

Paper 2

$$\begin{aligned} \textcircled{1} \text{ a) } M_{AC} &= \frac{-14+24}{-9-21} \\ &= \frac{10}{-30} \\ &= -\frac{1}{3} \end{aligned}$$

$$\therefore M_{alt} = 3$$

$$y - 20 = 3(x - a)$$

$$y - 20 = 3x - 27$$

$$\underline{\underline{y = 3x - 7}}$$

$$\text{b) mid} = (15, -2)$$

$$M_{med} = \frac{-2+14}{15+9}$$

$$= \frac{12}{24}$$

$$= \frac{1}{2}$$

$$y + 2 = \frac{1}{2}(x - 15)$$

$$2y + 4 = x - 15$$

$$\underline{\underline{2y = x - 19}}$$

$$\text{c) } 3x - y = 7 \quad \textcircled{1}$$

$$x - 2y = 19 \quad \textcircled{2}$$

$$\textcircled{1} \times 2 \quad 6x - 2y = 14 \quad \textcircled{3}$$

$$\textcircled{3} - \textcircled{2} \quad 5x = -5$$

$$\underline{\underline{x = -1}}$$

$$y = 3(-1) - 7$$

$$= -10$$

$$\therefore \underline{\underline{(-1, -10)}}$$

$$\begin{aligned} \textcircled{2} \quad p(x+q)^2 + r \\ &= p(x^2 + 2qx + q^2) + r \\ &= px^2 + 2pqx + pq^2 + r \end{aligned}$$

$$2x^2 + 16x + 5$$

$$\therefore \underline{p = 2}$$

$$2pq = 16$$

$$2(2)q = 16$$

$$4q = 16$$

$$\underline{q = 4}$$

$$pq^2 + r = 5$$

$$2(4)^2 + r = 5$$

$$2(16) + r = 5$$

$$32 + r = 5$$

$$\underline{r = -27}$$

$$\therefore \underline{\underline{2(x+4)^2 - 27}}$$

$$\textcircled{3} \int_2^4 (x^2 - 2x + 3) dx$$

$$= \left[\frac{x^3}{3} - \frac{2x^2}{2} + 3x \right]_2^4$$

$$= \left[\frac{x^3}{3} - x^2 + 3x \right]_2^4$$

$$= \left(\frac{4^3}{3} - 4^2 + 3(4) \right) - \left(\frac{2^3}{3} - 2^2 + 3(2) \right)$$

$$= \left(\frac{64}{3} - 16 + 12 \right) - \left(\frac{8}{3} - 4 + 6 \right)$$

$$= \left(\frac{64}{3} - 4 \right) - \left(\frac{8}{3} + 2 \right)$$

$$= \left(\frac{64}{3} - \frac{12}{3} \right) - \left(\frac{8}{3} + \frac{6}{3} \right)$$

$$= \frac{52}{3} - \frac{14}{3}$$

$$= \underline{\underline{\frac{38}{3} u^2}}$$

5) a) $\vec{AB} = \vec{b} - \vec{a}$
 $= \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$
 $= \begin{pmatrix} 9 \\ -3 \\ 6 \end{pmatrix} = 3 \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

$\vec{BC} = \vec{c} - \vec{b}$
 $= \begin{pmatrix} 12 \\ -3 \\ 9 \end{pmatrix} - \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix}$
 $= \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix} = 2 \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

$\vec{AB} = \frac{3}{2} \vec{BC} \therefore$ vectors are parallel

B is common \therefore points are collinear

b) 3:2

6) a) $k \cos(x+\alpha)$
 $= k \cos x \cos \alpha - k \sin x \sin \alpha$
 $= k \cos x \cos \alpha - k \sin x \sin \alpha$

$5 \cos x = 9 \sin x$

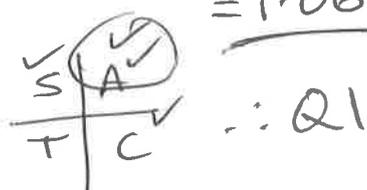
$-k \sin \alpha = -9$
 $k \sin \alpha = 9$
 $k \cos \alpha = 5$

$k^2 = 9^2 + 5^2$
 $= 81 + 25$

$k = \sqrt{106}$

$\tan \alpha = \frac{9}{5}$
 $\alpha = \tan^{-1}\left(\frac{9}{5}\right)$
 $= 1.06$

