Higher Portfolio

Higher

Logs and Exponentials

EF1. Logarithms and Exponentials

Section A - Revision Section

This section will help you revise previous learning which is required in this topic.

R1 Revision of Surds and Indices

1. Express each of the following in its simplest form.

(a) $\sqrt{8}$ **(b)** $\sqrt{12}$ **(c)** $\sqrt{50}$

(d) $\sqrt{45}$ **(e)** $3\sqrt{32}$ **(f)** $5\sqrt{40}$

2. Express each of the following with a *rational denominator*.

(a) $\frac{1}{\sqrt{2}}$ **(b)** $\frac{20}{\sqrt{2}}$ **(c)** $\frac{4}{5\sqrt{2}}$

3. Simplify the following writing the answers with positive indices only.

(a) $x^2 \times x^5$ **(b)** $y^{-3} \times y^7$ **(c)** $x^6 \div x^4$

(d) $y^{-3} \div y^{-1}$ **(e)** $(2a^4)^3$ **(f)** $(p^{-4})^{-2}$

(g) $\frac{x^3 \times y^5}{x^2 \times y^2}$ **(h)** $\frac{a^{-1} \times b^3}{a^{-2} \times b}$ **(i)** $5x^3 \times 2x^{-3}$

(j) $\frac{3x^5y^3}{6x^2y^5}$ **(k)** $3p^5 \times 2p^{\frac{1}{2}}$ **(l)** $4r^8 \div 2r^{-2}$

(m) $a^{-\frac{1}{2}} \times a^{\frac{3}{2}}$ **(n)** $r^{-\frac{1}{3}} \times r^{\frac{1}{3}}$ **(o)** $x^2(x^3 + 1)$

4. Write in the form $ax^m + bx^n + \cdots$

(a) $x^{-2}(x - 3)$ **(b)** $\frac{1}{x^2}(x^3 + 2x)$ **(c)** $\frac{1}{x}(3x^2 + 2x)$

(d) $\frac{1}{\sqrt{x}}(\frac{1}{\sqrt{x}} - 1)$ **(e)** $(2x^5 - 3)(x + 4x^{-2})$ **(f)** $(\frac{1}{x} + 1)^2$

(g) $\frac{1}{\sqrt[3]{x^2}}$ **(h)** $\frac{1}{2\sqrt[3]{x}}$ **(i)** $\frac{1}{5\sqrt[4]{x^3}}$

(j) $\frac{3}{5\sqrt[2]{x^5}}$ **(k)** $\frac{2}{7\sqrt[3]{x^2}}$ **(l)** $\frac{6}{\sqrt[3]{x}}$

(m) $\frac{x^2+3x+5}{x}$ **(n)** $\frac{2x^3+x^2+x}{\sqrt{x}}$ **(o)** $\frac{x^4-6x+x^3}{x^2}$

(p) $\frac{x+5}{\sqrt[2]{x^3}}$ **(q)** $\frac{3+x^3}{3x^2}$ **(r)** $\frac{x+2}{\sqrt{x}}$

(s) $\frac{(x+1)(x+2)}{x}$ **(t)** $\frac{(x-1)(x+3)}{5\sqrt[3]{x^4}}$ **(u)** $\frac{3x^2+5x+1}{2x^2}$

