dx$$

$$y^2 = \frac{x}{c_1 x + 18 - x^3 - 6x \ln x}$$

24

$$x^5 \frac{dy}{dx} = 2(x^6 - x^3 + 8)y^4$$

$$\int \frac{1}{2y^4} dy = \int \left(x - \frac{1}{x^2} + \frac{8}{x^5}\right) dx$$

$$y = \sqrt[3]{\frac{x^4}{c_1 x^4 - 3x^6 - 6x^3 + 12}}$$

25

$$\frac{dy}{dx} = \frac{(x + 2)^2 y^4}{x}$$

$$\int \frac{1}{y^4} dy = \int \left(x + 4 + \frac{4}{x}\right) dx$$

$$y = \sqrt[3]{\frac{2}{c_1 - 3x^2 - 24x - 24 \ln x}}$$

26

$$\frac{1}{(2x - 3)^2} \frac{dy}{dx} = \frac{y^5}{x}$$

$$\int \frac{1}{y^5} dy = \int \left(4x - 12 + \frac{9}{x}\right) dx$$

$$y^4 = \frac{1}{c_1 - 8x^2 + 48x - 36 \ln x}$$

27

$$\frac{dy}{dx} = \frac{(5x - 4)^2 y^3}{x^2}$$

$$\int \frac{1}{y^3} dy = \int \left(25 - \frac{40}{x} + \frac{16}{x^2}\right) dx$$

$$y^2 = \frac{x}{c_1 x - 50x^2 + 80x \ln x + 32}$$

28

$$x^3 \frac{dy}{dx} = (9x + 2)^2 y^6$$

$$\int \frac{1}{y^6} dy = \int \left(\frac{81}{x} + \frac{36}{x^2} + \frac{4}{x^3}\right) dx$$

$$y = \sqrt[5]{\frac{x^2}{c_1 x^2 - 405x^2 \ln x + 180x + 10}}$$

29

$$\frac{dy}{dx} = \frac{(3x + 2)^2 y^4}{x^3(3y - 2)^2}$$

$$\int \left(\frac{9}{y^2} - \frac{12}{y^3} + \frac{4}{y^4}\right) dy = \int \left(\frac{9}{x} + \frac{12}{x^2} + \frac{4}{x^3}\right) dx$$

$$-\frac{9}{y} + \frac{6}{y^2} - \frac{4}{3y^3} = c_1 + 9 \ln x - \frac{12}{x} - \frac{2}{x^2}$$

30

$$x^2(5y - 4)^2 \frac{dy}{dx} = (4x - 5)^2 y^5$$

$$\int \left(\frac{25}{y^3} - \frac{40}{y^4} + \frac{16}{y^5}\right) dy = \int \left(16 - \frac{40}{x} + \frac{25}{x^2}\right) dx$$

$$-\frac{25}{2y^2} + \frac{40}{3y^3} - \frac{4}{y^4} = c_1 + 16x - 40 \ln x - \frac{25}{x}$$

