Vectors Equation of a Line

1. Referred to a fixed origin O, the points A and B have position vectors $(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$ and $(5\mathbf{i} - 3\mathbf{j})$ respectively.

(a) Find, in vector form, an equation of the line l_1 which passes through A and B. **(2)**

The line l_2 has equation $\mathbf{r} = (4\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}) + \mu(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$, where μ is a scalar parameter.

(b) Show that A lies on l_2. **(1)**

(c) Find, in degrees, the acute angle between the lines l_1 and l_2. **(4)**

The point C with position vector $(2\mathbf{i} - \mathbf{k})$ lies on l_2.

(d) Find the shortest distance from C to the line l_1. **(4)**

2. Relative to a fixed origin O, the point A has position vector $5\mathbf{j} + 5\mathbf{k}$ and the point B has position vector $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

(a) Find a vector equation of the line L which passes through A and B. **(2)**

The point C lies on the line L and OC is perpendicular to L.

(b) Find the position vector of C. **(5)**

The points O, B and A, together with the point D, lie at the vertices of parallelogram $OBAD$.

(c) Find, the position vector of D. **(2)**

(d) Find the area of the parallelogram $OBAD$. **(4)**

3. Relative to a fixed origin O, the point A has position vector $(8\mathbf{i} + 13\mathbf{j} - 2\mathbf{k})$, the point B has position vector $(10\mathbf{i} + 14\mathbf{j} - 4\mathbf{k})$, and the point C has position vector $(9\mathbf{i} + 9\mathbf{j} + 6\mathbf{k})$.

The line l passes through the points A and B.

(a) Find a vector equation for the line l. **(3)**

(b) Find $|\overrightarrow{CB}|$. **(2)**

(c) Find the size of the acute angle between the line segment CB and the line l, giving your answer in degrees to 1 decimal place. **(3)**

(d) Find the shortest distance from the point C to the line l. **(3)**

The point X lies on l. Given that the vector $\overrightarrow{CX}$ is perpendicular to l,

(e) find the area of the triangle CXB, giving your answer to 3 significant figures. **(3)**

Solutions

1. **(a)** $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} \pm \lambda(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})$ or $\mathbf{r} = 5\mathbf{i} - 3\mathbf{j} \pm \lambda(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})$
(or any equivalent vector equation)

   (b) Show that $\mu = -3$

   (c) $\theta = 19.5°$

   (d) Shortest distance = 1 unit

2. (a) $\underline{r} = \begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix} + t\begin{pmatrix} 3 \\ -3 \\ -6 \end{pmatrix}$

   (b) $\overrightarrow{OC} = \begin{pmatrix} 2.5 \\ 2.5 \\ 0 \end{pmatrix}$

   (c) $\overrightarrow{OD} = \overrightarrow{BA} = \underline{a} - \underline{b}$ or $-\overrightarrow{AB}g = \begin{pmatrix} -3 \\ 3 \\ 6 \end{pmatrix}$

   (d) Area = $|\overrightarrow{OC}| \times |\overrightarrow{OD}|$ (o.e.), = $7.5\sqrt{12}$ or $15\sqrt{3}$ or AWRT 26.0

3. (a) $\mathbf{r} = \begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ *or* $\mathbf{r} = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

   (b) $CB = \sqrt{(1^2 + 5^2 + (-10)^2)} = \sqrt{(126)}\ (= 3\sqrt{14} \approx 11.2)$ awrt 11.2

   (c) $\cos\theta = \dfrac{3}{\sqrt{14}} \Rightarrow \theta \approx 36.7°$

   (d) $d = 3\sqrt{5}$ (≈ 6.7)

   (e) $CBX = \dfrac{1}{2} \times BX \times d = \dfrac{1}{2} \times 9 \times 3\sqrt{5} = \dfrac{27\sqrt{5}}{2}\ (\approx 30.2)$