R2. Revision of Straight Line

1. Find the gradient and equation each of the straight line between the following points

(a) A$(2,-1)$ and B$(4,7)$ **(b)** X$(-1,1)$ and Y$(5,13)$

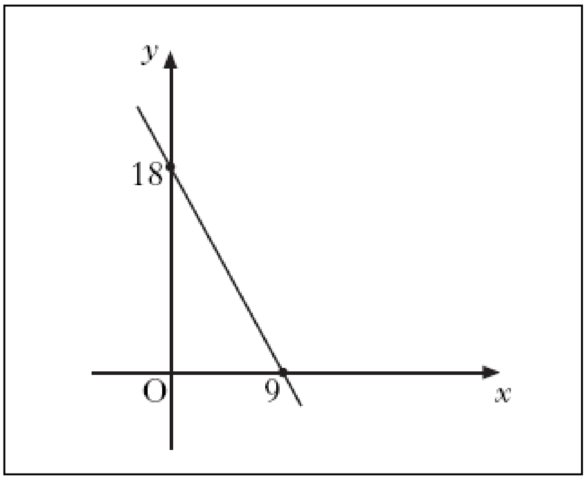
(c) R$(-2,-5)$ and S$(2,-7)$ **(d)** Q$(-1,3)$ and T$(-1,7)$

2. Write down the gradient and y-intercept of the following lines

(a) $y = 3x - 2$ **(b)** $y = x + 4$ **(c)** $y = 4x$

(d) $y = -2x - 1$ **(e)** $y - 2x = 3$ **(f)** $y - x + 3 = 0$

3. Find the equation of the straight line shown



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4. Find the equation of straight line, in the form $y = mx + c$, passing through each point with the given gradient.

(a) gradient = 5 passing through $(4, 7)$

(b) gradient = $\frac{2}{3}$ passing through $(2, -3)$

(c) gradient = -2 passing through $(-5, 1)$

5. Find the equation of the line parallel to $y = 3x + 5$ which passes through the point $(3,\ 7)$.

6. Find the equation of the line parallel to $y = \frac{1}{2}x - 7$ which passes through the point $(-2,\ 4)$.

7. Find the equation of the line parallel to $2x + y = -3$ which passes through the point $(-1, -3)$.

8. Find the equation of the line parallel to $5x - 2y = 7$ which passes through the point $(3,\ 7)$.

9. Find the equation of the line parallel to $2x - y - 7 = 0$ which passes through the point $(-1,\ 7)$.

10. Find the equation of the line parallel to $3x + 5y + 1 = 0$ which passes through the point $(4,\ 0)$.

Exponentials and Logs

Section B - Assessment Standard Section

This section will help you practise for your Assessment Standard Test for Exponentials and Logarithms (Expressions and Functions 1.1)

1. (a) Simplify $log_4 5r + log_4 7s$. (b) Simplify $log_5 3x + log_5 4y$.

(c) Simplify $log_3 2a + log_3 5b$.

2. (a) Express $log_a x^7 - log_a x^3$ in the form $klog_a x$.

(b) Express $log_a p^8 - log_a p^2$ in the form $klog_a p$.

(c) Express $log_a T^9 - log_a T^4$ in the form $klog_a T$.

3. Solve $log_4(x-2) = 1$.

4. Solve $log_5(x+3) = 2$.

5. Solve $log_{16}(x-5) = \frac{1}{2}$.

6. (a) Simplify $log_a 8 - log_a 2$ (b) Simplify $log_5 2 + log_5 50 - log_5 4$

(c) Simplify $3log_4 2 + log_4 8$

7. Solve $log_a x - log_a 7 = log_a 3$ for $x > 0$.

8. Find x if $4log_x 6 - 2log_x 4 = 1$.

9. (a) Simplify $log_b 10 + log_b 4$ (b) Simplify $log_4 320 - log_4 5$

(c) Simplify $2log_3 6 - log_3 4$

10. Given that $log_4 8 + log_4 q = 1$, what is the value of q?

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Section C - Operational Skills Section

This section provides problems with the operational skills associated with Exponentials and Logs

O1	***I can convert between exponential and logarithmic forms.***

1. Given $b = e^t$ which of the following is true:

(a) $log_t b = e$

(b) $log_e b = t$

2. Given $log_n x = y$ which of the following is true:

(a) $n^y = x$

(b) $x^y = n$

O2	***I can use the three main laws of logarithms to simplify expressions, including those involving natural logarithms.***

1. Simplify

a. $log_x 3 + log_x 5 - log_x 7$

b. $log_a 32 - 2log_a 4$

2. Show that (a) $\dfrac{log_3 8}{log_3 2} = 3$. (b) $\dfrac{log_b 9a^2}{log_b 3a} = 2$.

3. If $log_3 x = 2log_3 y - 3log_3 z$ find an expression for x in terms of y and z.

4. Find a if $log_a 64 = \frac{3}{2}$.

5. Simplify $3log_e(2e) - 2log_e(3e)$ expressing your answer in the form $A + log_e B - log_e C$ where A, B and C are whole numbers.

