National
Qualifications
2025

2025 Mathematics

Higher – Paper 2

Question Paper Finalised Marking Instructions

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General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$x = 1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x - 3)(x - 1) = 0$$
$$x = 1 \text{ or } 3$$

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x=2$	$x=-4$
$\bullet^6$	$y=5$	$y=-7$

Horizontal: $\bullet^5$ $x=2$ and $x=-4$ $\quad\bullet^6$ $y=5$ and $y=-7$

Vertical: $\bullet^5$ $x=2$ and $y=5$ $\quad\bullet^6$ $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\qquad$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0.3}$ must be simplified to 50 $\qquad$ $\frac{4\!\!\not/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*****

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $\left(x^3+2x^2+3x+2\right)(2x+1)$ written as

  $\left(x^3+2x^2+3x+2\right)\times 2x+1$

  $=2x^4+5x^3+8x^2+7x+2$ gains full credit
- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Note: Marking from Image (MFI) annotation change from 2025

A double cross-tick is used to indicate correct working which is irrelevant or insufficient to score any marks. In MFI marking instructions prior to 2025 this was shown as **ü₂** or ü2.

From 2025, the double cross-tick will no longer be used in MFI. A cross or omission symbol will be used instead.

Marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		$\bullet^1$ find gradient of AC	$\bullet^1$ $-\frac{1}{3}$ **OR** $-\frac{10}{30}$	3
			$\bullet^2$ use property of perpendicular lines	$\bullet^2$ 3 **OR** $\frac{30}{10}$	
			$\bullet^3$ determine equation of altitude	$\bullet^3$ $y=3x-7$	

Notes:

1. $\bullet^3$ is only available to candidates who find and use a perpendicular gradient.
2. The gradient of the altitude must appear in fully simplified form at the $\bullet^2$ or $\bullet^3$ stage for $\bullet^3$ to be awarded. See Candidate A.
3. At $\bullet^3$, accept any arrangement of a candidate's equation where constant terms have been simplified.

Commonly Observed Responses:

Candidate A - not simplifying the gradient

$10y=30x-70$ $\bullet^3$ ✗

Candidate B - $m = m_{perp}$

$m=-\frac{1}{3}=3$ $\bullet^1$ ✓ $\bullet^2$ ✗

$y=3x-7$ $\bullet^3$ ✓$_1$ - BOD

However

$m=-\frac{1}{3}$ $\bullet^1$ ✓

$=3$ $\bullet^2$ ✓ - BOD

$y=3x-7$ $\bullet^3$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
1.	(b)		$\bullet^4$ determine midpoint of BC $\bullet^5$ determine gradient of median $\bullet^6$ determine equation of median	$\bullet^4$ $(15,-2)$ $\bullet^5$ $\frac{1}{2}$ **OR** $\frac{12}{24}$ $\bullet^6$ $2y = x - 19$	**3**

Notes:

4. $\bullet^5$ is only available to candidates who use a midpoint to find a gradient.
5. $\bullet^6$ is only available as a consequence of using a 'midpoint' of BC and the point A. See Candidates B to E.
6. The gradient of the median must appear in fully simplified form at the $\bullet^5$ or $\bullet^6$ stage for $\bullet^6$ to be awarded.
7. At $\bullet^6$, accept any arrangement of a candidate's equation where constant terms have been simplified.
8. $\bullet^6$ is not available as a consequence of using a perpendicular gradient.

Commonly Observed Responses:

Candidate B - Perpendicular bisector of BC

$(15,-2)$ $\bullet^4$ ✓

$m_{BC} = -\frac{11}{3}$, $m_{\perp} = \frac{3}{11}$ $\bullet^5$ ×

$11y = 3x - 67$ $\bullet^6$ ×

For other perpendicular bisectors award 0/3

Candidate C - Altitude through A

$\bullet^4$ ^

$m_{BC} = -\frac{11}{3}$, $m_{\perp} = \frac{3}{11}$ $\bullet^5$ ×

$11y = 3x - 127$ $\bullet^6$ ×

Candidate D - Median through B

$\text{Midpoint}_{AC} = (6,-19)$ $\bullet^4$ ×

$m_{BM} = 13$ $\bullet^5$ $\checkmark_1$

$y = 13x - 97$ $\bullet^6$ ×

Candidate E - Median through C

$\text{Midpoint}_{AB} = (0,3)$ $\bullet^4$ ×

$m_{CM} = -\frac{9}{7}$ $\bullet^5$ $\checkmark_1$

$7y = -9x + 21$ $\bullet^6$ ×

	(c)		$\bullet^7$ determine x-coordinate $\bullet^8$ determine y-coordinate	$\bullet^7$ -1 $\bullet^8$ -10	**2**

Notes:

9. For (–1,–10) without working, award 2/2.

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
2.	**Method 1** $\bullet^1$ identify common factor $\bullet^2$ complete the square $\bullet^3$ process for r and write in required form	**Method 1** $\bullet^1$ $2\left(x^2+8x\ldots\right.$ stated or implied by $\bullet^2$ $\bullet^2$ $2\left(x+4\right)^2\ldots$ $\bullet^3$ $2\left(x+4\right)^2-27$	**3**
	Method 2 $\bullet^1$ expand completed square form $\bullet^2$ equate coefficients $\bullet^3$ process for q and r and write in required form	**Method 2** $\bullet^1$ $px^2+2pqx+pq^2+r$ stated or implied by $\bullet^2$ $\bullet^2$ $p=2,\ 2pq=16,\ pq^2+r=5$ $\bullet^3$ $2\left(x+4\right)^2-27$	

Notes:

1. $2\left(x+4\right)^2-27$ with no working gains $\bullet^1$ and $\bullet^2$ only. However, see Candidate E.
2. Do not penalise candidates who do not work with p, q and r.

