FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$

represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$

represents a circle centre (a, b) and radius r .

Scalar Product: $\quad a.b = |a||b|\cos\theta$, where θ is the angle between a and b

$\quad$ or $\quad a.b = a_1b_1 + a_2b_2 + a_3b_3$ where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

Trigonometric formulae:
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

derivatives:

Table of standard

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\dfrac{1}{a}\cos ax + C$
$\cos ax$	$\dfrac{1}{a}\sin ax + c$

integrals:

Algebra Unit Practice

1.(a)(i) Simplify $\log_5 7a + \log_5 2b$

  (ii) Simplify $\log_6 4b + \log_6 3c$

  (iii) Simplify $\log_4 9d + \log_4 5a$

  (iv) Simplify $\log_8 7y + \log_8 3s$ [1]

1.(b)(i) Express $\log_a x^3 - \log_a x^2$ in the form $k \log_a x$

  (ii) Express $\log_a x^5 - \log_a x^2$ in the form $k \log_a x$

  (iii) Express $\log_a x^3 - \log_a x$ in the form $k \log_a x$

  (iv) Express $\log_a x^6 - \log_a x^5$ in the form $k \log_a x$ [2]

  2. (a) Solve $\log_2(x - 5) = 5$

  (b) Solve $\log_5(y + 2) = 2$

  (c) Solve $\log_3(z - 1) = 3$

  (d) Solve $\log_3(d + 2) = 2$

3. The diagram shows the graph of $y = f(x)$ with a maximum turning point   at (−2, 3) and a minimum turning point at (1, −2).
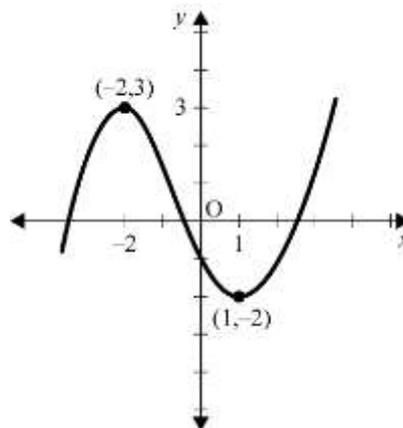
  (a) Sketch the graph of $y = f(x + 3) - 2$

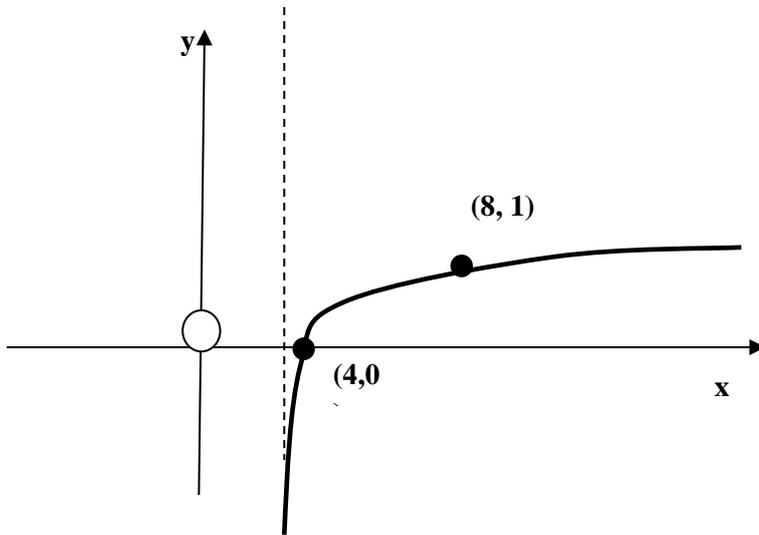 **(b)** Sketch the graph of $y = f(x + 4) - 3$

  (c) Sketch the graph of $y = f(x - 2) + 3$

  (d) Sketch the graph of $y = f(x - 3) - 6$

[[3]]

4. **(a)** The diagram shows the graph of $y = \log_b(x - a)$

Determine the values of a and b

(8, 1)