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O3	***I can solve logarithmic and exponential equations using the laws of logarithms.***

1. Given the equation $y = 3 \times 4^x$ find the value of x when $y = 10$ giving your answer to 3 significant figures.

2. Given the equation $A = A_0 e^{-kt}$, find, to 3 significant figures:

(a) A when $A_0 = 5$, $k = 0 \cdot 23$ and $t = 20$.

(b) k when $A = 70$, $A_0 = 35$ and $t = 20$.

(c) t when $A = 1000$, $A_0 = 10$ and $k = 0.01$.

3. Solve $log_4 x + log_4(x + 6) = 2,\ x > 0$.

4. Solve the equation $log_5(3 - 2x) + log_5(2 + x) = 1,\ -2 < x < \frac{3}{2}$.

5. (a) Given that $log_4 x = P$, show that $log_{16} x = \frac{1}{2}P$.

(b) Solve $log_3 x + log_9 x = 12$.

6. The curve with equation $y = log_3(x - 1) - 2 \cdot 2$, where $x > 1$, cuts the x-axis at the point $(a, 0)$. Find the value of a.

7. If $log_4 8 + log_4 q = 1$, find the value of q.

8. Solve the equation $log_2(x + 1) - 2log_2 3 = 3$.

9. Find x if $4log_x 6 - 2log_x 4 = 1$.

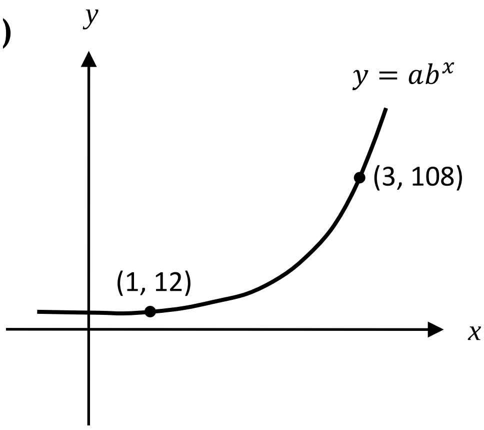
Exponentials and Logs

O4	***I can solve for a and b equations of the following forms, given two pairs of corresponding values of x and y:*** $\log y = b\log x + \log a,\ y = ax^b$ ***and,*** $\log y = x\log b + \log a,\ y = ab^x$

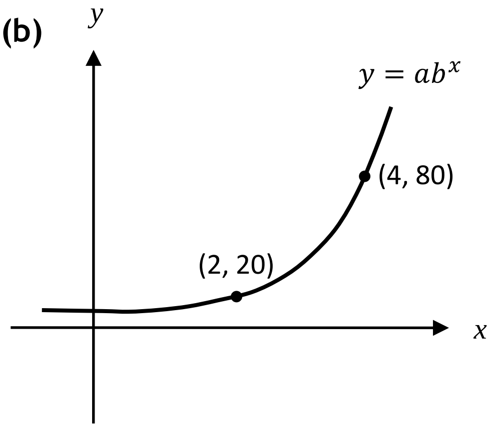
1. Each graph below is in the form $y = ab^x$.

In each case state the values of a and b.

