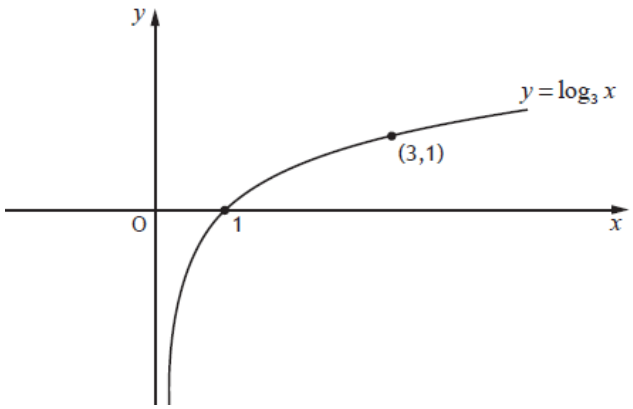
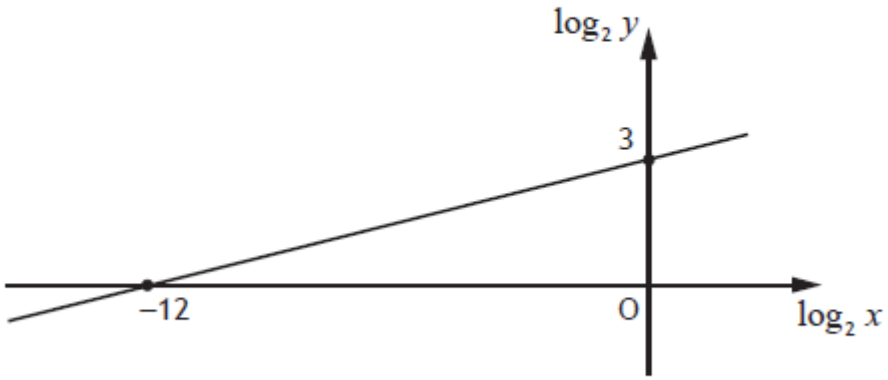
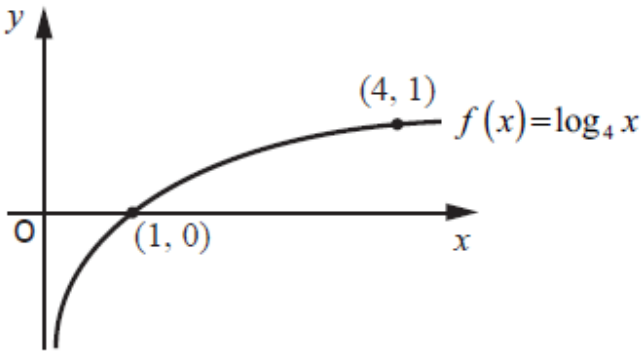
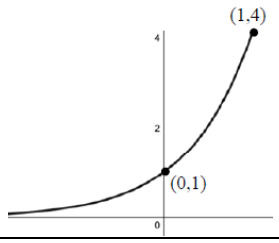
Higher : Exponential & Logs

2018 P1 Q6	Find the value of $\log_5 250 - \frac{1}{3}\log_5 8$.	3
Ans	3	
2018 P1 Q11	The diagram shows the curve with equation $y = \log_3 x$.  (a) On the diagram in your answer booklet, sketch the curve with equation $y = 1 - \log_3 x$. (b) Determine the exact value of the x-coordinate of the point of intersection of the two curves.	2 3
Ans	(a) Curve reflected in x-axis and moved vertically up by 1 unit. (b) $\sqrt{3}$	
2018 P2 Q11	A supermarket has been investigating how long customers have to wait at the checkout. During any half hour period, the percentage, P%, of customers who wait for less than t minutes, can be modelled by $P = 100\left(1 - e^{kt}\right)$, where k is a constant. (a) If 50% of customers wait for less than 3 minutes, determine the value of k. (b) Calculate the percentage of customers who wait for 5 minutes or longer.	4 2
Ans	(a) k = -0·231 (b) 31·5%	