31

$$\frac{dy}{dx} = \frac{\sqrt{x}}{y^8}$$

$$\int y^8 dy = \int \sqrt{x} dx$$

$$y = \sqrt[9]{c_1 + 6x^{\frac{3}{2}}}$$

32

$$x^8 \frac{dy}{dx} = \sqrt{y}$$

$$\int \frac{1}{\sqrt{y}} dy = \int \frac{1}{x^8} dx$$

$$y = \left(c_1 - \frac{1}{14x^7} \right)^2$$

33

$$\frac{dy}{dx} = \sqrt{\frac{x}{y}}$$

$$\int \sqrt{y} dy = \int \sqrt{x} dx$$

$$y = \left(c_1 + x^{\frac{3}{2}} \right)^{\frac{2}{3}}$$

34

$$\frac{dy}{dx} = \sqrt{\frac{y}{x}}$$

$$\int \frac{1}{\sqrt{y}} dy = \int \frac{1}{\sqrt{x}} dx$$

$$y = (c_1 + \sqrt{x})^2$$

35

$$\frac{dy}{dx} = \sqrt[3]{\frac{x}{y}}$$

$$\int \sqrt[3]{y} dy = \int \sqrt[3]{x} dx$$

$$y^{\frac{4}{3}} = c_1 + x^{\frac{4}{3}}$$

36

$$\frac{dy}{dx} = \sqrt[3]{\frac{y}{x}}$$

$$\int \frac{1}{\sqrt[3]{y}} dy = \int \frac{1}{\sqrt[3]{x}} dx$$

$$y^{\frac{2}{3}} = c_1 + x^{\frac{2}{3}}$$

37

$$\frac{dy}{dx} = \frac{e^x}{y}$$

$$\int y dy = \int e^x dx$$

$$y^2 = c_1 + 2e^x$$

38

$$\frac{dy}{dx} = \frac{e^y}{x}$$

$$\int \frac{1}{e^y} dy = \int \frac{1}{x} dx$$

$$y = \ln\left(\frac{1}{c_1 - \ln x}\right)$$

39

$$\frac{1}{e^x} \frac{dy}{dx} = \frac{1}{e^y}$$

$$\int e^y dy = \int e^x dx$$

$$y = \ln(c_1 + e^x)$$

40

$$\frac{dy}{dx} = e^{2x-y}$$

$$\int e^y dy = \int e^{2x} dx$$

$$y = \ln(c_1 + e^{2x})$$

41

$$\frac{dy}{dx} = \frac{e^y}{e^x}$$

$$\int \frac{1}{e^y} dy = \int \frac{1}{e^x} dx$$

$$y = \ln\left(\frac{e^x}{c_1 e^x + 1}\right)$$

42

$$\frac{dy}{dx} = e^{2y-x}$$

$$\int \frac{1}{e^{2y}} dy = \int \frac{1}{e^x} dx$$

$$y = \frac{1}{2} \ln\left(\frac{e^x}{c_1 e^x + 2}\right)$$

43

$$\frac{dy}{dx} = e^x e^y$$

$$\int \frac{1}{e^y} dy = \int e^x dx$$

$$y = \ln\left(\frac{1}{c_1 - e^x}\right)$$

44

$$\frac{dy}{dx} = e^{2x+y}$$

$$\int \frac{1}{e^y} dy = \int e^{2x} dx$$

$$y = \ln\left(\frac{2}{c_1 - e^{2x}}\right)$$

45

$$\frac{dy}{dx} = \frac{e^{-3x}}{e^{-4y}}$$

$$\int e^{-4y} dy = \int e^{-3x} dx$$

$$y = \frac{1}{4} \ln\left(\frac{3e^{3x}}{c_1 e^{3x} + 4}\right)$$

46

$$\frac{dy}{dx} = \frac{e^{-4y}}{e^{-2x}}$$

$$\int e^{4y} dy = \int e^{2x} dx$$

$$y = \frac{1}{4} \ln(c_1 + 2e^{2x})$$

47

$$\frac{dy}{dx} = \frac{\sin x}{y}$$

$$\int y dy = \int \sin x dx$$

$$y^2 = c_1 - 2 \cos x$$

48

$$\frac{dy}{dx} = \frac{\cos x}{y}$$

$$\int y dy = \int \cos x dx$$

$$y^2 = c_1 + 2 \sin x$$

49

$$\frac{dy}{dx} = \frac{\sin 2x}{2y}$$

$$\int 2y dy = \int \sin 2x dx$$

$$y^2 = c_1 - \frac{1}{2} \cos 2x$$

50

$$\sec 3x \frac{dy}{dx} = \frac{1}{3y}$$

$$\int 3y dy = \int \cos 3x dx$$

$$y^2 = c_1 + \frac{2}{9} \sin 3x$$

<p>51</p> $\frac{dy}{dx} = \frac{\cos x}{\sin y}$ $\int \sin y \, dy = \int \cos x \, dx$ $\cos y = c_1 - \sin x$	<p>52</p> $\frac{dy}{dx} = \frac{\sin x}{\cos y}$ $\int \cos y \, dy = \int \sin x \, dx$ $\sin y = c_1 - \cos x$
<p>53</p> $\frac{dy}{dx} = \frac{\cos 4x}{\sin 6y}$ $\int \sin 6y \, dy = \int \cos 4x \, dx$ $\cos 6y = c_1 - \frac{3}{2} \sin 4x$	<p>54</p> $\operatorname{cosec} 5x \frac{dy}{dx} = \frac{2}{\cos 10y}$ $\int \cos 10y \, dy = \int 2 \sin 5x \, dx$ $\sin 10y = c_1 - 4 \cos 5x$
<p>55</p> $\frac{dy}{dx} = \frac{e^{3x}}{\sin 3y}$ $\int \sin 3y \, dy = \int e^{3x} \, dx$ $\cos 3y = c_1 - e^{3x}$	<p>56</p> $e^{-4x} \frac{dy}{dx} = \frac{1}{\cos 8y}$ $\int \cos 8y \, dy = \int e^{4x} \, dx$ $\sin 8y = c_1 + 2e^{4x}$
<p>57</p> $\frac{dy}{dx} = \frac{\sin^2 x}{y}$ $\int y \, dy = \int \sin^2 x \, dx$ $y^2 = c_1 + x - \frac{1}{2} \sin 2x$	<p>58</p> $\frac{dy}{dx} = \frac{\cos^2 x}{y}$ $\int y \, dy = \int \cos^2 x \, dx$ $y^2 = c_1 + x + \frac{1}{2} \sin 2x$
<p>59</p> $\frac{dy}{dx} = \frac{\sin^2 x}{\cos^2 y}$ $\int \cos^2 y \, dy = \int \sin^2 x \, dx$ $2y + \sin 2y = c_1 + 2x - \sin 2x$	<p>60</p> $\sec^2 x \frac{dy}{dx} = \operatorname{cosec}^2 y$ $\int \sin^2 y \, dy = \int \cos^2 x \, dx$ $2y - \sin 2y = c_1 + 2x + \sin 2x$