Commonly Observed Responses:

Candidate A

$2\left(x^2+8\right)+5$

$2\left(\left(x+4\right)^2-16\right)+5$ $\bullet^1$ ✓ $\bullet^2$ ✓

$2\left(x+4\right)^2-27$ $\bullet^3$ ✓

See exception to general marking principle (h)

Candidate B - not using required form

$px^2+2pqx+pq^2+r$ $\bullet^1$ ✓

$p=2,\ 2pq=16,\ pq^2+r=5$ $\bullet^2$ ✓

$q=4,\ r=-27$ $\bullet^3$ ^

$\bullet^3$ is lost as answer is not in completed square form

Candidate C

$2\left(x^2+16x\right)+5$ $\bullet^1$ ×

$2\left(\left(x+8\right)^2-64\right)+5$ $\bullet^2$ ✓1

$2\left(x+8\right)^2-123$ $\bullet^3$ ✓1

Candidate D

$2\left(\left(x+8\right)^2-64\right)+5$ $\bullet^1$ × $\bullet^2$ ×

$2\left(x+8\right)^2-123$ $\bullet^3$ ✓1

Candidate E

$2\left(x+4\right)^2-27$ $\bullet^1$ ✓ $\bullet^2$ ✓

Check:

$=2\left(x^2+8x+16\right)-27$

$=2x^2+16x+32-27$

$=2x^2+16x+5$ $\bullet^3$ ✓

Candidate F

$2\left(x+4\right)^2=2x^2+16x+32$ $\bullet^1$ ✓ $\bullet^2$ ✓

$\therefore 2\left(x+4\right)^2-27=2x^2+16x+5$ $\bullet^3$ ✓

Question	Generic scheme	Illustrative scheme	Max mark
3.	$\bullet^1$ state appropriate definite integral	$\bullet^1$ $\int_2^4 \left(x^2-2x+3\right)dx$	**4**
	$\bullet^2$ integrate	$\bullet^2$ $\frac{x^3}{3}-\frac{2x^2}{2}+3x$	
	$\bullet^3$ substitute limits	$\bullet^3$ $\left(\frac{4^3}{3}-\frac{2(4)^2}{2}+3(4)\right)$ $-\left(\frac{2^3}{3}-\frac{2(2)^2}{2}+3(2)\right)$	
	$\bullet^4$ calculate area	$\bullet^4$ $\frac{38}{3}$	

Notes:

1. $\bullet^1$ is not available to candidates who omit 'dx'.
2. Limits must appear at the $\bullet^1$ stage for $\bullet^1$ to be awarded.
3. Where candidates differentiate one or more terms at $\bullet^2$, then $\bullet^3$ and $\bullet^4$ are unavailable.
4. Candidates who substitute limits without integrating do not gain $\bullet^3$ or $\bullet^4$.
5. Do not penalise the inclusion of '$+c$' at $\bullet^2$ or $\bullet^3$.
6. Do not penalise the continued appearance of the integral sign or 'dx' after $\bullet^1$.
7. Do not penalise rounded or truncated answers with at least one decimal place.

Commonly Observed Responses:

Candidate A - missing working

$\int_2^4 x^2-2x+3$ $\bullet^1$ ^

$=\frac{x^3}{3}-\frac{2x^2}{2}+3x$ $\bullet^2$ ✓

$\bullet^3$ ^

$=\frac{38}{3}$ $\bullet^4$ ✓1

Candidate B - evidence of substitution using a calculator

$\bullet^1$ ^

$=\frac{x^3}{3}-\frac{2x^2}{2}+3x$ $\bullet^2$ ✓

$=\frac{52}{3}-\frac{14}{3}$ $\bullet^3$ ✓

$=\frac{38}{3}$ $\bullet^4$ ✓

Candidate C - limits not stated at $\bullet^1$

$\bullet^1$ ^

$=\frac{x^3}{3}-\frac{2x^2}{2}+3x$ $\bullet^2$ ✓

$=\left(\frac{4^3}{3}-\frac{2(4)^2}{2}+3(4)\right)$

$-\left(\frac{2^3}{3}-\frac{2(2)^2}{2}+3(2)\right)$ $\bullet^3$ ✓

$=\frac{38}{3}$ $\bullet^4$ ✓

Candidate D - reversed limits

$\int_4^2 \left(x^2-2x+3\right)dx$

... $\bullet^2$ ✓ $\bullet^3$ ✓

$=-\frac{38}{3}$, hence area is $\frac{38}{3}$ $\bullet^1$ ✓ $\bullet^4$ ✓

However,

$\int_4^2 \left(x^2-2x+3\right)dx$

... $\bullet^2$ ✓ $\bullet^3$ ✓

$-\frac{38}{3}$

$=\frac{38}{3}$ units^2 $\bullet^1$ ✗ $\bullet^4$ ✗

Question	Generic scheme	Illustrative scheme	Max mark
4.	**Method 1** $\bullet^1$ equate composite function to x $\bullet^2$ write $g\left(g^{-1}(x)\right)$ in terms of $g^{-1}(x)$ $\bullet^3$ state inverse function	**Method 1** $\bullet^1$ $g\left(g^{-1}(x)\right)=x$ $\bullet^2$ $\left(g^{-1}(x)-4\right)^3=x$ $\bullet^3$ $g^{-1}(x)=\sqrt[3]{x}+4$	**3**
	Method 2 $\bullet^1$ write as $y=(x-4)^3$ and start to rearrange $\bullet^2$ complete rearrangement $\bullet^3$ state inverse function	**Method 2** $\bullet^1$ $y=g(x)\Rightarrow x=g^{-1}(y)$ $\sqrt[3]{y}=x-4$ $\bullet^2$ $x=4+\sqrt[3]{y}$ $\bullet^3$ $g^{-1}(y)=\sqrt[3]{y}+4$ $\Rightarrow g^{-1}(x)=\sqrt[3]{x}+4$	

Notes:

1. In method 1, accept $x=\left(g^{-1}(x)-4\right)^3$ for $\bullet^1$ and $\bullet^2$.
2. In method 2, accept '$\sqrt[3]{y}=x-4$' without reference to $y=g(x)\Rightarrow x=g^{-1}(y)$ at $\bullet^1$.
3. In method 2, accept $g^{-1}(x)=\sqrt[3]{x}+4$ without reference to $g^{-1}(y)$ at $\bullet^3$.
4. In method 2, beware of candidates with working where each line is not mathematically equivalent. See Candidates A, B and C.
5. At $\bullet^3$ stage, accept g^{-1} written in terms of any dummy variable. For example $g^{-1}(y)=\sqrt[3]{y}+4$.
6. $y=\sqrt[3]{x}+4$ does not gain $\bullet^3$.
7. $g^{-1}(x)=\sqrt[3]{x}+4$ with no working gains 3/3.
8. In method 2, where candidates make multiple algebraic errors at the $\bullet^2$ stage, $\bullet^3$ is still available.
9. Marks should only be awarded for using a valid strategy to find the inverse of $g(x)$.