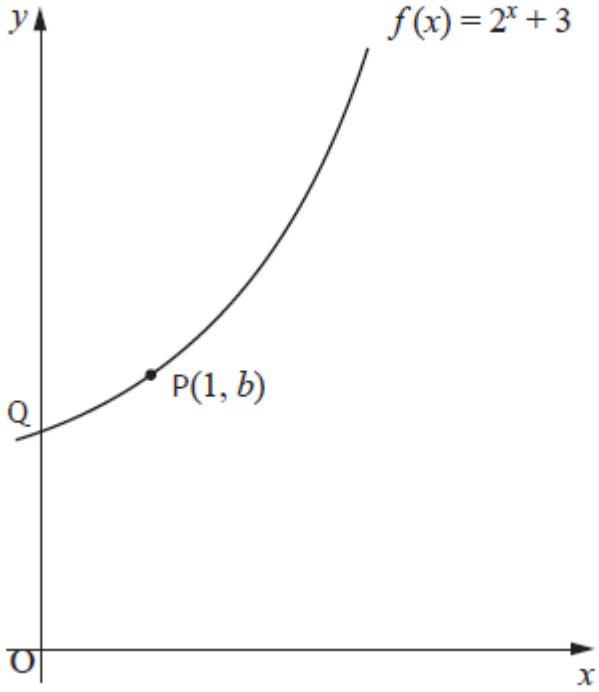
2017 P1 Q12	Given that $\log_a 36 - \log_a 4 = \frac{1}{2}$, find the value of a.	3
Ans	$a = 81$	
2017 P2 Q9	Two variables, x and y, are connected by the equation $y = kx^n$. The graph of $\log_2 y$ against $\log_2 x$ is a straight line as shown.  Find the values of k and n.	5
Ans	$k = 8$, $n = \frac{1}{4}$	
2016 P1 Q10	The diagram below shows the graph of the function $f(x) = \log_4 x$, where $x > 0$.  The inverse function, f^{-1}, exists. On the diagram in your answer booklet, sketch the graph of the inverse function.	2

Ans		
2016 P1 Q14	(a) Evaluate $\log_5 25$. (b) Hence solve $\log_4 x + \log_4 (x-6) = \log_5 25$, where $x > 6$.	1 5
Ans	*(a) 2 (b) x = 8*	
2016 P2 Q6	Scientists are studying the growth of a strain of bacteria. The number of bacteria present is given by the formula $$B(t) = 200\,e^{0{\cdot}107t},$$ where *t* represents the number of hours since the study began. (a) State the number of bacteria present at the start of the study. (b) Calculate the time taken for the number of bacteria to double.	 1 4
Ans	*(a) 200 (b) t = 6.478…*	
2015 NH P1 Q6	Evaluate $\log_6 12 + \frac{1}{3}\log_6 27$.	3
Ans	*2*	

2015 NH P1 Q13

The function $f(x) = 2^x + 3$ is defined on $\mathbb{R}$, the set of real numbers.

The graph with equation $y = f(x)$ passes through the point P$(1, b)$ and cuts the y-axis at Q as shown in the diagram.



(a) What is the value of b? 1

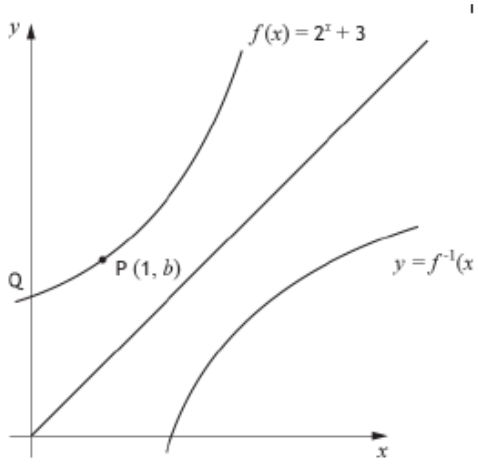
(b) (i) Copy the above diagram.

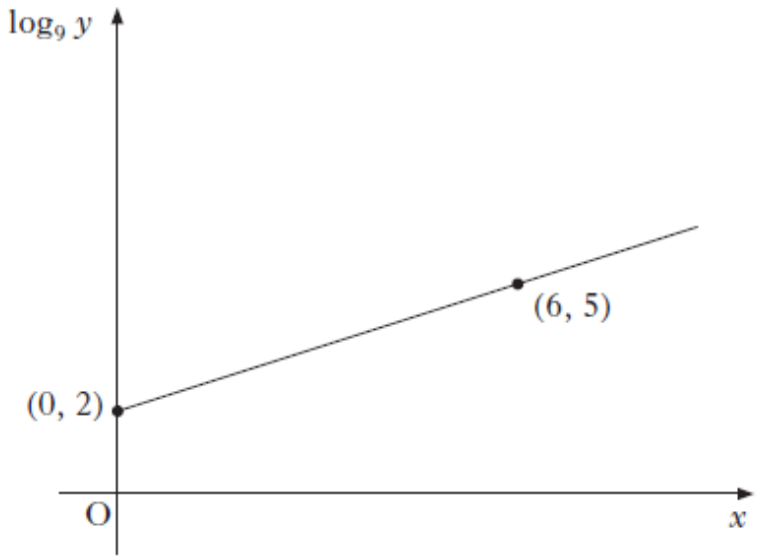
On the same diagram, sketch the graph with equation $y = f^{-1}(x)$. 1

