Name _Solutions_____

Teacher _____

Mathematics

Paper 2

National 5 Booster Paper B2

Duration: 1 hour 50 minutes

Total Marks – 60

Attempt **ALL** questions.

You may use a calculator

To earn full marks, you must show your working in your answers.

State the units for your answer where appropriate.

Write your answers clearly in the spaces provided in this booklet.

Use **blue** or **black** ink.

Notes:

- This is a Booster Paper. Your May exam will be (a bit) harder than this.
- The Booster Papers get **more challenging** as you work through them.
- The final Booster Paper will be as challenging as your May exam.
- The number of marks indicated beside each question is intended as a guide and may differ slightly from SQA marking instructions.
- These original papers are **produced independently of the SQA** and are free of charge.
- All Booster Papers and answers can be found at www.maths180.com/BoosterPapers

Total marks – 60

Attempt ALL questions

1.   Solve $5\cos x° + 1 = 3$ where $0 \leq x \leq 360°$ 3

$5\cos x + 1 = 3$

$5\cos x = 2$

$\cos x = \dfrac{2}{5}$

S	A ✓
T	C ✓

$x = 66.4 , \ 360 - 66.4$

$x = 66.4° , \ 293.6°$

2.   Solve, algebraically, $3x + 38 \geq 7x + 6$ 3

$3x + 38 \geqslant 7x + 6$

$-3x \qquad\qquad -3x$

$38 \geqslant 4x + 6$

$-6 \qquad\qquad -6$

$32 \geqslant 4x$

$8 \geqslant x$

$x \leq 8$

3. The number of electric cars using a charging point was counted and recorded every day for 5 days in 2015. The results are shown below.

3	3	2	3	4

(a) Calculate (correct to 1 decimal place) the mean and standard deviation for this data. **4**

$$\text{mean} = \frac{3+3+2+3+4}{5} = \frac{15}{5} = 3$$

Standard deviation

Method 1

x	$x-\bar{x}$	$(x-\bar{x})^2$
3	3-3=0	$0^2 = 0$
3	3-3=0	$0^2 = 0$
2	2-3=-1	$(-1)^2 = 1$
3	3-3=0	$0^2 = 0$
4	4-3=1	$1^2 = 1$
		$\overline{2}$

$$s.d. = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{2}{4}}$$

$$= 0.707$$

$$= 0.7 \ (1d.p)$$

Method 2

x	x^2
3	9
3	9
2	4
3	9
4	16
15	47

$$s.d = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}}$$

$$= \sqrt{\frac{47 - \frac{15^2}{5}}{4}}$$

$$= \sqrt{\frac{2}{4}}$$

$$= 0.707$$

$$= 0.7 \ (1d.p)$$

The same survey was conducted over a week in 2019.
The mean number of cars was 7 and the standard deviation was 1.5

(b) Make two valid comparisons between the data for 2015 and 2019. **2**

On average the number of cars using a charging point has increased from 2015 to 2019 (7 > 3).

The number of cars using the charge point was more consistant in 2015 than 2019 (0.7 < 1.5)

4. Solve, algebraically, $\dfrac{x+9}{2}+\dfrac{x-3}{3}=5$ 3

$$\frac{x+9}{2}+\frac{x-3}{3}=5$$

$$\frac{6(x+9)}{2}+\frac{6(x-3)}{3}=5\times6$$

Multiply each term by 6 (LCM of 2+3) and simplify

$$3(x+9)+2(x-3)=30$$

$$3x+27+2x-6=30$$

$$5x+21=30$$

$$5x=9$$

$$x=\frac{9}{5}$$

5. Find the resultant vector $3\boldsymbol{u}-2\boldsymbol{v}$ when $u=\begin{pmatrix}4\\-2\\3\end{pmatrix}$ and $v=\begin{pmatrix}-5\\3\\-2\end{pmatrix}$.

Express your answer in component form. 3

$$3\underline{u}-2\underline{v}=3\begin{pmatrix}4\\-2\\3\end{pmatrix}-2\begin{pmatrix}-5\\3\\-2\end{pmatrix}$$

$$=\begin{pmatrix}12\\-6\\9\end{pmatrix}-\begin{pmatrix}-10\\6\\-4\end{pmatrix}$$

$$=\begin{pmatrix}22\\-12\\13\end{pmatrix}$$

6.   A house has reduced in value by 20% to £86000.

       Calculate the original value.   3

$$100\% - 20\% = 80\%$$

$$\div 8 \left\{ \begin{array}{l} 80\% \longrightarrow 86000 \\ 10\% \longrightarrow 10750 \\ 100\% \longrightarrow 107500 \end{array} \right. \begin{array}{l} \}\div 8 \\ \\ \} \times 10 \end{array}$$

$$\text{Original value} = £107500$$

7.   Simplify $\dfrac{24x^6}{5x^3 \times 4x^2}$ 2

$$\dfrac{24x^6}{5x^3 \times 4x^2}$$

$$= \dfrac{24x^6}{20x^5}$$ ⟨ add indices when multiplying ⟩

$$= \dfrac{6}{5}x$$ ⟨ subtract indices when dividing ⟩

8.   This sector has an area of 28.6 square centimetres and radius 5 centimetres.

Calculate the size of angle $x°$

Give your answer to the nearest degree.