$k \sin \alpha = \text{Pos}$
 $k \cos \alpha = \text{Pos}$



5

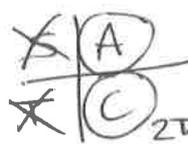
$\therefore 5 \cos x - 9 \sin x$
 $= \sqrt{106} \cos(x+1.06)$

b) $\sqrt{106} \cos(x+1.06) = 7$
 $\cos(x+1.06) = \frac{7}{\sqrt{106}}$
 $x+1.06 = \cos^{-1}\left(\frac{7}{\sqrt{106}}\right)$

$x+1.06 = 0.82, 5.46$

$x = -0.24, 1.40$

$x = 1.4, 6.04$



$2\pi - 0.82$
 $= 5.46$

7) $\int (3x+2)^7 dx$
 $= \frac{(3x+2)^8}{8 \times 3} + C$
 $= \frac{1}{24} (3x+2)^8 + C$

8) $\vec{BE} = \vec{BC} + \vec{CD} + \vec{DE}$
 $= \vec{AD} - \vec{DC} + \vec{DE}$
 $= \begin{pmatrix} 6 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix} + \begin{pmatrix} -4 \\ -3 \\ 4 \end{pmatrix}$
 $= \begin{pmatrix} -8 \\ 5 \\ 4 \end{pmatrix}$

$\therefore -8\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$

$$(9) a) L = \frac{b}{1-a}$$

$$10 = \frac{4}{1-a}$$

$$1-a = \frac{4}{10}$$

$$1-a = 0.4$$

$$\underline{a = 0.6}$$

$$b) U_{n+1} = 0.6U_n + 4$$

$$19 = 0.6U_0 + 4$$

$$0.6U_0 = 15$$

$$U_0 = \frac{15}{0.6}$$

$$\underline{U_0 = 25}$$

$$(10) a) \text{ Triangle } A = \frac{1}{2}bh$$

$$= \frac{1}{2}(4x)(3x)$$

$$= \frac{1}{2}(12x^2)$$

$$= \underline{6x^2}$$

$$\text{Rectangle } A = U_0$$

$$= \underline{5xy}$$

$$\therefore \text{Area} = 150$$

$$5xy + 6x^2 = 150$$

$$5y = 150 - 6x^2$$

$$\underline{y = \frac{30}{x} - \frac{6}{5}x}$$

$$\text{Perimeter} = 4x + 3x + 5x + 2y$$

$$= 12x + 2\left(\frac{30}{x} - \frac{6}{5}x\right)$$

$$= 12x + \frac{60}{x} - \frac{12}{5}x$$

$$= 9.6x + \frac{60}{x}$$

(as required)

$$b) \text{ SP's @ } \frac{dP}{dx} = 0$$

$$P = 9.6x + 60x^{-1}$$

$$\therefore \frac{dP}{dx} = 9.6 - 60x^{-2}$$

$$= 9.6 - \frac{60}{x^2}$$

$$\therefore 9.6 - \frac{60}{x^2} = 0$$

$$9.6 = \frac{60}{x^2}$$

$$x^2 = \frac{60}{9.6}$$

$$x = \sqrt{\frac{60}{9.6}}$$

$$\underline{x = 2.5, \quad \cancel{-2.5}}$$

x	$\xrightarrow{2}$	2.5	$\xrightarrow{3}$
$\frac{dP}{dx}$	-	0	+
shape	\	-	/

\therefore minimum @ $x = 2.5$

$$\therefore P = 9.6(2.5) + \frac{60}{2.5}$$

$$= \underline{48 \text{ cm}}$$

$$(11) 3 \sin 2x + 4 \cos x = 0$$

$$3(2 \sin x \cos x) + 4 \cos x = 0$$

$$6 \sin x \cos x + 4 \cos x = 0$$

$$2 \cos x (3 \sin x + 2) = 0$$

$$2 \cos x = 0 \quad 3 \sin x + 2 = 0$$

$$\cos x = 0$$

$$\sin x = -\frac{2}{3}$$

$$x = 90^\circ, 270^\circ$$

$$x = 41.8^\circ$$

$$x = 221.8^\circ, 318.2^\circ$$

$$(12) \therefore \underline{x = 90^\circ, 221.8^\circ, 270^\circ, 318.2^\circ}$$