(a)



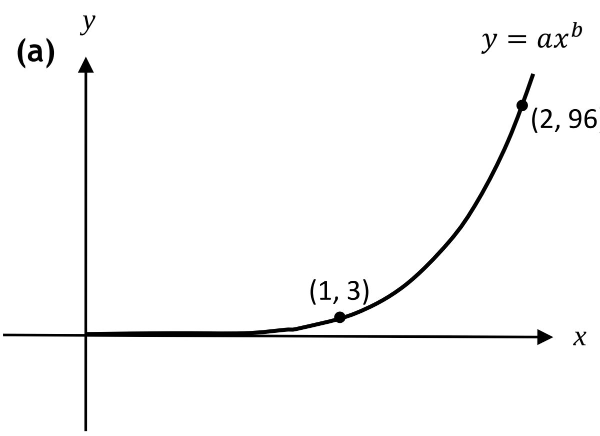
(b)



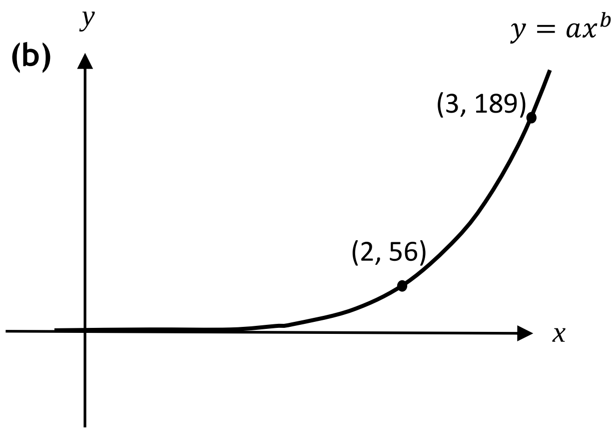
2. Each graph below is in the form $y = ax^b$.

In each case state the values of a and b.

(a)



(b)



3. Given that $y = px^q$, and that $x = 3$ when $y = 162$ and that $x = 5$ when $y = 1250$, find p and q.

4. An investment (£A) grows according to the relationship $A = ab^t$ where t is the time after the investment is made in years. If after 3 years the investment is worth £1157·63 and after 10 years it is worth £1628·89, find the values of a and b.

O5	***I can plot and extract information from straight line graphs with logarithmic axes (axis).***

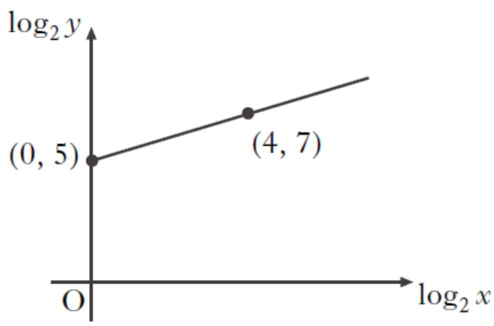
1. Given that $y = kx^n$, where k and n are constants, what would you plot in order to get a straight line graph?

2. Given that $y = Ae^{kx}$ where k and A are constants, what would you plot in order to get a straight line graph?

3. Variables x and y are related by the equation $y = kx^n$

The graph of $log_2 y$ against $log_2 x$ is a straight line through the points $(0, 5)$ and $(4, 7)$, as shown in the diagram.

Find the values of k and n.



4. Two variables x and y satisfy the equation $y = 3 \times 4^x$.

(a) Find the values of a if $(a, 6)$ lies on the graph with equation $y = 3 \times 4^x$.

(b) If $(-\frac{1}{2}, b)$ also lies on the graph, find b.

(c) A graph is drawn of $log_{10} y$ against x. Show that its equations will be of the form $log_{10} y = Px + Q$ and state the gradient of this line.