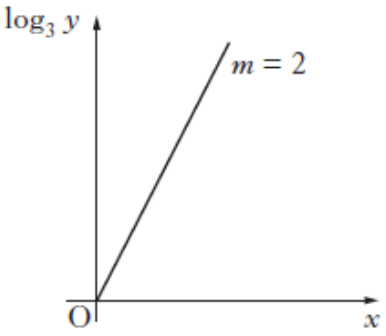
(ii) Write down the coordinates of the images of P and Q. 3

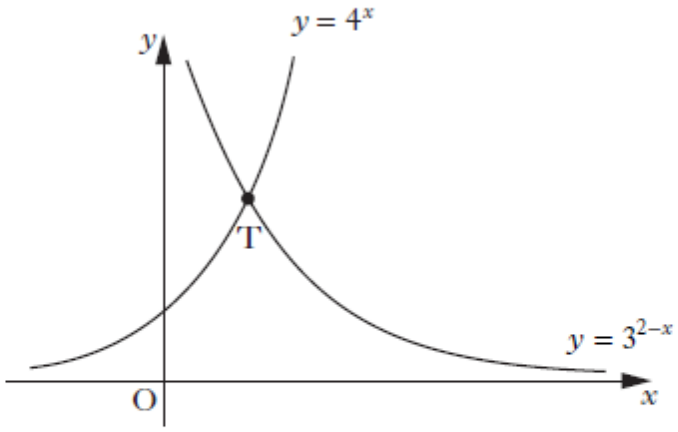
(c) R $(3,11)$ also lies on the graph with equation $y = f(x)$.

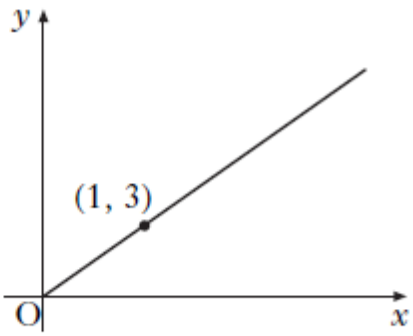
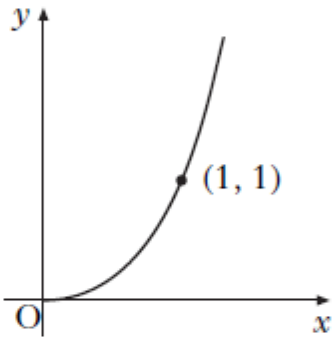
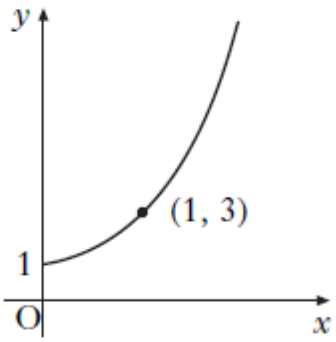
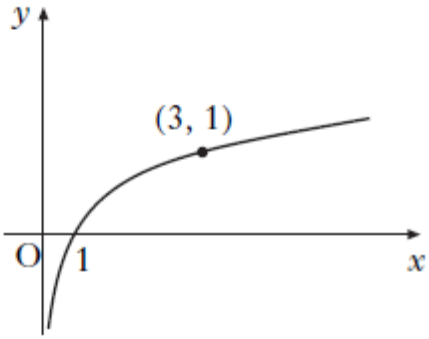
Find the coordinates of the image of R on the graph with equation $y = 4 - f(x + 1)$. 2

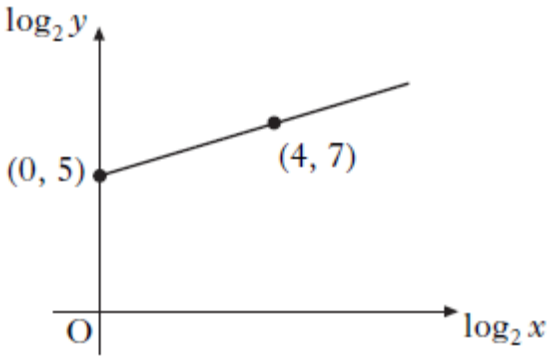
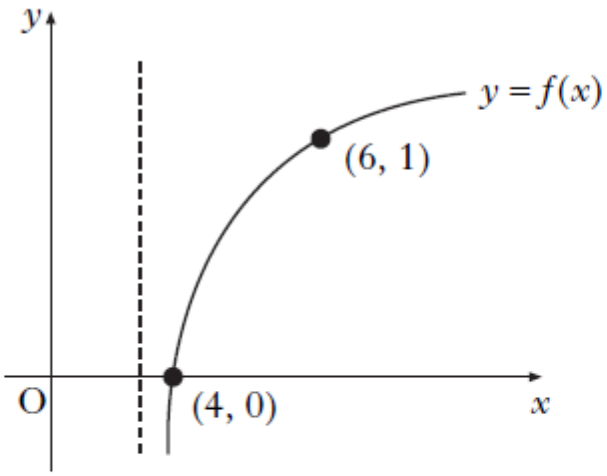
Ans	*(a) b = 5* *(b)(i)*  *(ii) Q'(4, 0) , P'(5, 1)*	
2015 P1 Q16	If $e^{4t} = 6$, find an expression for t. A $t = \log_e \frac{3}{2}$ B $t = \frac{\log_e 6}{4}$ C $t = \frac{6}{\log_e 4}$ D $t = \frac{\log_e 6}{\log_e 4}$	2
Ans	*B*	
2015 P1 Q23	Solve $\log_2(3x + 7) = 3 + \log_2(x - 1),\ x > 1$.	4
Ans	*x = 3*	
2014 P1 Q3	If $\log_4 12 - \log_4 x = \log_4 6$, what is the value of x? A 2 B 6 C 18 D 72	2
Ans	*A*	