<p>61</p> $\frac{dy}{dx} = \frac{4 \sin^2 2x}{y^2}$ $\int y^2 dy = \int 4 \sin^2 2x dx$ $y = \sqrt[3]{c_1 + 6x - \frac{3}{2} \sin 4x}$	<p>62</p> $\frac{dy}{dx} = \frac{6 \cos^2 3x}{y^3}$ $\int y^3 dy = \int 6 \cos^2 3x dx$ $y^4 = c_1 + 12x + 2 \sin 6x$
<p>63</p> $\frac{dy}{dx} = \frac{\sec^2 x}{\sec^2 y}$ $\int \cos^2 y dy = \int \sec^2 x dx$ $2y + \sin 2y = c_1 + 4 \tan x$	<p>64</p> $\frac{dy}{dx} = \frac{\cos^2 2y}{\cos^2 2x}$ $\int \sec^2 2y dy = \int \sec^2 2x dx$ $\tan 2y = c_1 + \tan 2x$
<p>65</p> $\frac{dy}{dx} = \frac{\ln x}{y}$ $\int y dy = \int \ln x dx$ $y^2 = c_1 + 2x \ln x - 2x$	<p>66</p> $\frac{1}{\ln x} \frac{dy}{dx} = \frac{x}{y}$ $\int y dy = \int x \ln x dx$ $y^2 = c_1 + x^2 \ln x - \frac{1}{2} x^2$
<p>67</p> $\frac{dy}{dx} = \frac{x}{\ln y}$ $\int \ln y dy = \int x dx$ $y \ln y - y = c_1 + \frac{1}{2} x^2$	<p>68</p> $\frac{1}{x^2} \frac{dy}{dx} = \frac{10}{y^2 \ln y}$ $\int y^2 \ln y dy = \int 10x^2 dx$ $3x^3 \ln x - x^3 = c_1 + 30x^3$
<p>69</p> $\frac{dy}{dx} = \frac{x^5 \ln x}{y^3 \ln y}$ $\int y^3 \ln y dy = \int x^5 \ln x dx$ $36y^4 \ln y - 9y^4 = c_1 + 24x^6 \ln x - 4x^6$	<p>70</p> $\frac{x^5}{\ln x} \frac{dy}{dx} = \frac{y^3}{\ln y}$ $\int \frac{\ln y}{y^3} dy = \int \frac{\ln x}{x^5} dx$ $\frac{8 \ln y}{y^2} + \frac{4}{y^2} = c_1 + \frac{4 \ln x}{x^4} + \frac{1}{x^4}$