Commonly Observed Responses:

Candidate A

$g(x)=(x-4)^3$

$y=(x-4)^3$

$x=\sqrt[3]{y}+4$

$y=\sqrt[3]{x}+4$

$g^{-1}(x)=\sqrt[3]{x}+4$

$\bullet^1$ ✓ $\bullet^2$ ✓

$\bullet^3$ ✗

Candidate B

$g(x)=(x-4)^3$

$y=(x-4)^3$

$x=(y-4)^3$

$y=\sqrt[3]{x}+4$

$g^{-1}(x)=\sqrt[3]{x}+4$

$\bullet^1$ ✗

$\bullet^2$ ✓$_1$

$\bullet^3$ ✓$_1$

Question	Generic scheme	Illustrative scheme	Max mark

4. **(continued)**

Candidate C

$x=\left(g\left(x\right)-4\right)^3$ •1 ✗

$g\left(x\right)=\sqrt[3]{x}+4$ •2 ✓1

$g^{-1}\left(x\right)=\sqrt[3]{x}+4$ •3 ✓1

Candidate D - Method 1

$g\left(g^{-1}\left(x\right)\right)=\left(g^{-1}\left(x\right)-4\right)^3$ •2 ✓

$x=\left(g^{-1}\left(x\right)-4\right)^3$ •1 ✓

$g^{-1}\left(x\right)=\sqrt[3]{x}+4$ •3 ✓

Candidate E - BEWARE of incorrect notation

$g'\left(x\right)=\ldots$ •3 ✗

$f^{-1}\left(x\right)=\ldots$ •3 ✗

Candidate F

$x\rightarrow x-4\rightarrow\left(x-4\right)^3=g\left(x\right)$

$-4\rightarrow\ ^3$

$\therefore\sqrt[3]{\ }\rightarrow\ +4$ •1 ✓

$\sqrt[3]{x}+4$ •2 ✓

$g^{-1}\left(x\right)=\sqrt[3]{x}+4$ •3 ✓

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		$\bullet^1$ find an appropriate vector eg $\overrightarrow{AB}$ $\bullet^2$ find a second vector eg $\overrightarrow{BC}$ **AND** compare $\bullet^3$ appropriate conclusion	$\bullet^1$ eg $\overrightarrow{AB} = \begin{pmatrix} 9 \\ -3 \\ 6 \end{pmatrix}$ $\bullet^2$ eg $\overrightarrow{BC} = \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$ $\therefore \overrightarrow{AB} = \frac{3}{2}\overrightarrow{BC}$ $\bullet^3$ $\ldots \Rightarrow$ AB is parallel to BC (common direction) **AND** B is a common point $\Rightarrow$ A,B and C are collinear.	**3**

Notes:

1. Do not penalise inconsistent vector notation (for example lack of arrows or brackets).
2. If no comparison of vectors or the trivial comparison '$\overrightarrow{AB} = \overrightarrow{BC}$' is made at $\bullet^2$, then $\bullet^3$ is not available.
3. $\bullet^3$ can only be awarded if a candidate has stated 'parallel', 'common point' and 'collinear'.
4. Candidates who state that 'points are parallel' or 'vectors are collinear' or 'parallel and share a common point $\Rightarrow$ collinear' do not gain $\bullet^3$. There must be a reference to the points.
5. Do not accept 'a, b and c are collinear' at $\bullet^3$.

Commonly Observed Responses:

Candidate A - missing labels

$\begin{pmatrix} 9 \\ -3 \\ 6 \end{pmatrix}$ $\bullet^1$ ^

$\begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$ $\therefore \overrightarrow{AB} = \frac{3}{2}\overrightarrow{BC}$ $\bullet^2$ ✓$_1$

Missing labels at $\bullet^2$ is a repeated error

$\Rightarrow$ AB is parallel to BC
and B is a common point
$\Rightarrow$ A, B and C are collinear $\bullet^3$ ✓$_1$

Candidate B

$\overrightarrow{AB} = \begin{pmatrix} 9 \\ -3 \\ 6 \end{pmatrix}$ $\bullet^1$ ✓

$\overrightarrow{BC} = \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$

$\begin{pmatrix} 9 \\ -3 \\ 6 \end{pmatrix} = 3\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ **AND** $\begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix} = 2\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ $\bullet^2$ ✓

$\therefore \overrightarrow{AB} = \frac{2}{3}\overrightarrow{BC}$

Ignore working subsequent to correct statement made on previous line

$\Rightarrow$ AB is parallel to BC
and B is a common point
$\Rightarrow$ A, B and C are collinear $\bullet^3$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
5.	(b)		$\bullet^4$ state ratio	$\bullet^4$ 3:2	**1**

Notes:

6. Answers in (b) must be consistent with the components of the vectors in (a) or the comparison of the vectors in (a). See Candidates C and D. However, award $\bullet^4$ for '3:2' with no working.
7. In this question, the answer for $\bullet^4$ must be stated explicitly in part (b).
8. The only acceptable variations for $\bullet^4$ must be related explicitly to AB and BC. For $\frac{BC}{AB}=\frac{2}{3}$, $\frac{AB}{BC}=\frac{3}{2}$ or $BC:AB=2:3$ stated in part (b) award $\bullet^4$. See Candidate E.
9. Accept unitary ratios for $\bullet^4$, for example $\frac{3}{2}:1$ or $1:\frac{2}{3}$.
10. Where candidates state multiple ratios which are not equivalent, award 0/1.

Commonly Observed Responses:

Candidate C - using components of vectors

(a) $\overrightarrow{AB}=\begin{pmatrix}9\\-3\\6\end{pmatrix}$ $\bullet^1$ ✓

$\overrightarrow{BC}=\begin{pmatrix}6\\-2\\4\end{pmatrix}$

$\overrightarrow{BC}=\frac{3}{2}\overrightarrow{AB}$ $\bullet^2$ ✗

(b) $3:2$ $\bullet^4$ ✓

Candidate D - using comparison of vectors

(a) $\overrightarrow{AB}=\begin{pmatrix}9\\-3\\6\end{pmatrix}$ $\bullet^1$ ✓

$\overrightarrow{BC}=\begin{pmatrix}6\\-2\\4\end{pmatrix}$

$\overrightarrow{BC}=\frac{3}{2}\overrightarrow{AB}$ $\bullet^2$ ✗

(b) $2:3$ $\bullet^4$ $\checkmark_1$

Candidate E - acceptable variation

$\frac{AB}{BC}=\frac{3}{2}$ $\bullet^4$ ✓

$\text{Ratio}=2:3$

Ignore working subsequent to correct statement made on previous line

Candidate F - trivial ratio

$\text{Ratio}=1:1$ $\bullet^4$ ✗

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		$\bullet^1$ use compound angle formula $\bullet^2$ compare coefficients $\bullet^3$ process for k $\bullet^4$ process for a and express in required form	$\bullet^1$ $k\cos x\cos a - k\sin x\sin a$ **stated explicitly** $\bullet^2$ $k\cos a = 5$ and $k\sin a = 9$ **stated explicitly** $\bullet^3$ $\sqrt{106}$ $\bullet^4$ $\sqrt{106}\cos(x+1.06\ldots)$	4