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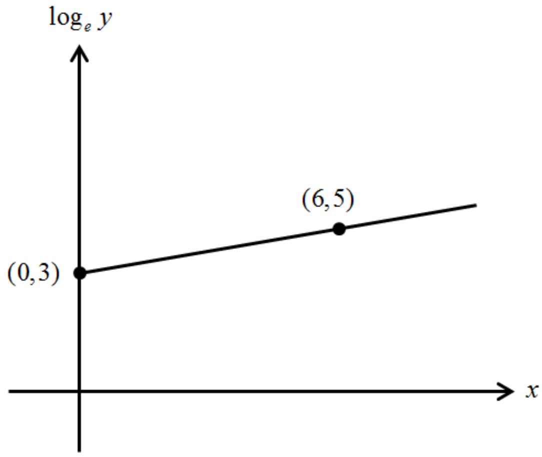
5. Variables x and y are related by the equation $y = Ae^{kx}$

The graph of $log_e y$ against x is a straight line through the points $(0, 3)$ and $(6, 5)$, as shown in the diagram.

Find the values of k and A.



O6 *I can solve logarithmic and exponential equations in real life contexts.*

1. Radium decays exponentially and its half-life is 1600 years.

If A_0 represents the amount of radium in a sample to start with and $A(t)$ represents the amount remaining after t years, then $(t) = A_0e^{-kt}$.

(a) Determine the value of k, correct to 4 significant figures.

(b) Hence find what percentage, to the nearest whole number, of the original amount of radium will be remaining after 4000 years.

2. The concentration of the pesticide, Xpesto, in soil can be modelled by the equation

$$P = P_0e^{-kt}$$

Where

- P_0 is the initial concentration;
- P_t is the concentration at time t;
- t is the time, in days, after the application of the pesticide.

(a) Once in the soil, the half-life of pesticide is the time taken for its concentration to be reduced to one half of its initial value.

If the half-life of *Xpesto* is 25 day, find the value of k to 2 significant figures.

(b) Eighty days after the initial application, what is the percentage decrease in the concentration of *Xpesto*?

3. Before a forest fire was brought under control, the spread of the fire was described by a law of the form $A(t) = A_0 e^{-kt}$ where A_0 is the area covered by the fire when it was first detected and A is the area covered by the fire t hours later.

If it takes one and a half hours for the area of the forest fire to double, find the value of the constant k.

Exponentials and Logs

Answers

Section A

R1

1. **(a)** $2\sqrt{2}$ **(b)** $2\sqrt{3}$ **(c)** $5\sqrt{2}$

(d) $3\sqrt{5}$ **(e)** $12\sqrt{2}$ **(f)** $10\sqrt{10}$

2. **(a)** $\frac{\sqrt{2}}{2}$ **(b)** $10\sqrt{2}$ **(c)** $\frac{2\sqrt{2}}{5}$

3. **(a)** x^7 **(b)** y^4 **(c)** x^2 **(d)** $\frac{1}{y^2}$ **(e)** $8a^{12}$

(f) p^8 **(g)** xy^3 **(h)** ab^2 **(i)** 10 **(j)** $\frac{x^3}{2y^2}$

(k) $6p^{\frac{11}{2}}$ **(l)** $2r^{10}$ **(m)** a **(n)** 1 **(o)** $x^5 + x^2$

4. **(a)** $x^{-1} + 3x^{-2}$ **(b)** $x + 2x^{-1}$ **(c)** $3x + 2$ **(d)** $x^{-1} - x^{-\frac{1}{2}}$

(e) $2a^6 + 8x^3 - 3x - 12x^{-2}$ **(f)** $x^{-2} + 2x^{-1} + 1$ **(g)** $x^{-\frac{2}{3}}$ **(h)** $\frac{1}{2}x^{-\frac{1}{3}}$

(i) $\frac{1}{5}x^{-\frac{3}{4}}$ **(j)** $\frac{3}{5}x^{-\frac{5}{2}}$ **(k)** $\frac{2}{7}x^{-\frac{2}{3}}$ **(l)** $6x^{-\frac{1}{3}}$

(m) $x + 3 + 5x^{-1}$ **(n)** $2x^{\frac{5}{2}} + x^{\frac{3}{2}} + x^{\frac{1}{2}}$