<p>71</p> $\frac{dy}{dx} = \frac{x \sin x}{y \sin y}$ $\int y \sin y \, dy = \int x \sin x \, dx$ $-y \cos y + \sin y = c_1 - x \cos x + \cos x$	<p>72</p> $\frac{dy}{dx} = \frac{x \cos x}{y \cos y}$ $\int y \cos y \, dy = \int x \cos x \, dx$ $y \sin y + \cos y = c_1 + x \sin x + \cos x$
<p>73</p> $\frac{dy}{dx} = \frac{(x+1)^3}{y^2}$ $\int y^2 \, dy = \int (x+1)^3 \, dx$ $y = \sqrt[3]{c_1 + \frac{3}{4}(x+1)^4}$	<p>74</p> $\frac{dy}{dx} = \frac{(x+2)^5}{y^4}$ $\int y^4 \, dy = \int (x+2)^5 \, dx$ $y = \sqrt[5]{c_1 + \frac{5}{6}(x+2)^6}$
<p>75</p> $\frac{dy}{dx} = \frac{(2x-3)^4}{(3y+4)^3}$ $\int (3y+4)^3 \, dy = \int (2x-3)^4 \, dx$ $5(3y+4)^4 = c_1 + 6(2x-3)^5$	<p>76</p> $(3x+4)^3 \frac{dy}{dx} = (2y-3)^4$ $\int \frac{1}{(2y-3)^4} \, dy = \int \frac{1}{(3x+4)^3} \, dx$ $y = \frac{1}{2} \left(\sqrt[3]{\frac{(3x+4)^2}{c_1(3x+4)^2 + 1}} + 3 \right)$
<p>77</p> $\frac{dy}{dx} = \frac{(4x+5)^6}{(5y-3)^4}$ $\int (5y-3)^4 \, dy = \int (4x+5)^6 \, dx$ $y = \frac{1}{5} \left(\sqrt[5]{c_1 + \frac{25}{28}(4x+5)^7} + 3 \right)$	<p>78</p> $(5x-3)^4 \frac{dy}{dx} = (4y+5)^6$ $\int \frac{1}{(4y+5)^6} \, dy = \int \frac{1}{(5x-3)^4} \, dx$ $y = \frac{1}{4} \left(\sqrt[5]{\frac{3(5x-3)^3}{c_1(5x-3)^3 + 4}} - 5 \right)$
<p>79</p> $\frac{dy}{dx} = \frac{\sqrt{x-4}}{y^3}$ $\int y^3 \, dy = \int \sqrt{x-4} \, dx$ $y^4 = c_1 + \frac{8}{3}(x-4)^{\frac{3}{2}}$	<p>80</p> $\frac{dy}{dx} = \frac{\sqrt{y-4}}{x^3}$ $\int \frac{1}{\sqrt{y-4}} \, dy = \int \frac{1}{x^3} \, dx$ $y = \left(c_1 - \frac{1}{4x^2} \right)^2 + 4$

<p>81</p> $\frac{dy}{dx} = \frac{\sqrt{2x-5}}{y^4}$ $\int y^4 dy = \int \sqrt{2x-5} dx$ $y = \sqrt[5]{c_1 + \frac{5}{3}(2x-5)^{\frac{3}{2}}}$	<p>82</p> $x^4 \frac{dy}{dx} = \sqrt{2y-5}$ $\int \frac{1}{\sqrt{2y-5}} dy = \int \frac{1}{x^4} dx$ $y = \frac{1}{2} \left(\left(c_1 - \frac{1}{3x^3} \right)^2 + 5 \right)$
<p>83</p> $\frac{dy}{dx} = \frac{y}{(x+1)(x+2)}$ $\int \frac{1}{y} dy = \int \frac{1}{(x+1)(x+2)} dx$ $y = \frac{c_1(x+1)}{x+2}$	<p>84</p> $x \frac{dy}{dx} = (y+1)(y+2)$ $\int \frac{1}{(y+1)(y+2)} dy = \int \frac{1}{x} dx$ $y = \frac{2c_1x - 1}{1 - c_1x}$
<p>85</p> $(x+2)(x+3) \frac{dy}{dx} = y^2$ $\int \frac{1}{y^2} dy = \int \frac{1}{(x+2)(x+3)} dx$ $y = \frac{1}{c_1 - \ln\left(\frac{x+2}{x+3}\right)}$	<p>86</p> $x^2 \frac{dy}{dx} = (y+2)(y+3)$ $\int \frac{1}{(y+2)(y+3)} dy = \int \frac{1}{x^2} dx$ $y = \frac{3c_1 e^{-\frac{1}{x}} - 2}{1 - c_1 e^{-\frac{1}{x}}}$
<p>87</p> $\frac{dy}{dx} = \frac{12y^3}{(x+1)(x+4)}$ $\int \frac{1}{y^3} dy = \int \frac{12}{(x+1)(x+4)} dx$ $y^2 = \frac{1}{c_1 - 8 \ln\left(\frac{x+1}{x+4}\right)}$	<p>88</p> $\frac{dy}{dx} = \frac{(y+1)(y+4)}{18x^4}$ $\int \frac{1}{(y+1)(y+4)} dy = \int \frac{1}{18x^4} dx$ $y = \frac{4c_1 e^{-\frac{1}{18x^3}} - 1}{1 - c_1 e^{-\frac{1}{18x^3}}}$
<p>89</p> $\frac{dy}{dx} = \frac{x-17}{(x-3)(x+4)y}$ $\int y dy = \int \frac{x-17}{(x-3)(x+4)} dx$ $y^2 = c_1 + 6 \ln(x+4) - 4 \ln(x-3)$	<p>90</p> $x(y-17) \frac{dy}{dx} = (y-3)(y+4)$ $\int \frac{y-17}{(y-3)(y+4)} dy = \int \frac{1}{x} dx$ $\frac{(y+4)^3}{(y-3)^2} = c_1 x$