Notes:

1. Accept $k(\cos x\cos a - \sin x\sin a)$ for $\bullet^1$.
2. Treat $k\cos x\cos a - \sin x\sin a$ as bad form only if the equations at the $\bullet^2$ stage both contain k.
3. $\sqrt{106}\cos x\cos a - \sqrt{106}\sin x\sin a$ **or** $\sqrt{106}(\cos x\cos a - \sin x\sin a)$ are acceptable for $\bullet^1$ and $\bullet^3$.
4. $\bullet^2$ is not available for $k\cos x = 5$ and $k\sin x = 9$, however $\bullet^4$ may still be gained. See Candidate E.
5. Accept $-k\sin a = -9$ and $k\cos a = 5$ for $\bullet^2$.
6. $\bullet^4$ is not available for a value of a given in degrees.
7. Accept values of a which round to 1.1.
8. Candidates may state and use any form of the wave function for $\bullet^1$, $\bullet^2$ and $\bullet^3$. However, $\bullet^4$ is only available if the wave is interpreted in the form $k\cos(x+a)$.
9. Evidence for $\bullet^4$ may not appear until part (b) and must appear by the $\bullet^5$ stage.

Commonly Observed Responses:

Candidate A

$\bullet^1$ ^

$\sqrt{106}\cos a = 5$

$\sqrt{106}\sin a = 9$ $\bullet^2$ ✓ $\bullet^3$ ✓

$\tan a = \frac{9}{5}$

$a = 1.06\ldots$

$\sqrt{106}\cos(x+1.06\ldots)$ $\bullet^4$ ✓

Candidate B

$k\cos x\cos a - k\sin x\sin a$ $\bullet^1$ ✓

$\cos a = 5$

$\sin a = 9$ $\bullet^2$ ✗

$\tan a = \frac{9}{5}$

$a = 1.06\ldots$ (Not consistent with equations at $\bullet^2$)

$\sqrt{106}\cos(x+1.06\ldots)$ $\bullet^3$ ✓ $\bullet^4$ ✗

Candidate C

$\cos x\cos a - \sin x\sin a$ $\bullet^1$ ✗

$\cos a = 5$

$\sin a = 9$ $\bullet^2$ ✗

$k = \sqrt{106}$ $\bullet^3$ ✓

$\tan a = \frac{9}{5}$

$a = 1.06\ldots$ (Not consistent with equations at $\bullet^2$)

$\sqrt{106}\cos(x+1.06\ldots)$ $\bullet^4$ ✗

Candidate D - errors at $\bullet^2$

$k\cos x\cos a - k\sin x\sin a$ $\bullet^1$ ✓

$k\cos a = 9$

$k\sin a = 5$ $\bullet^2$ ✗

$\tan a = \frac{5}{9}$

$a = 0.507\ldots$

$\sqrt{106}\cos(x+0.507\ldots)$ $\bullet^3$ ✓ $\bullet^4$ ✓1

Question	Generic scheme	Illustrative scheme	Max mark
6.(a) (continued)			

Candidate E - use of x at $\bullet^2$

$k\cos x\cos a - k\sin x\sin a$ $\bullet^1$ ✓

$k\cos x = 5$
$k\sin x = 9$ $\bullet^2$ ✗

$\tan a = \frac{9}{5}$
$a = 1.06\ldots$
$\sqrt{106}\cos(x + 1.06\ldots)$ $\bullet^3$ ✓ $\bullet^4$ ✓$_1$

Candidate F

$k\cos A\cos B - k\sin A\sin B$ $\bullet^1$ ✗

$k\cos A = 5$
$k\sin A = 9$ $\bullet^2$ ✗

$\tan A = \frac{9}{5}$
$a = 1.06\ldots$
$\sqrt{106}\cos(x + 1.06\ldots)$ $\bullet^3$ ✓ $\bullet^4$ ✓$_1$

Question			Generic scheme	Illustrative scheme	Max mark
6.	(b)		$\bullet^5$ link to (a) $\bullet^6$ solve for $(x+a)$ $\bullet^7$ solve for x	$\bullet^5$ $\sqrt{106}\cos(x+1.06\ldots)=7$ $\bullet^6$ $0.82\ldots$ $(7.106\ldots)$, $\bullet^7$ $5.46\ldots$ $\bullet^7$ $6.04\ldots$, $4.396\ldots$	**3**

Notes:

10. In part (b), where candidates work in degrees throughout, the maximum mark available is 2/3.
11. $\bullet^7$ is only available for two solutions within the stated range. Ignore 'solutions' outwith the range.
12. At $\bullet^7$ accept values of x which round to 6.0, 6.05 or 4.4.

Commonly Observed Responses:

Candidate G - converting to radians

$\vdots$ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓

$\sqrt{106}\cos(x+60.9\ldots)$ $\bullet^4$ ✗

$\sqrt{106}\cos(x+60.9\ldots)=7$ $\bullet^5$ $\checkmark_1$

$x+60\otimes 9\ldots=312.8\ldots,\ 407.1\ldots$

$x=251.8\ldots,\ 346.2\ldots$ $\bullet^6$ $\checkmark_1$

$x=\frac{251.9\pi}{180},\ \frac{346.2.\pi}{180}$ $\bullet^7$ $\checkmark_1$

Candidate H - working in degrees

$\vdots$ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓

$\sqrt{106}\cos(x+60.9\ldots)$ $\bullet^4$ ✗

$\sqrt{106}\cos(x+60.9\ldots)=7$ $\bullet^5$ $\checkmark_1$

$x+60.9\ldots=312.8\ldots,\ 407.1\ldots$

$x=251.8\ldots,\ 346.2\ldots$ $\bullet^6$ $\checkmark_1$ $\bullet^7$ ^

Candidate I - working in degrees

$\vdots$ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓

$\sqrt{106}\cos(x+60.9\ldots)$ $\bullet^4$ ✗

$\sqrt{106}\cos(x+60.9\ldots)=7$ $\bullet^5$ $\checkmark_1$

$x+60.9\ldots=312.8\ldots,$

$x=251.8\ldots$ $\bullet^6$ ^ $\bullet^7$ ^

Candidate J - working in degrees

$\vdots$ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓

$\sqrt{106}\cos(x+60.9\ldots)$ $\bullet^4$ ✗

$\sqrt{106}\cos(x+60.9\ldots)=7$ $\bullet^5$ $\checkmark_1$

$x+60.9\ldots=312.8\ldots,\ 407.1\ldots$ $\bullet^6$ ^ $\bullet^7$ ^

Question	Generic scheme	Illustrative scheme	Max mark
7.	$\bullet^1$ start to integrate $\bullet^2$ complete integration	$\bullet^1$ $\dfrac{(3x+2)^8}{8}$ $\bullet^2$ $\cdots\times\dfrac{1}{3}+c$	**2**

Notes:

1. Award $\bullet^1$ for any appearance of $\dfrac{(3x+2)^8}{8}$ regardless of any constant multiplier.
2. Where candidates **differentiate throughout** or make **no attempt to integrate**, award 0/2.
3. Where candidates start to integrate individual terms within the bracket or use another invalid approach, award 0/2.
4. Do not penalise the continued appearance of the integral sign or 'dx'.

Commonly Observed Responses:

Candidate A

$\dfrac{(3x+2)^8}{8}+c$ $\bullet^1$ ✓ $\bullet^2$ ^

Candidate B - NOT Differentiating throughout

$7(3x+2)^6\times\dfrac{1}{3}+c$ $\bullet^1$ ✗ $\bullet^2$ ✓$_1$

Candidate C - 'Integrating' over two lines

$\dfrac{(3x+2)^8}{8}$ $\bullet^1$ ✓ $\bullet^2$ ^

$\dfrac{(3x+2)^8}{8}\times\dfrac{1}{3}+c$ SEEN

Candidate D - integration incomplete at $\bullet^2$ stage

$\dfrac{(3x+2)^8}{8}\times\dfrac{1}{3}$ $\bullet^1$ ✓ $\bullet^2$ ^

$\dfrac{(3x+2)^8}{8}\times\dfrac{1}{3}+c$ SEEN

Candidate E - integration by substitution

$\int u^7\times\dfrac{1}{3}du$ where $u=3x+2$ and $du=3dx$ $\bullet^1$ ✓

$\dfrac{u^8}{24}+c$

$\dfrac{(3x+2)^8}{24}+c$ $\bullet^2$ ✓

However, for $\int u^7du$ or $\int u^7dx$ leading to $\dfrac{(3x+2)^8}{24}+c$ award 0/2.

Question			Generic scheme	Illustrative scheme	Max mark
8.			$\bullet^1$ identify an appropriate pathway $\bullet^2$ find $\overrightarrow{BE}$	$\bullet^1$ eg $\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}$ stated or implied by $\bullet^2$ $\bullet^2$ $(0\mathbf{i}+)\ 5\mathbf{j}+4\mathbf{k}$	**2**

Notes:

1. Do not penalise inconsistent vector notation (for example lack of arrows or brackets).
2. $\bullet^1$ is not available for $\overrightarrow{BD}+\overrightarrow{DE}$ or $\overrightarrow{BC}+\overrightarrow{CE}$ or similar. However, see Candidate C.
3. $\bullet^2$ is only available where a valid pathway has been stated.
4. Do not accept $\begin{pmatrix}0\mathbf{i}\\5\mathbf{j}\\4\mathbf{k}\end{pmatrix}$ or $\begin{pmatrix}0\\5\\4\end{pmatrix}$ for $\bullet^2$.
5. Where an invalid pathway 'leads' to $5\mathbf{j}+4\mathbf{k}$ award 0/2.

Commonly Observed Responses:

Candidate A - using given vectors

$\overrightarrow{BE}=-\overrightarrow{DC}+\overrightarrow{AD}+\overrightarrow{DE}$ $\bullet^1$ ✓

$5\mathbf{j}+4\mathbf{k}$ $\bullet^2$ ✓

Candidate B - using given vectors

$\overrightarrow{BE}=(6\mathbf{i}+4\mathbf{j}+2\mathbf{k})-(2\mathbf{i}-4\mathbf{j}+2\mathbf{k})+(-4\mathbf{i}-3\mathbf{j}+4\mathbf{k})$ $\bullet^1$ ✓

$\overrightarrow{BE}=\begin{pmatrix}6\\4\\2\end{pmatrix}-\begin{pmatrix}2\\-4\\2\end{pmatrix}+\begin{pmatrix}-4\\-3\\4\end{pmatrix}$ $\bullet^1$ ✓

Candidate C

$\overrightarrow{BD}+\overrightarrow{DE}$

$\overrightarrow{BD}=4\mathbf{i}+8\mathbf{j}$ $\bullet^1$ ✓

$\overrightarrow{BE}=5\mathbf{j}+4\mathbf{k}$ $\bullet^2$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		$\bullet^1$ substitute $\bullet^2$ process	$\bullet^1$ $10 = 10m + 4$ **OR** $10 = \dfrac{4}{1-m}$ $\bullet^2$ $m = \dfrac{3}{5}$	**2**

Notes:

1. Correct answer with no working, award 1/2.
2. Where candidates state '$m = \frac{3}{5}$' or equivalent, and then verify the result from the given recurrence relationship, award 1/2.
3. Where candidates work in terms of a, a link to m must be made for $\bullet^1$ to be awarded. See Candidates A, B and C.

Commonly Observed Responses:

Candidate A - working not in terms of *m*

(a) $10 = \dfrac{4}{1-a}$ $\bullet^1$ ^

$a = \dfrac{3}{5}$ $\bullet^2$ ✓1

Candidate B - working not in terms of *m*

(a) $10 = \dfrac{4}{1-a}$

$a = \dfrac{3}{5}$ $\bullet^2$ ✓

$m = \dfrac{3}{5}$ $\bullet^1$ ✓

Candidate C - working not in terms of *m*

(a) $10 = \dfrac{4}{1-a}$

$a = \dfrac{3}{5}$ $\bullet^2$ ✓

(b) $19 = \dfrac{3}{5}u_0 + 4$ $\bullet^1$ ✓

$\bullet^1$ is awarded when the calculated value of *a* is used in place of *m*.