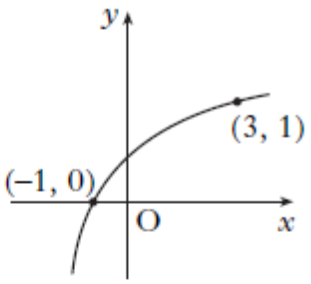
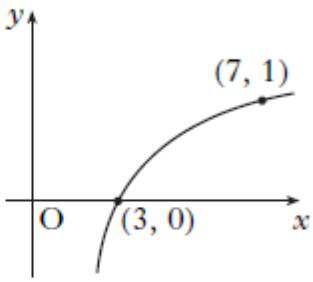
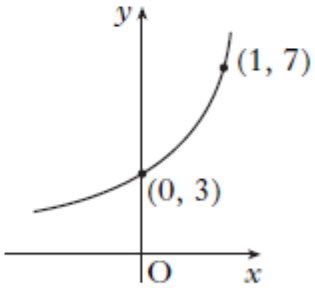
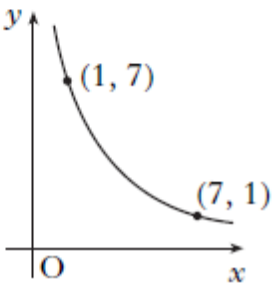
2014 P1 Q20	Evaluate $2 - \log_5 \frac{1}{25}$. A -3 B 0 C $\frac{3}{2}$ D 4	2
Ans	*D*	
2014 P1 Q24	Two variables, x and y, are related by the equation $y = ka^x$. When $\log_9 y$ is plotted against x, a straight line passing through the points $(0, 2)$ and $(6, 5)$ is obtained, as shown in the diagram.  Find the values of k and a.	5
Ans	*a = 3 , k = 81*	

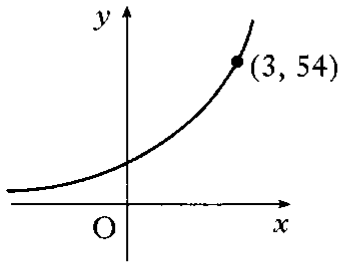
2013 P1 Q20	The graph of $\log_3 y$ plotted against x is a line through the origin with gradient 2, as shown.  Express y in terms of x. A $y = 2x$ B $y = 9x$ C $y = 6^x$ D $y = 9^x$	2
Ans	*D*	
2013 P2 Q5	Solve the equation $\log_5(3-2x)+\log_5(2+x)=1$, where x is a real number.	4
Ans	*x = 0·5 , x = -1*	
2013 P2 Q9	The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation $P_t = P_0 e^{-kt}$ where: • P_0 is the initial concentration; • P_t is the concentration at time t; • t is the time, in days, after the application of the pesticide. (*a*) Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value. If the half-life of *Xpesto* is 25 days, find the value of k to 2 significant figures. (*b*) Eighty days after the initial application, what is the percentage decrease in concentration of *Xpesto*?	4 3

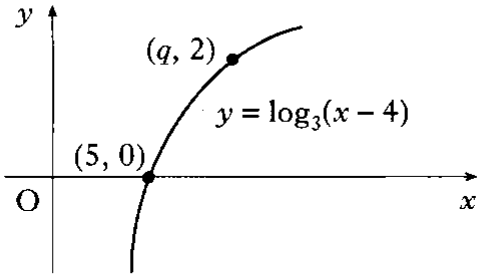
Ans	*(a) k = 0·028 (b) 89%*	
2012 P1 Q20	Simplify $\frac{\log_b 9a^2}{\log_b 3a}$, where $a > 0$ and $b > 0$. A 2 B $3a$ C $\log_b 3a$ D $\log_b(9a^2 - 3a)$	2
Ans	*A*	
2012 P2 Q7	The diagram shows the curves with equations $y = 4^x$ and $y = 3^{2-x}$.  The graphs intersect at the point T. (*a*) Show that the x – coordinate of T can be written in the form $\frac{\log_a p}{\log_a q}$, for all $a > 1$. (*b*) Calculate the y – coordinate of T.	6 2
Ans	*(a) Proof (b) y = 3·4*	