(o) $x^2 - 6x^{-1} + x$ **(p)** $x^{-\frac{1}{2}} + 5x^{-\frac{3}{2}}$

(q) $x^{-2} + \frac{x}{3}$ **(r)** $x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}$ **(s)** $x + 3 + 2x^{-1}$

(t) $\frac{1}{5}x^{\frac{2}{3}} + \frac{2}{5}x^{-\frac{1}{3}} - \frac{3}{5}x^{-\frac{4}{3}}$ **(u)** $\frac{3}{2} + \frac{5}{2}x^{-1} + \frac{1}{2}x^{-2}$

R2

1. **(a)** $m = 4,\ y = 4x - 9$ **(b)** $m = 2,\ y = 2x + 3$

(c) $m = -\frac{1}{2},\ y = -\frac{1}{2}x - 6$ **(d)** undefined, $x = -1$

2. **(a)** $m = 3,$ y-intercept (0, -2) **(b)** $m = 1,$ y-intercept (0, 4)

(c) $m = 4,$ y-intercept (0, 0) **(d)** $m = -2,$ y-intercept (0, -1)

(e) $m = 2,$ y-intercept (0, 3) **(f)** $m = 1,$ y-intercept (0, -3)

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3. $y = 18 - 2x$

4. **(a)** $y = 5x - 13$ **(b)** $y = \frac{2}{3}x - \frac{13}{3}$ (c) $y = -2x - 9$

5. $y = 3x - 2$ **6.** $y = \frac{1}{2}x + 5$ **7.** $y = -2x - 5$

8. $y = \frac{5}{2}x - \frac{1}{2}$ **9.** $y = 2x + 9$ **10.** $y = -\frac{3}{5}x + \frac{12}{5}$

Section B Answers

1. **(a)** $log_4 35rs$ **(b)** $log_5 12xy$ **(c)** $log_3 10ab$

2. **(a)** $4log_a x$ **(b)** $6log_a p$ **(c)** $5log_a T$

3. $x = 6$ **4.** $x = 22$ **5.** $x = 9$

6. **(a)** $log_a 4$ **(b)** 2 **(c)** 3

7. $x = 21$ **8.** $x = 81$

9. **(a)** $log_b 40$ **(b)** 3 **(c)** 2

10. $q = \frac{1}{2}$

Section C

O1

1. (b) **2.** (a)

O2

1. **(a)** $log_x \frac{15}{7}$ **(b)** $log_a 2$ **2.** $(a), (b) Proof$ **3.** $x = \frac{y^2}{z^3}$

4. $a = 16$ **5.** $1 + log_e 8 - log_e 9$

O3

1. $x = 0 \cdot 868$ **2.** **(a)** $A = 0 \cdot 0503$ **(b)** $k = -0.0347$ **(c)** $t = 461$

3. $x = 2$ (discard $x = -8$) **4.** $x = -1$ and $x = \frac{1}{2}$

5. **(a)** $Proof$ **(b)** $x = 3^8$ **6.** $a = 3^{2 \cdot 2} + 1$

7. $q = \frac{1}{2}$ **8.** $x = 71$ **9.** $x = 81$

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O4

1. **(a)** $a = 4,\ b = 3$ **(b)** $a = 5,\ b = 2$

2. **(a)** $a = 3,\ b = 5$ **(b)** $a = 7,\ b = 3$ **3.** $p = 2,\ q = 4$

4. $a = 1000,\ b = 1 \cdot 05$

O5

1. $log_a y$ against $log_a x$ **2.** $log_a y$ against x

3. $k = 32,\ m = \frac{1}{2}$

4. **(a)** $a = \frac{1}{2}$ **(b)** $b = \frac{3}{2}$ **(c)** $m = log_{10} 4$ and $(0, log_{10} 3)$

5. $A = e^3,\ k = \frac{1}{3}$

O6

1. **(a)** $k = 0 \cdot 0004332$ **(b)** 18% **2.** **(a)** $k = 0 \cdot 028$ **(b)** $89 \cdot 4\%$

3. $k = -0 \cdot 462$