Question			Generic scheme	Illustrative scheme	Max mark
9.	(b)		$\bullet^3$ calculate u_0	$\bullet^3$ 25	**1**
Notes:					
4. Where candidates use an incorrect value of m without supporting working in part (a) or which is inconsistent with their answer in part (a), $\bullet^3$ is not available.					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		$\bullet^1$ express P in terms of x and y $\bullet^2$ express y in terms of x $\bullet^3$ substitute for y and complete proof	$\bullet^1$ $P = 12x + 2y$ $\bullet^2$ $y = \dfrac{150 - 6x^2}{5x}$ $\bullet^3$ $P = 12x + 2\left(\dfrac{150 - 6x^2}{5x}\right)$ leading to $P = 9.6x + \dfrac{60}{x}$	**3**

Notes:

1. Accept $P = 4x + 3x + y + 5x + y$ or equivalent for $\bullet^1$.
2. The substitution for y at $\bullet^3$ must be clearly shown for $\bullet^3$ to be available.
3. Do not penalise the omission of brackets at $\bullet^3$ leading to the correct solution. See Candidate A.

Commonly Observed Responses:

Candidate A - missing brackets

$\vdots$

$P = 12x + 2 \times \dfrac{150 - 6x^2}{5x}$

$P = 9.6x + \dfrac{60}{x}$ $\bullet^3$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
10.	(b)		$\bullet^4$ express P in differentiable form	$\bullet^4$ $9.6x+60x^{-1}$ stated or implied by $\bullet^5$	**6**
			$\bullet^5$ differentiate	$\bullet^5$ $9.6-60x^{-2}$	
			$\bullet^6$ equate expression for derivative to 0	$\bullet^6$ $9.6-60x^{-2}=0$	
			$\bullet^7$ process for x	$\bullet^7$ 2.5 or $\frac{5}{2}$	
			$\bullet^8$ verify nature	$\bullet^8$ table of signs for a derivative $\therefore$ minimum **OR** $P''(x)=120x^{-3}$ and $P''(2.5)=7.68\ldots>0$ $\therefore$ minimum	
			$\bullet^9$ find minimum value of P	$\bullet^9$ 48(cm)	

Notes:

4. For a numerical approach, award 0/6.
5. $\bullet^6$ can be awarded for $60x^{-2}=9.6$.
6. Where candidates integrate any term at the $\bullet^5$ stage, only $\bullet^6$ is available on follow through for setting their 'derivative' to 0.
7. $\bullet^7$, $\bullet^8$ and $\bullet^9$ are only available for working with a derivative which contains an index ≤ -2.
8. $\left(\frac{60}{9.6}\right)^{0.5}$ or $\sqrt[-2]{\frac{9.6}{60}}$ must be simplified at $\bullet^7$ or $\bullet^8$ for $\bullet^7$ to be awarded.
9. Ignore the appearance of −2.5 at $\bullet^7$.
10. Notation for the derivative is only assessed at $\bullet^8$.
11. $\bullet^8$ is not available to candidates who consider a value of $x\leq 0$ in the neighbourhood of 2.5.
12. $\bullet^9$ is still available in cases where a candidate's table of signs does not lead legitimately to a minimum at $\bullet^8$.
13. $\bullet^8$ and $\bullet^9$ are not available to candidates who state that the minimum exists at a value of x where $x\leq 0$.

Commonly Observed Responses:

Candidate B - differentiating over multiple lines

$P'(x)=9.6+60x^{-1}$ $\bullet^4$ ✗

$P'(x)=9.6-60x^{-2}$ $\bullet^5$ ✗

$9.6-60x^{-2}=0$ $\bullet^6$ ✓$_1$

Candidate C - differentiating over multiple lines

$P'(x)=9.6x+60x^{-1}$ $\bullet^4$ ✓

$P'(x)=9.6-60x^{-2}$ $\bullet^5$ ✗

$9.6-60x^{-2}=0$ $\bullet^6$ ✓$_1$

Candidate D

Stationary points when $P'(x)=0$

$P'(x)=9.6-60x^{-2}$ $\bullet^4$ ✓ $\bullet^5$ ✓ $\bullet^6$ ✓

Question	Generic scheme	Illustrative scheme	Max mark

10.(b) (continued)

For the table of signs for a derivative, **accept**:

x	2.5^-	2.5	2.5^+
$P'(x)$	$-$	0	$+$
Slope or shape			

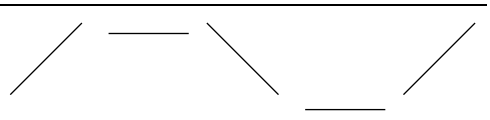
x	$\rightarrow$	2.5	$\rightarrow$
$P'(x)$	$-$	0	$+$
Slope or shape			

Arrows are taken to mean 'in the neighbourhood of'

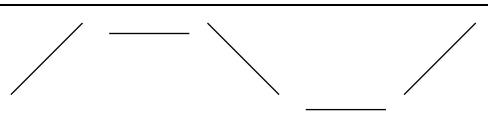
x	a	2.5	b
$P'(x)$	$-$	0	$+$
Slope or shape			

Where $0 < a < 2.5$ and $b > 2.5$

For the table of signs for a derivative, do **NOT accept**:

x	$\rightarrow$	-2.5	$\rightarrow$	2.5	$\rightarrow$
$P'(x)$	$+$	0	$-$	0	$+$
Slope or shape					

Since the function is discontinuous
$-2.5 \rightarrow 2.5$ is NOT acceptable

x	a	-2.5	b	2.5	c
$P'(x)$	$+$	0	$-$	0	$+$
Slope or shape					

Since the function is discontinuous
$-2.5 < b < 2.5$ is NOT acceptable

- For this question, do not penalise the omission of 'x' or the word 'shape'/'slope'
- Stating values of $P'(x)$ is an acceptable alternative to writing '$+$' or '$-$' signs
- Acceptable variations of $P'(x)$ are: P', $\frac{dP}{dx}$, and $9.6-60x^{-2}$. Accept $f'(x)$ only where candidates have previously used $f(x)=9.6x+60x^{-1}$ in their working
- Do not accept $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$

