Vector Lines and Planes Worksheet

1. Find the perpendicular distance of $(1,2,1)$ from the plane $x-y+2z=5$

2. Find the coordinates of the point of intersection of the line L and the plane Π where L: $\mathbf{r}=\mathbf{i}-2\mathbf{j}+\mathbf{k}+\lambda(-\mathbf{i}+2\mathbf{k})$ and Π: $\mathbf{r}.(2\mathbf{i}+\mathbf{j}+2\mathbf{k})=6$

3. Two non-parallel planes have equations $\mathbf{r}.(\mathbf{i}+\mathbf{j}+2\mathbf{k})=3$ and $\mathbf{r}.(\mathbf{i}+2\mathbf{j}+\mathbf{k})=4$

Find the equation of the line of intersection in Cartesian form and vector form.

4. Three points $A(1,-1,0)$, $B(0,1,1)$ and $C(4,1,-2)$ lie in a plane Π.

(a) Find the equation of the plane in the form $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}+\mu\mathbf{c}$

(b) Find the equivalent Cartesian form for the equation of this plane.

5. Find in the form $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}+\mu\mathbf{c}$ an equation of the plane Π containing the points $P(-2,1,3)$, $Q(1,1,-1)$ and $R(2,4,-2)$.

Express the equation of this plane in Cartesian form $a_1x+a_2y+a_3z=a_4$

Furthermore, find in the form $(\mathbf{r}-\mathbf{u})\times\mathbf{v}=\mathbf{0}$ the equation of the line through P perpendicular to Π.

6. The point $A(1,2,3)$ lies in the plane Π. The vector $-\mathbf{i}+3\mathbf{j}+2\mathbf{k}$ is perpendicular to Π.

Find: *(a)* The equation of the plane in scalar product form

(b) The equation of the plane in Cartesian form

Solutions

1. Distance between point (1, 2, 1) and plane $x - y + 2z = 5$ is $\dfrac{4}{\sqrt{6}}$ or $\dfrac{2\sqrt{6}}{3}$
2. $x = -1, \quad y = -2, \quad z = 5$
3. $\underline{r} = 2\underline{i} + \underline{j} + \lambda(-3\underline{i} + \underline{j} + \underline{k})$
4. (a) $\underline{r} = (\underline{i} - \underline{j}) + \lambda(-\underline{i} + 2\underline{j} + \underline{k}) + \mu(3\underline{i} + 2\underline{j} - 2\underline{k})$ (b) $-6x + y - 8z = -7$
5. $12x - y + 9z = 2$ $(\underline{r} - (-2\underline{i} + \underline{j} + 3\underline{k})) \times (12\underline{i} - \underline{j} + 9\underline{k}) = 0$
6. (a) $\underline{r}.(-\underline{i} + 3\underline{j} + 2\underline{k}) = 11$ (b) $-x + 3y + 2z = 11$