(4,0

y

x

4.(b) The diagram shows the graph of $y = \log_b(x - a)$

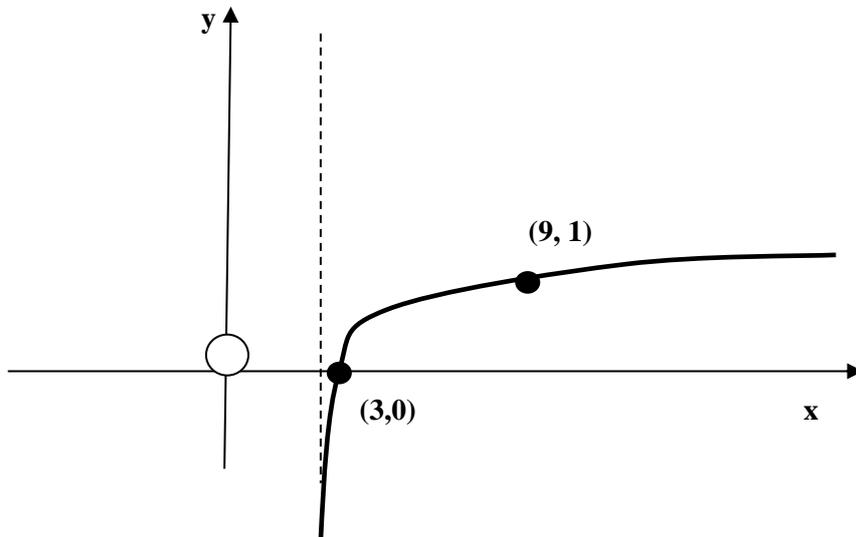Determine the values of a and b
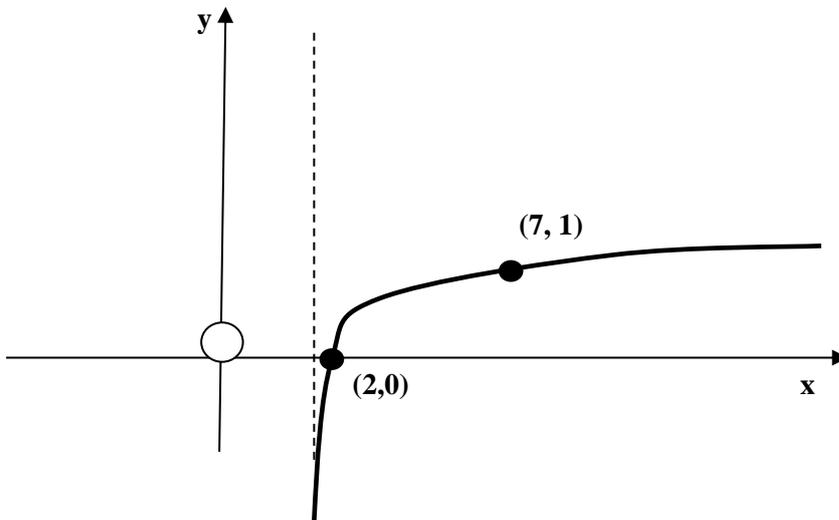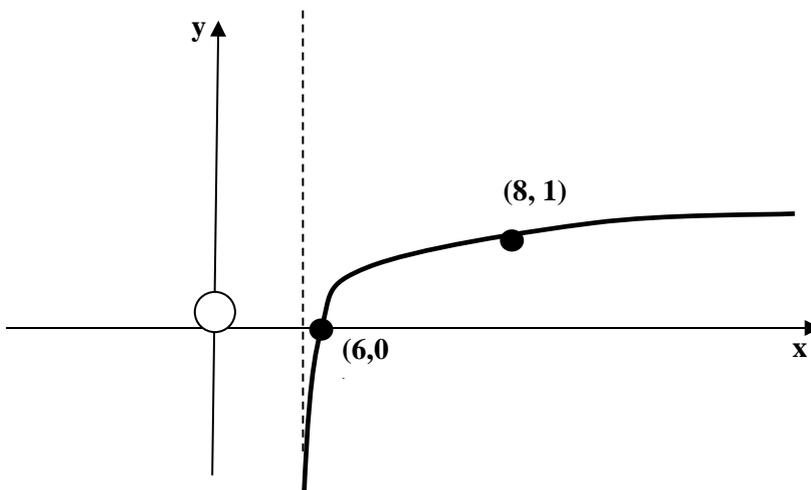