Question	Generic scheme	Illustrative scheme	Max mark
11.	$\bullet^1$ substitute double angle formula for $\sin 2x°$ $\bullet^2$ factorise $\bullet^3$ solve for $\cos x°$ and $\sin x°$ $\bullet^4$ solve for x	$\bullet^1$ $3(2\sin x°\cos x°)+4\cos x°(=0)$ $\bullet^2$ eg $2\cos x°(3\sin x°+2)=0$ $\bullet^3$ $\cos x°=0$, $\bullet^4$ $\sin x°=-\frac{2}{3}$ $\bullet^4$ 90, 270, 221.8..., 318.1...	**4**

Notes:

1. Substituting $2\sin A\cos A$ for $\sin 2x°$ at the $\bullet^1$ stage should be treated as bad form provided the equation is written in terms of x at the $\bullet^2$ stage. Otherwise, $\bullet^1$ is not available.
2. '$=0$' must appear by the $\bullet^2$ stage for $\bullet^2$ to be awarded.
3. Do not penalise the absence of '2' as a common factor at $\bullet^2$.
4. Award $\bullet^3$ for $x=\cos^{-1}(0)$ **AND** $x=\sin^{-1}\left(-\frac{2}{3}\right)$.
5. Do not penalise the omission of degree signs.
6. Candidates who leave their answer in radians do not gain $\bullet^4$ (if marking horizontally) or $\bullet^3$ (if marking vertically).
7. Where equations for $\sin x°$ and/or $\cos x°$ do not have solutions, marks are unavailable for stating 'no solutions'.

Commonly Observed Responses:

Candidate A - dividing by cos x

$6\sin x°\cos x°=-4\cos x°$ $\bullet^1$ ✓
$6\sin x°=-4$ $\bullet^2$ ^ $\bullet^3$ ^
$x=221.8\ldots, 318.1\ldots$ $\bullet^4$ ✓1

Candidate B - insufficient evidence for $\bullet^3$

$6\sin x°\cos x°+4\cos x°=0$ $\bullet^1$ ✓
$2\cos x°(3\sin x°+2)=0$ $\bullet^2$ ✓
$2\cos x°=0,\ \sin x°=-\frac{2}{3}$ $\bullet^3$ ^ $\bullet^4$ ^
However,
$x=90, 270, 221.8\ldots, 318.1\ldots$ $\bullet^3$ ✓ $\bullet^4$ ✓

Candidate C

$\cos x°=0$, $\sin x°=-\frac{2}{3}$ $\bullet^3$ ✓
$x=90, 270$ $x=41$
$x=221.8..., 318.1...$ $\bullet^4$ ×

However, where the final solution(s) are clearly identified by the candidate award $\bullet^4$.

Question			Generic scheme	Illustrative scheme	Max mark
12.	(a)		•1 interpret notation •2 state expression for $f(g(x))$	•1 $f(1-x^3)$ **OR** $(g(x))^5+3$ •2 $(1-x^3)^5+3$	**2**

Notes:

1. For $(1-x^3)^5+3$ without working, award 2/2.

Commonly Observed Responses:

Candidate A

$f(g(x))=(1-x^3)^5+3$ •1 ✓ •2 ✓

$h(x)=4-x^{15}$

Candidate B - two 'attempts'

$f(g(x))=x^5+3$ •1 ✗ •2 ✗

$f(g(x))=(1-x^3)^5+3$

Candidate C

$f(g(x))=1-(x^5+3)^3$ •1 ✗ •2 $✓_1$

Question			Generic scheme	Illustrative scheme	Max mark
12.	(b)		$\bullet^3$ start to differentiate $\bullet^4$ complete differentiation	$\bullet^3$ $5\left(1-x^3\right)^4\ldots$ $\bullet^4$ $\cdots\times\left(-3x^2\right)$	**2**

Notes:

2. For $-15x^2\left(1-x^3\right)^4$, award 2/2.
3. $\bullet^3$ and $\bullet^4$ are not available for working with $4-x^{15}$.
4. Accept '$5u^4$ where $u=1-x^3$' for $\bullet^3$.
5. Do not award $\bullet^4$ where the answer includes '$+c$'.

Commonly Observed Responses:

Candidate D - differentiating over two lines

$5\left(1-x^3\right)^4$ $\bullet^3$ ✓

$5\left(1-x^3\right)^4\times\left(-3x^2\right)$ $\bullet^4$ ^

Candidate E - poor notation

$y=\left(1-x^3\right)^5$ $y=1-x^3$

$\frac{dy}{dx}=-3x^2$

$\frac{dy}{dx}=5\left(1-x^3\right)^4\times\left(-3x\right)^2$ $\bullet^3$ ✓ $\bullet^4$ ✓

Candidate F - poor communication

$y=\left(1-x^3\right)^5+3$

$y=5\left(1-x^3\right)^4\times\left(-3x^2\right)$ $\bullet^3$ ✓ $\bullet^4$ ✓

Candidate G - insufficient evidence for $\bullet^3$

$-15\left(1-x^3\right)^4$ $\bullet^3$ ✗ $\bullet^4$ ✗

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		$\bullet^1$ identify initial mass	$\bullet^1$ 150 (micrograms)	**1**
Notes:					
Commonly Observed Responses:					
	(b)		$\bullet^2$ interpret information	$\bullet^2$ $120 = 150e^{-0.0054t}$ stated or implied by $\bullet^3$	**4**
			$\bullet^3$ process equation	$\bullet^3$ $e^{-0.0054t} = \dfrac{120}{150}$	
			$\bullet^4$ write in logarithmic form	$\bullet^4$ $\log_e\left(\dfrac{120}{150}\right) = -0.0054t$	
			$\bullet^5$ process for t	$\bullet^5$ 41.32… (years)	

Notes:

1. Where values other than 120 are used in the substitution, $\bullet^3$, $\bullet^4$ and $\bullet^5$ are still available.
2. $\bullet^3$ may be implied by $\bullet^4$.
3. Evidence for $\bullet^4$ must be stated explicitly. See Candidate B.
4. At $\bullet^4$ all exponentials must be processed.
5. Any base may be used at $\bullet^4$ stage. See Candidate A.
6. Accept $\ln 0.8 = -0.0054t \ln e$ and $-0.223\ldots = -0.0054t$ for $\bullet^4$.
7. $\bullet^5$ is unavailable where candidates round the value of $\ln 0.8$ to fewer than 2 decimal places.
8. Accept answers where $40.7 \le t \le 42$ at $\bullet^5$.
9. The calculation at $\bullet^5$ must follow from the valid use of exponentials and logarithms at $\bullet^3$ and $\bullet^4$.
10. Where candidates show no working or take an iterative approach to arrive at $t = 41$ or $t = 42$, award 1/4. However, if, in any iterations M is evaluated for $t = 41$ and $t = 42$ leading to a final answer of $t = 42$ (years), then award 4/4.