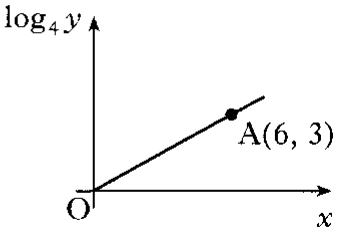
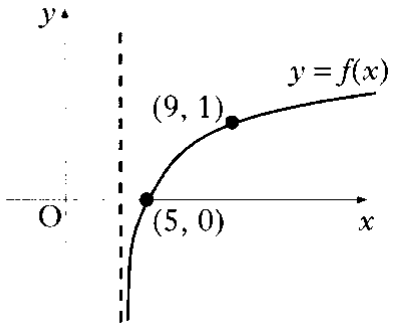
2011 P1 Q19	Which of the following diagrams represents the graph with equation $\log_3 y = x$? A  B  C  D  	2
Ans	*C*	

2011 P2 Q5	Variables x and y are related by the equation $y = kx^n$. The graph of $\log_2 y$ against $\log_2 x$ is a straight line through the points (0, 5) and (4, 7), as shown in the diagram. Find the values of k and n. 	5
Ans	*k =32 , n =0·5*	
2010 P1 Q19	The diagram shows the graph of $y = f(x)$ where f is a logarithmic function.  What is $f(x)$? A $f(x) = \log_6(x - 3)$ B $f(x) = \log_3(x + 3)$ C $f(x) = \log_3(x - 3)$ D $f(x) = \log_6(x + 3)$	2
Ans	C	
2010 P2 Q7	(*a*) Given that $\log_4 x = P$, show that $\log_{16} x = \frac{1}{2}P$. (*b*) Solve $\log_3 x + \log_9 x = 12$.	3 3
Ans	*(b) $x = 9^4$ (= 6561)*	

2009 P1 Q10	Which of the following graphs has equation $y = \log_5(x - 2)$? A  B  C  D  	2
Ans	B	
2009 P2 Q3b	Solve $\log_2(x + 3) + \log_2(x^2 + 5x - 4) = 3$.	5
Ans	*x = 1*	

2009 P2 Q6	The size of the human population, N, can be modelled using the equation $N = N_0e^{rt}$ where N_0 is the population in 2006, t is the time in years since 2006, and r is the annual rate of increase in the population. (*a*) In 2006 the population of the United Kingdom was approximately 61 million, with an annual rate of increase of 1·6%. Assuming this growth rate remains constant, what would be the population in 2020? (*b*) In 2006 the population of Scotland was approximately 5·1 million, with an annual rate of increase of 0·43%. Assuming this growth rate remains constant, how long would it take for Scotland's population to double in size?	2 3
Ans	*(a) 76 million (b) t = 161·2 years*	
2008 P1 Q19	The diagram shows part of the graph whose equation is of the form $y = 2m^x$. What is the value of m?  A 2 B 3 C 8 D 18	2
Ans	B	

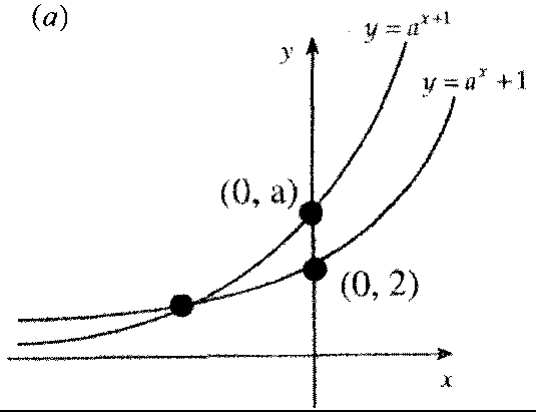
2008 P1 Q20	The diagram shows part of the graph of $y = \log_3(x - 4)$. The point $(q, 2)$ lies on the graph.  What is the value of q? A 6 B 7 C 8 D 13	2
Ans	D	
2008 P1 Q23	Functions f, g and h are defined on suitable domains by $f(x) = x^2 - x + 10$, $g(x) = 5 - x$ and $h(x) = \log_2 x$. (*a*) Find expressions for $h(f(x))$ and $h(g(x))$. (*b*) Hence solve $h(f(x)) - h(g(x)) = 3$.	3 5
Ans	(*a*) $h(f(x)) = \log_2(x^2 - x + 10)$ $h(g(x)) = \log_2(5 - x)$ (*b*) $x = 3, -10$	
2007 P2 Q8	The curve with equation $y = \log_3(x - 1) - 2{\cdot}2$, where $x > 1$, cuts the x-axis at the point $(a, 0)$. Find the value of a.	4
Ans	12·2	
2007 P2 Q11	Two variables x and y satisfy the equation $y = 3 \times 4^x$. (*a*) Find the value of a if $(a, 6)$ lies on the graph with equation $y = 3 \times 4^x$. (*b*) If $(-\frac{1}{2}, b)$ also lies on the graph, find b. (*c*) A graph is drawn of $\log_{10} y$ against x. Show that its equation will be of the form $\log_{10} y = Px + Q$ and state the gradient of this line.	1 1 4