(9, 1)

(3,0)

y

x

4.(c)   The diagram shows the graph of $y = \log_b(x - a)$
Determine the values of a and b

4.(d)   The diagram shows the graph of $y = \log_b(x - a)$
Determine the values of a and b

[2]

5.(a) The functions f and g defined on suitable domains, are given by

$f(x) = 2x + 5$ and $g(x) = \sqrt{x}$.

A third function $h(x)$ is defined as $h(x) = g(f(x))$.

(i) Find an expression for $h(x)$.

(ii) Explain why the largest domain for $h(x)$ is given by $x \geq -2 \cdot 5$.

(b) The functions f and g defined on suitable domains, are given by

$f(x) = 3x + 6$ and $g(x) = \sqrt{x}$.

A third function $h(x)$ is defined as $h(x) = g(f(x))$.

(i) Find an expression for $h(x)$.

(ii) Explain why the largest domain for $h(x)$ is given by $x \geq -2$

(c) The functions f and g defined on suitable domains, are given by

$f(x) = 4x + 10$ and $g(x) = \sqrt{x}$.

A third function $h(x)$ is defined as $h(x) = g(f(x))$.

(i) Find an expression for $h(x)$.

(ii) Explain why the largest domain for $h(x)$ is given by $x \geq -2 \cdot 5$

(d) The functions f and g defined on suitable domains, are given by

$f(x) = 2x + 7$ and $g(x) = \sqrt{x}$.

A third function $h(x)$ is defined as $h(x) = g(f(x))$.

(i) Find an expression for $h(x)$.

(ii) Explain why the largest domain for $h(x)$ is given by $x \geq -3 \cdot 5$

[2,#2.2]

6. (a) A function is given by f (x) = 6x + 7. Find the inverse function f⁻¹(x).

(b) A function is given by f (x) = 5x + 8. Find the inverse function f⁻¹(x).

(c) A function is given by f (x) = 8x + 9. Find the inverse function f⁻¹(x).

(d) A function is given by f (x) = 2x + 1. Find the inverse function f⁻¹(x).

[3]

7a) A function f is defined by the formula $f(x) = x^3 - 3x^2 - 6x + 8$
where x is a real number.

    i) Show that $x - 1$ is a factor of $f(x)$.

    ii) Hence factorise $f(x)$ fully.

    iii) Solve $f(x) = 0$.

b) A function f is defined by the formula $f(x) = x^3 - 4x^2 + x + 6$
where x is a real number.

    i) Show that $x - 3$ is a factor of $f(x)$.

    ii) Hence factorise $f(x)$ fully.

    iii) Solve $f(x) = 0$.

c) A function f is defined by the formula
$f(x) = x^3 - 2x^2 - 11x + 12$ where x is a real number.

    i) Show that $x - 1$ is a factor of $f(x)$.

    ii) Hence factorise $f(x)$ fully.

    iii) Solve $f(x) = 0$.

d) A function f is defined by the formula
$f(x) = x^3 + 9x^2 + 24x + 16$ where x is a real number.

    i) Show that $x + 4$ is a factor of $f(x)$.

    ii) Hence factorise $f(x)$ fully.

    iii) Solve $f(x) = 0$.