12) a) $f(g(x))$
 $= f(1-x^3)$
 $= (1-x^3)^5 + 3$

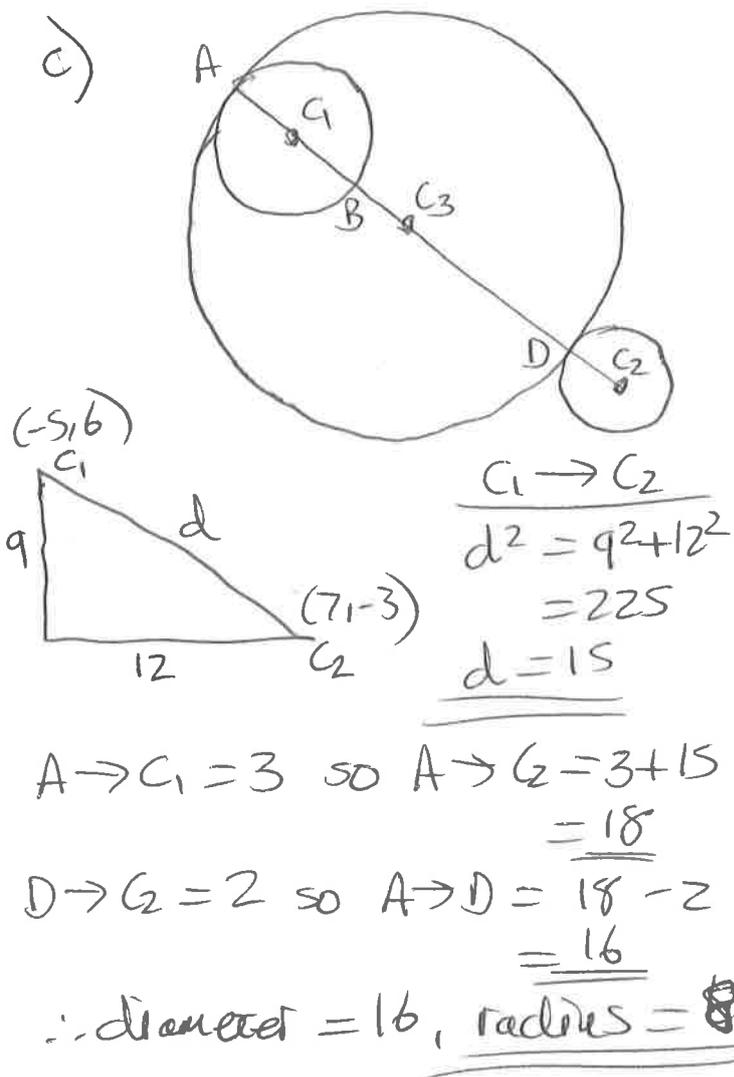
b) $h(x) = (1-x^3)^5 + 3$
 $h'(x) = 5(1-x^3)^4 \times (-3x^2)$
 $= -15x^2(1-x^3)^4$

13) a) 150 μ g
 b) $120 = 150e^{-0.0054t}$
 $\frac{120}{150} = e^{-0.0054t}$
 $\ln\left(\frac{120}{150}\right) = -0.0054t$
 $t = \frac{\ln\left(\frac{120}{150}\right)}{-0.0054}$
 $t = 41.32 \text{ years}$

14) a) $C_1 = (-5, 6)$
 $r_1 = 3$

b) $2g = -14$ $2f = 6$ $c = 54$
 $g = -7$ $f = 3$

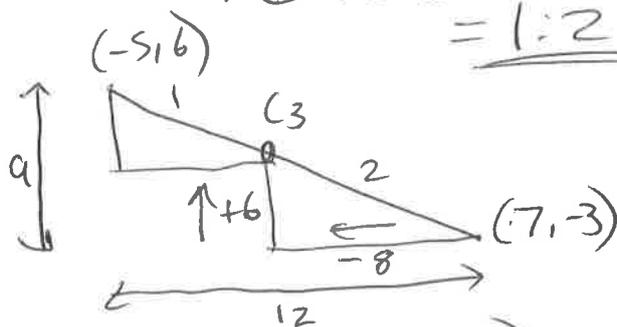
$C_2 = (7, -3)$
 $r_2 = \sqrt{(-7)^2 + 3^2 - 54}$
 $= \sqrt{4}$
 $= 2$



$A \rightarrow B = 6$ and $A \rightarrow C_3 = 8$
 so $B \rightarrow C_3 = 2$
 and $C_1 \rightarrow C_3 = 5$

$C_3 \rightarrow D = 8$ and $D \rightarrow C_2 = 2$
 so $C_3 \rightarrow C_2 = 10$

$\therefore C_3$ splits $C_1 \rightarrow C_2$ in
 the ratio $5:10$
 $= 1:2$



$\therefore C_3 = (-1, 3)$

$\therefore (x+1)^2 + (y-3)^2 = 64$

17