Ans	(*a*) $a = \frac{1}{2}$ (*b*) $b = \frac{3}{2}$	(*c*) $y = 3 \times 4^x$ $\log_{10} y = \log_{10} 3 + \log_{10}(4^x)$ $= \log_{10} 3 + x \log_{10}(4)$ So gradient of line = $\log_{10}(4)$
2006 P1 Q10	Two variables, x and y, are connected by the law $y = a^x$. The graph of $\log_4 y$ against x is a straight line passing through the origin and the point A(6, 3). Find the value of a.	 4
Ans	$a = 2$	
2006 P2 Q11	It is claimed that a wheel is made from wood which is over 1000 years old. To test this claim, carbon dating is used. The formula $A(t) = A_0 e^{-0.000124t}$ is used to determine the age of the wood, where A_0 is the amount of carbon in any living tree, $A(t)$ is the amount of carbon in the wood being dated and t is the age of the wood in years. For the wheel it was found that $A(t)$ was 88% of the amount of carbon in a living tree. Is the claim true?	5
Ans	$t = 1031$ years so claim valid	
2005 P1 Q7	The function f is of the form $f(x) = \log_b (x - a)$. The graph of $y = f(x)$ is shown in the diagram. (*a*) Write down the values of a and b. (*b*) State the domain of f.	 2 1
Ans	(*a*) $a = 4$ $b = 5$ (*b*) domain is $x > 4$	
2005 P2 Q7	Solve the equation $\log_4(5 - x) - \log_4(3 - x) = 2$, $x < 3$.	4
Ans	$x = \frac{43}{15}$	

2005 P2 Q9	The value V (in £ million) of a cruise ship t years after launch is given by the formula $V = 252e^{-0.06335t}$. (*a*) What was its value when launched? (*b*) The owners decide to sell the ship once its value falls below £20 million. After how many years will it be sold?	1 4
Ans	(*a*) 252(£m) (*b*) $t = 40$	
2004 P1 Q9	Solve the equation $\log_2(x + 1) - 2\log_2(3) = 3$.	4
Ans	$x = 71$	
2004 P2 Q10	The amount A_t micrograms of a certain radioactive substance remaining after t years decreases according to the formula $A_t = A_0e^{-0.002t}$, where A_0 is the amount present initially. (*a*) If 600 micrograms are left after 1000 years, how many micrograms were present initially? (*b*) The half-life of a substance is the time taken for the amount to decrease to half of its initial amount. What is the half-life of this substance?	3 4
Ans	(*a*) 4433 (*b*) 347 years	
2003 P1 Q12	Simplify $3\log_e(2e) - 2\log_e(3e)$ expressing your answer in the form $A + \log_e B - \log_e C$ where A, B and C are whole numbers.	4
Ans	$1 + \log_e 8 - \log_e 9$	
2003 P2 Q11	(*a*) (i) Sketch the graph of $y = a^x + 1$, $a > 2$. (ii) On the same diagram, sketch the graph of $y = a^{x+1}$, $a > 2$. (*b*) Prove that the graphs intersect at a point where the x-coordinate is $\log_a\left(\frac{1}{a-1}\right)$.	2 3

Ans

(*a*)



(*b*) $a^{x+1} = a^x + 1$

$a \times a^x - a^x = 1$

$(a - 1) \times a^x = 1$

$a^x = \frac{1}{a-1}$

$x = \log_a \left(\frac{1}{a-1}\right)$

2002W P1 Q12

If $\log_a p = \cos^2 x$ and $\log_a r = \sin^2 x$, show that $pr = a$. — 3

Ans

- $\log_a p + \log_a r = \cos^2 x + \sin^2 x$
- $\log_a p + \log_a r = \log_a pr$
- $\log_a pr = 1$ and so $pr = a$

Alternative

- $p = a^{\cos^2 x}$ $r = a^{\sin^2 x}$
- $pr = a^{\cos^2 x + \sin^2 x}$
- $pr = a^1 = a$

2002W P2 Q9

A researcher modelled the size N of a colony of bacteria t hours after the beginning of her observations by $N(t) = 950 \times (2{\cdot}6)^{0{\cdot}2t}$.