8.a) Solve the cubic equation $f(x) = 0$ given the following:

- when $f(x)$ is divided by $x + 2$, the remainder is zero
- when the graph of $y = f(x)$ is drawn, it passes through the point (-6, 0)
- $(x + 3)$ is a factor of $f(x)$.

b) Solve the cubic equation $f(x) = 0$ given the following:

- when $f(x)$ is divided by $x + 4$, the remainder is zero
- when the graph of $y = f(x)$ is drawn, it passes through the point (2, 0)
- $(x - 5)$ is a factor of $f(x)$.

c) Solve the cubic equation $f(x) = 0$ given the following:

- when $f(x)$ is divided by $x + 7$, the remainder is zero
- when the graph of $y = f(x)$ is drawn, it passes through the point (-1, 0)
- $(x + 11)$ is a factor of $f(x)$.

d) Solve the cubic equation $f(x) = 0$ given the following:

- when $f(x)$ is divided by $x - 6$, the remainder is zero
- when the graph of $y = f(x)$ is drawn, it passes through the point (10, 0)
- $(x - 12)$ is a factor of $f(x)$.

9 a) The graph of the function $f(x) = kx^2 + 3x + 3$ touches the x-axis at one point.

What is the range of values for k ?

b) The graph of the function $f(x) = kx^2 + 2x - 5$ touches the x-axis at two points.

What is the range of values for k ?

c) The graph of the function $f(x) = kx^2 - 8x + 2$ does not touch the x-axis.

What is the range of values for k ?

d) The graph of the function $f(x) = kx^2 - 2x + 7$ touches the x-axis at one point.

What is the range of values for k ?

10. (a) A sequence is defined by the recurrence relation $u_{n+1} = mu_n + c$

Where m and c are constants.

It is known that $u_1 = 2, u_2 = 4$ and $u_3 = 14$.

Find the recurrence relation described by the sequence and use it to find the value of u_6.

(b) A sequence is defined by the recurrence relation $u_{n+1} = mu_n + c$

Where m and c are constants.

It is known that $u_1 = 10, u_2 = 35$ and $u_3 = 47.5$.

Find the recurrence relation described by the sequence and use it to find the value of u_6.

(c) A sequence is defined by the recurrence relation $u_{n+1} = mu_n + c$

Where m and c are constants.

It is known that $u_1 = 5, u_2 = 9.5$ and $u_3 = 20.75$

Find the recurrence relation described by the sequence and use it to find the value of u_6.

(d) A sequence is defined by the recurrence relation $u_{n+1} = mu_n + c$

Where m and c are constants.

It is known that $u_1 = 12, u_2 = 10$ and $u_3 = 8$.

Find the recurrence relation described by the sequence and use it to find the value of u_6.

11. (a) On a particular day at 07:00, a doctor injects a first dose of 300mg of medicine into a patients bloodstream. The doctor then continues to administer the medicine in this way at 07:00 each day.

The doctor knows that at the end of the 24-hour period after an injection, the amount of medicine in the bloodstream will only be 20% of what it was at the start.

(i) Set up a recurrence relation which shows the amount of medicine in the bloodstream immediately after an injection.

The patient will overdose if the amount of medicine in their bloodstream exceeds 390mg.

(ii) In the long term, if a patient continues with this treatment, is there a danger they will overdose?
Explain your answer.

(b) On a particular day at 06:00, a doctor injects a first dose of 150mg of medicine into a patients bloodstream. The doctor then continues to administer the medicine in this way at 06:00 each day.

The doctor knows that at the end of the 24-hour period after an injection, the amount of medicine in the bloodstream will only be 10% of what it was at the start.

(i) Set up a recurrence relation which shows the amount of medicine in the bloodstream immediately after an injection.

The patient will overdose if the amount of medicine in their bloodstream exceeds 170mg.

(ii) In the long term, if a patient continues with this treatment, is there a danger they will overdose?
Explain your answer.

(c) On a particular day at 09:00, a doctor injects a first dose of 50mg of medicine into a patients bloodstream. The doctor then continues to administer the medicine in this way at 09:00 each day.

The doctor knows that at the end of the 24-hour period after an injection, the amount of medicine in the bloodstream will only be 25% of what it was at the start.

(i)Set up a recurrence relation which shows the amount of
medicine in the bloodstream immediately after an injection.