Commonly Observed Responses:

Candidate A - using other bases

$120 = 150e^{-0.0054t}$ $\bullet^2$ ✓

$0.8 = e^{-0.0054t}$ $\bullet^3$ ✓

$\log_{10} 0.8 = -0.0054t \log_{10} e$ $\bullet^4$ ✓

$t = 41.32\ldots$ (years) $\bullet^5$ ✓

Candidate B - missing working

$120 = 150e^{-0.0054t}$ $\bullet^2$ ✓

$0.8 = e^{-0.0054t}$ $\bullet^3$ ✓

$\bullet^4$ ^

$t = 41$ $\bullet^5$ ✓

Candidate C - iterative approach

$t = 40 \Rightarrow M = 120.86\ldots$

$t = 41 \Rightarrow M = 120.20\ldots$

$t = 42 \Rightarrow M = 119.56\ldots$

So $t = 42$ (years) **Award 4/4**

Candidate D - taking logarithms of both sides

$120 = 150e^{-0.0054t}$ $\bullet^2$ ✓

$\log_e 120 = \log_e\left(150e^{-0.0054t}\right)$

$\log_e 120 = \log_e 150 + \log_e e^{-0.0054t}$ $\bullet^3$ ✓

$\log_e 120 - \log_e 150 = -0.0054t$ $\bullet^4$ ✓

$t = 41.32\ldots$ (years) $\bullet^5$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
14.	(a)		$\bullet^1$ state coordinates of centre $\bullet^2$ state radius	$\bullet^1$ $(-5,6)$ $\bullet^2$ 3	**2**

Notes:

1. Accept $x=-5$, $y=6$ for $\bullet^1$.
2. Do not accept '$a=\ldots, b=\ldots$' or '−5,6' for $\bullet^1$.

Commonly Observed Responses:

	(b)		$\bullet^3$ find coordinates of centre $\bullet^4$ find radius	$\bullet^3$ $(7,-3)$ $\bullet^4$ 2	**2**

Notes:

3. Accept $x=7$, $y=-3$ for $\bullet^3$.
4. Do not accept '$g=\ldots$, $f=\ldots$' or '7,−3' for $\bullet^3$.
5. Do not penalise candidates who treat negatives with a lack of rigour when calculating the radius.
   For example, accept $\sqrt{7^2+3^2-54}=2$ or $\sqrt{7^2+-3^2-54}=2$ or $\sqrt{-7^2+3^2-54}=2$ for $\bullet^4$.
   However, do not accept $\sqrt{7^2-3^2-54}=2$ for $\bullet^4$.

Commonly Observed Responses:

Candidate A - repeated error within a question		**Candidate B - two errors**	
(a) −5,6	$\bullet^1$ ✗	(a) 5,−6	$\bullet^1$ ✗
(b) 7,−3	$\bullet^3$ ✓$_1$	(b) 7,−3	$\bullet^3$ ✗

	(c)		$\bullet^5$ find distance between centres of C_1 and C_2 $\bullet^6$ calculate radius of C_3 $\bullet^7$ find centre of C_3 and state equation of C_3	$\bullet^5$ 15 $\bullet^6$ 8 $\bullet^7$ $(-1,3)$ **stated explicitly** and $(x+1)^2+(y-3)^2=64$	**3**

Notes:

6. $\bullet^5$ may be awarded for $\sqrt{(-5-7)^2+(6+3)^2}$ within a valid calculation for the radius of C_3.
7. $\bullet^6$ is only available where a valid approach to finding the distance between the centres of C_1 and C_2 has been used.
8. Where candidates use a radius without valid supporting working, $\bullet^7$ is not available.
9. Accept the centre of C_3 written as a position vector.

Question	Generic scheme	Illustrative scheme	Max mark

14.(c) (continued)

Commonly Observed Responses:

Valid approaches for finding r

$r=\dfrac{15+3-2}{2}\ \therefore\ r=8$ •6 ✓

$r=\dfrac{\sqrt{(7+5)^2+(6+3)^2}+3-2}{2}\ \therefore\ r=8$ •5 ✓ •6 ✓

$r=3+3+\dfrac{(13-9)}{2}\ \therefore\ r=8$ •6 ✓

Let $x=r_3-r_1$. Since $r_3=13-x$,
$x=(13-x)-3\ \Rightarrow\ x=5\ \therefore\ r=8$ •6 ✓
(may also use $y+3=r_3-r_1$)

This list is not exhaustive

Invalid approaches and/or insufficient communication for finding r

$r=3+3+2\ \therefore\ r=8$ •6 ^

gap $=2\ \therefore\ r=10-2\ \therefore\ r=8$ •6 ^

$r=\dfrac{15}{3}+3\ \therefore\ r=8$ •6 ^

Distance = 15 $\therefore\ r=8$ •6 ^
(with no supporting working)

This list is not exhaustive

Valid approaches for finding centre of C_3

Ratio $=1:2\ \therefore\ C_3(-1,3)$

$C_3\left(-5+12\times\dfrac{1}{3},6-9\times\dfrac{1}{3}\right)\ \therefore\ C_3(-1,3)$

Ratio $=15:10$ & stepping out $\therefore\ C_3(-1,3)$
(may be drawn as similar triangles)

Let $x=r_3-r_1$. Since $r_3=13-x$,
$x=(13-x)-3\ \Rightarrow\ x=5$
$\Rightarrow$ Ratio $=5:10\ \therefore\ C_3(-1,3)$
(may also use $y+3=r_3-r_1$)

$2\overrightarrow{C_1C_3}=\overrightarrow{C_3C_2}\ \therefore\ C_3=\begin{pmatrix}-1\\3\end{pmatrix}$

This list is not exhaustive

Invalid approaches and/or insufficient communication for finding centre of C_3

$m_{C_1C_2}=-\dfrac{3}{4}\ \therefore\ C_1(-5,6)\rightarrow C_3(-1,3)$ •7 ×

$\dfrac{y-6}{x+5}=\dfrac{-3-y}{7-x}=-\dfrac{3}{4}\ \therefore\ C_3(-1,3)$ •7 ×

$\{3,4,5\}$ triangle with no evidence
of a ratio $\therefore\ C_3(-1,3)$ •7 ×

This list is not exhaustive

[END OF MARKING INSTRUCTIONS]