(*a*) What was the size of the colony when observations began? — 1

(*b*) How long does it take for the size of the colony to be multiplied by 10? — 4

Ans

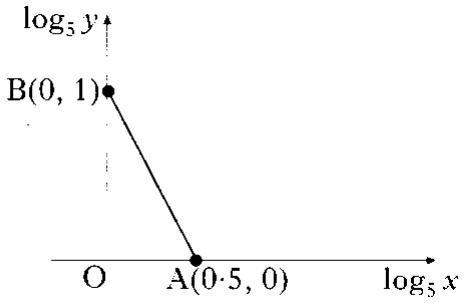
(*a*) 950

(*b*) approx 12 hours

2002 P1 Q11

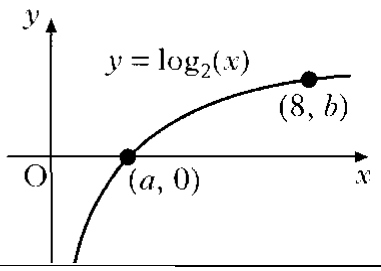
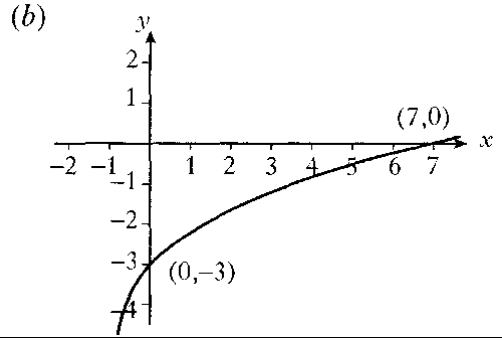
The graph illustrates the law $y = kx^n$.

If the straight line passes through A(0·5, 0) and B(0, 1), find the values of k and n. — 4



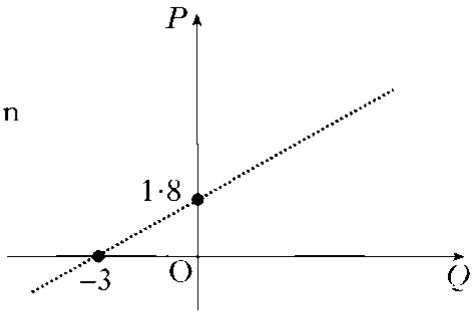
Ans

$\log_5 y = -2 (\log_5 x) + 1$
$\log_5 y = \log_5 x^{-2} + \log_5 5$
$\log_5 y = \log_5 5x^{-2}$
$k = 5$, $n = -2$
$(y = 5x^{-2})$

2002 P2 Q7	Find the x-coordinate of the point where the graph of the curve with equation $y = \log_3 (x - 2) + 1$ intersects the x-axis.	3
Ans	$\log_3 (x - 2) = -1$ $x = 2\frac{1}{3}$	
2001 P1 Q8	Find x if $4 \log_x 6 - 2 \log_x 4 = 1$.	3
Ans	$x = 81$	
2001 P1 Q10	The diagram shows a sketch of part of the graph of $y = \log_2(x)$. (*a*) State the values of a and b. (*b*) Sketch the graph of $y = \log_2(x + 1) - 3$. 	1 3
Ans	(*a*) $a = 1$, $b = 3$ (*b*) 	
2001 P2 Q9	Before a forest fire was brought under control, the spread of the fire was described by a law of the form $A = A_0 e^{kt}$ where A_0 is the area covered by the fire when it was first detected and A is the area covered by the fire t hours later. If it takes one and half hours for the area of the forest fire to double, find the value of the constant k.	3
Ans	$k = 0{\cdot}46$	
2000 P1 Q9	Evaluate $\log_5 2 + \log_5 50 - \log_5 4$.	3
Ans	2	

2000 P2 Q11

The results of an experiment give rise to the graph shown.



(*a*) Write down the equation of the line in terms of P and Q. 2

It is given that $P = \log_e p$ and $Q = \log_e q$.

(*b*) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b. 4

Ans

(*a*) $P = 0{\cdot}6Q + 1{\cdot}8$

(*b*) $a = 6{\cdot}05,\ b = 0{\cdot}6$

Specimen 2 P2 Q7

The intensity I_t of light is reduced as it passes through a filter according to the law $I_t = I_0 e^{-kt}$ where I_0 is the initial intensity and I_t is the intensity after passing through a filter of thickness t cm. k is a constant.