The patient will overdose if the amount of medicine in their
bloodstream exceeds 70mg.

(ii)   In the long term, if a patient continues with this treatment, is
there a danger they will overdose?
Explain your answer.

(d)   On a particular day at 08:30, a doctor injects a first dose of
225mg of medicine into a patients bloodstream. The doctor then
continues to administer the medicine in this way at 08:30 each
day.

The doctor knows that at the end of the 24-hour period after an
injection, the amount of medicine in the bloodstream will only
be17% of what it was at the start.

(i)   Set up a recurrence relation which shows the amount of
medicine in the bloodstream immediately after an injection.

The patient will overdose if the amount of medicine in their
bloodstream exceeds 275mg.

(ii)   In the long term, if a patient continues with this treatment, is
there a danger they will overdose?
Explain your answer.

Answers

1(a) (i) $\log_5 14ab$ (ii) $\log_6 12bc$ (iii) $\log_4 45ad$ (iv) $\log_8 21sy$

1(b) (i) $\log_a x$ (ii) $3\log_a x$ (iii) $2\log_a x$ (iv) $\log ax$

2. (a) $x = 37$ (b) $y = 23$

     (c) $z = 28$ (d) $d = 7$

3. Correct x coordinates, correct y coordinates and correct shape and annotation

4. (a) $a = 3, b = 5$ (b) $a = 2, b = 7$ (c) $a = 1, b = 6$ (d) $a = 5, b = 3$

5. (a) $g(f(x)) = \sqrt{2x + 5}$,   Square root of a negative cannot be found

     (b) $g(f(x)) = \sqrt{3x + 6}$ (c) $g(f(x)) = \sqrt{4x + 10}$ (d) $g(f(x)) = \sqrt{2x + 7}$

6. (a) $f^{-1}(x) = \frac{x-7}{6}$ (b) $f^{-1}(x) = \frac{x-8}{5}$

     (c) $f^{-1}(x) = \frac{x-9}{8}$ (d) $f^{-1}(x) = \frac{x-1}{2}$

7a) **i)** remainder $= 0$ **ii)** $(x - 1)(x + 2)(x - 4)$

        iii) $x = 1, x = -2, x = 4$

    b) **i)** remainder $= 0$ **ii)** $(x - 3)(x - 2)(x + 1)$

        iii) $x = 3, x = 2, x = -1$

    c) **i)** remainder $= 0$ **ii)** $(x - 1)(x - 4)(x + 3)$

        iii) $x = 1, x = 4, x = -3$

    d) **i)** remainder $= 0$ **ii)** $(x + 4)(x + 4)(x + 1)$

        iii) $x = -4, x = -4, x = -1$

8. **a)** $x = -2, x = -3, x = -6$ **b)** $x = 2, x = 5, x = -4$

    c) $x = -1, x = -7, x = -11$ **d)** $x = 6, x = 10, x = 12$

9. **a)** $k = \frac{3}{4}$ **b)** $k > -\frac{1}{5}$ **c)** $k > 8$ **(d)** $k = \frac{1}{7}$

10 (a) $U_{n+1} = 5U_n - 6$ (ii) $U_6 = 1564$

   (b) $U_{n+1} = 0 \cdot 5U_n + 30$ (ii) $U_6 = 58 \cdot 44$

   (c) $U_{n+1} = 2 \cdot 5U_n - 3$ (ii) $U_6 = 294 \cdot 97$

   (d) $U_{n+1} = U_n - 2$ (ii) $U_6 = 2$

11 (a) $U_{n+1} = 0 \cdot 2U_n + 300$ (ii) $L = 375 \therefore$ No danger

   (b) $U_{n+1} = 0 \cdot 1U_n + 150$ (ii) $L = 166.67 \therefore$ No danger

   (c) $U_{n+1} = 0 \cdot 25U_n + 50$ (ii) $L = 66.67 \therefore$ No danger

   (d) $U_{n+1} = 0 \cdot 17U_n + 225$ (ii) $L = 271\frac{7}{83} \therefore$ No danger