<p>91</p> $\frac{dy}{dx} = \frac{(8x + 1)e^{2y}}{x(2x - 1)}$ $\int \frac{1}{e^{2y}} dy = \int \frac{8x + 1}{x(2x - 1)} dx$ $y = \frac{1}{2} \ln \left(\frac{1}{c_1 - 10 \ln(2x - 1) + 2 \ln x} \right)$	<p>92</p> $e^{3x} \frac{dy}{dx} = \frac{y(4y - 1)}{32y - 6}$ $\int \frac{32y - 6}{y(4y - 1)} dy = \int \frac{1}{e^{3x}} dx$ $6 \ln y + 2 \ln(4y - 1) = c_1 - \frac{1}{3e^{3x}}$
<p>93</p> $\frac{dy}{dx} = \frac{8x^2 + 25x + 27}{x(x + 3)^2 e^{4y}}$ $\int e^{4y} dy = \int \frac{8x^2 + 25x + 27}{x(x + 3)^2} dx$ $y = \frac{1}{4} \ln \left(c_1 + 12 \ln x + 20 \ln(x + 3) + \frac{32}{x + 3} \right)$	<p>94</p> $\frac{dy}{dx} = \frac{x^2 y^2 (y + 1)}{5y^2 - 6y - 9}$ $\int \frac{5y^2 - 6y - 9}{y^2 (y + 1)} dy = \int x^2 dx$ $3 \ln y + \frac{9}{y} + 2 \ln(y + 1) = c_1 + \frac{1}{3} x^3$
<p>95</p> $(x - 3)(x + 2)(x + 5) \frac{dy}{dx} = 24(x + 7) \sec y$ $\int \cos y dy = \int \frac{24(x + 7)}{(x - 3)(x + 2)(x + 5)} dx$ $\sin y = c_1 + 6 \ln(x - 3) - 8 \ln(x + 2) + 2 \ln(x + 5)$	<p>96</p> $\frac{dy}{dx} = \frac{(y - 6)(y + 1)(y + 2)}{(79y + 198) \operatorname{cosec} x}$ $\int \frac{79y + 198}{(y - 6)(y + 1)(y + 2)} dy = \int \sin x dx$ $12 \ln(y - 6) - 17 \ln(y + 1) + 5 \ln(y + 2) = c_1 - \cos x$
<p>97</p> $\frac{1}{13x + 46} \frac{dy}{dx} = \frac{\sqrt{4y - 5}}{(x + 7)(2x + 5)}$ $\int \frac{1}{\sqrt{4y - 5}} dy = \int \frac{13x + 46}{(x + 7)(2x + 5)} dx$ $y = \frac{1}{4} ((c_1 + 10 \ln(x + 7) + 3 \ln(2x + 5))^2 + 5)$	<p>98</p> $(8x - 5)^{\frac{3}{5}} \frac{dy}{dx} = \frac{(y - 3)(3y - 4)}{15y - 5}$ $\int \frac{15y - 5}{(y - 3)(3y - 4)} dy = \int \frac{1}{(8x - 5)^{\frac{3}{5}}} dx$ $8 \ln(y - 3) - 3 \ln(3y - 4) = c_1 + \frac{5}{16} (8x - 5)^{\frac{2}{5}}$
<p>99</p> $\frac{dy}{dx} = \frac{(90x + 140)e^y}{y(20 + 9x - 18x^2)}$ $\int \frac{y}{e^y} dy = \int \frac{90x + 140}{20 + 9x - 18x^2} dx$ $\frac{y}{e^y} + \frac{1}{e^y} = c_1 - \frac{5}{3} \ln(6x + 5) + \frac{20}{3} \ln(4 - 3x)$	<p>100</p> $\frac{1}{x^2} \frac{dy}{dx} = \frac{(4 - 13y - 35y^2) \sin 8x}{40 - 11y}$ $\int \frac{40 - 11y}{4 - 13y - 35y^2} dy = \int x^2 \sin 8x dx$ $\frac{12}{7} \ln(7y + 4) - \frac{7}{5} \ln(1 - 5y) = c_1 - x^2 \cos x + 2x \sin x + 2 \cos x$