(*a*) A filter of thickness 4 cm reduces the intensity from 120 candle-power to 90 candle-power. Find the value of k. 4

(*b*) The light is passed through a filter of thickness 10 cm. Find the percentage reduction in its intensity. 3

Ans

(*a*) $90 = 120e^{-4k} \Rightarrow e^{-4k} = 0{\cdot}75$

$-4k = \ln 0{\cdot}75 \Rightarrow k = 0{\cdot}0719$

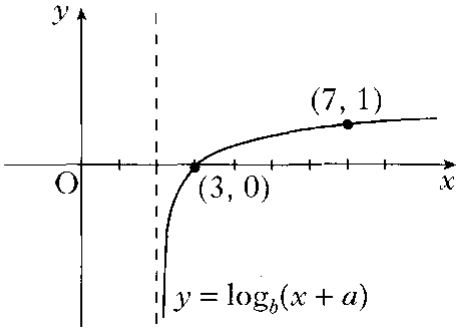
(*b*) $I_{10} = I_0 e^{-10\times 0{\cdot}0719}$

$\frac{I_{10}}{I_0} = 0{\cdot}487$ so a 51·3% reduction

Specimen 2 P2 Q8

The diagram shows part of the graph of $y = \log_b(x + a)$.

Determine the values of a and b. 3



Ans

translation $\Rightarrow a = -2$

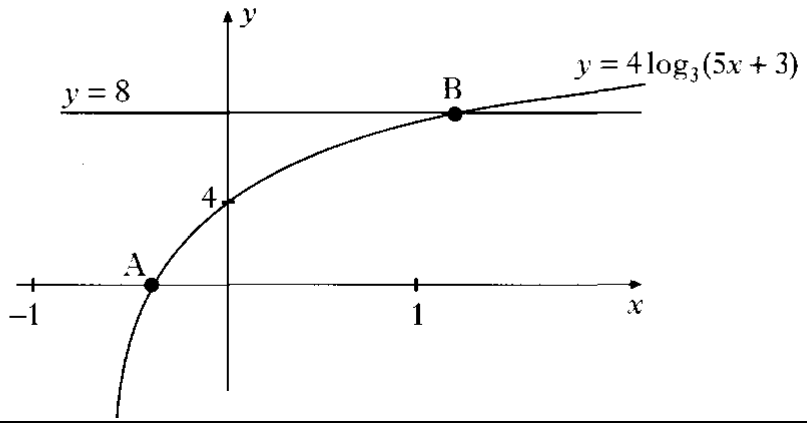
$(7,1) \Rightarrow 1 = \log_b(7 - 2) \Rightarrow b = 5$

Specimen 1 P1 Q10

Part of the graph of $y = 4\log_3(5x + 3)$ is shown in the diagram. This graph crosses the x-axis at the point A and the straight line $y = 8$ at the point B.

Find the x-coordinate of B.

3



Ans

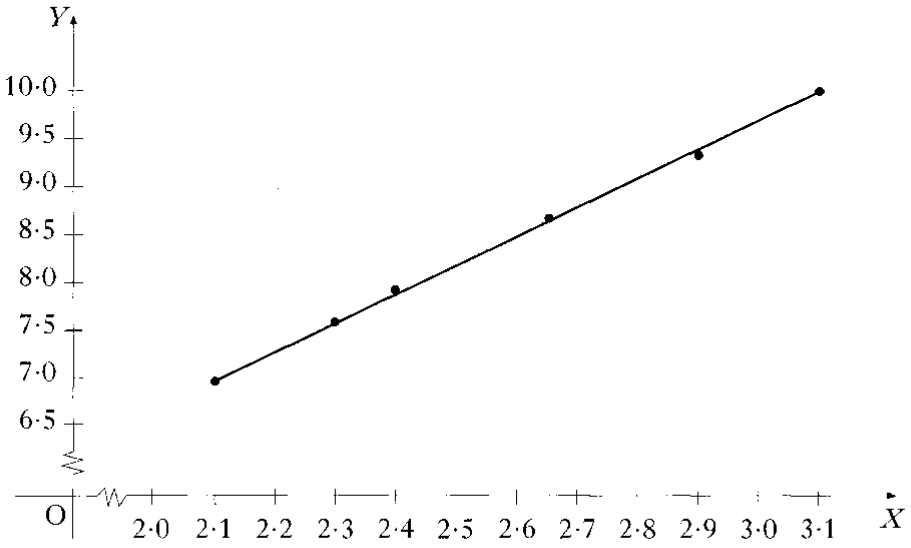
Specimen 1 P2 Q10

Six spherical sponges were dipped in water and weighed to see how much water each could absorb. The diameter (x millimetres) and the gain in weight (y grams) were measured and recorded for each sponge. It is thought that x and y are connected by a relationship of the form $y = ax^b$.

By taking logarithms of the values of x and y, the table below was constructed.

$X (= \log_e x)$	2·10	2·31	2·40	2·65	2·90	3·10
$Y (= \log_e y)$	7·00	7·60	7·92	8·70	9·38	10·00

A graph was drawn and is shown below.



Find the equation of the line in the form $Y = mX + c$.

3

Ans	$Y = 3X + 0{\cdot}7$