5 cm
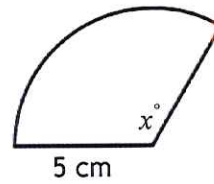
4

$$\text{area of sector} = \frac{\text{angle}}{360} \times \pi r^2$$

$$28.6 = \frac{x}{360} \times \pi \times 5^2$$

$$x = \frac{28.6 \times 360}{(\pi \times 5^2)}$$

$$= 131.09\ldots$$

$$= 131° \text{ (to nearest degree)}$$

9. Use the discriminant to determine the nature of the roots of $y = 9x^2 - 12x + 4$. **3**

$$a = 9 \quad b = -12 \quad c = 4$$

$$b^2 - 4ac = (-12)^2 - 4 \times 9 \times 4$$

$$= 144 - 144$$

$$= 0$$

• As $b^2 - 4ac = 0$ there are two equal real roots

OR

• As $b^2 - 4ac = 0$ there is one repeated real root.

10. These two juice bottles are mathematically similar.

8 cm

4 cm

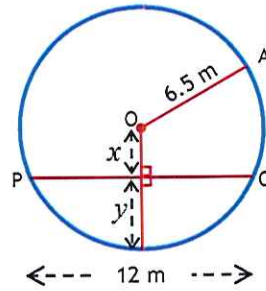The volume of the smaller bottle is 28 cubic centimetres.

Find the volume of the larger bottle. **3**

Linear Scale Factor $= \dfrac{8}{4} = 2$

Volume Scale Factor $= 2^3 = 8$
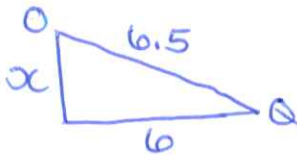
Volume of large bottle $= 8 \times 28 = 224 \, cm^3$

11. OA is a radius with length 6.5 metres.

PQ is a chord with length 12 metres.

All lengths are in metres.

The line marked x joins the centre to the chord.

(a)　Find the length of the line marked x .　　　　　　　　　　3

$$x^2 = 6.5^2 - 6^2$$
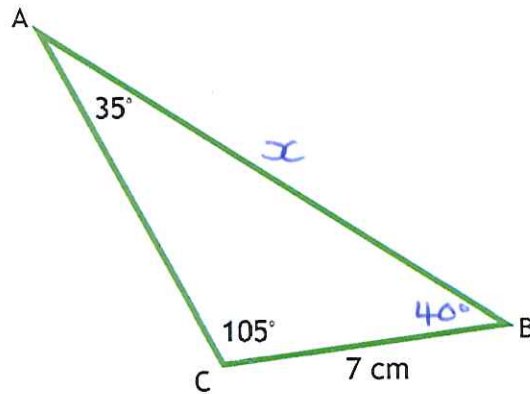$$= 42.25 - 36$$
$$= 6.25$$
$$x = \sqrt{6.25}$$
$$x = 2.5m.$$

The line marked y meets the chord PQ at 90° and joins the chord to the circumference.

(b)　Find the length of the line marked y .　　　　　　　　　　1

$$y = radius - x$$
$$= 6.5 - 2.5$$
$$= 4m$$

12. Calculate the length of side AB.

Give your answer to 3 significant figures.

$\angle ABC = 180 - 35 - 105 = 40$

$$\frac{x}{\sin 105} = \frac{7}{\sin 35}$$

$$x \sin 35 = 7 \sin 105$$

$$x = \frac{7 \sin 105}{\sin 35}$$

$$= 11.7882\cdots$$

$$= 11.8 \text{ cm} \quad (\text{to } 3 \text{ sig fig})$$

3

13.   Solve the equation

$$5x^2 - 3x - 7 = 0$$

Give your answer(s) correct to two decimal places.   4

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 5$
$b = -3$
$c = -7$

$$= \frac{3 \pm \sqrt{(-3)^2 - 4 \times 5 \times (-7)}}{2 \times 5}$$

$$= \frac{3 \pm \sqrt{149}}{10}$$

$$= \frac{3 + \sqrt{149}}{10} \quad , \quad \frac{3 - \sqrt{149}}{10}$$

$$= 1.5206 \quad , \quad -0.9206\ldots$$

$$= 1.52 \quad , \quad -0.92 \quad \text{(to 2 decimal places)}$$

14. Solve the equation $\dfrac{2x+3}{2} = 4x$

Give your answer in its simplest form. 3

$$\frac{2x+3}{2} = 4x$$
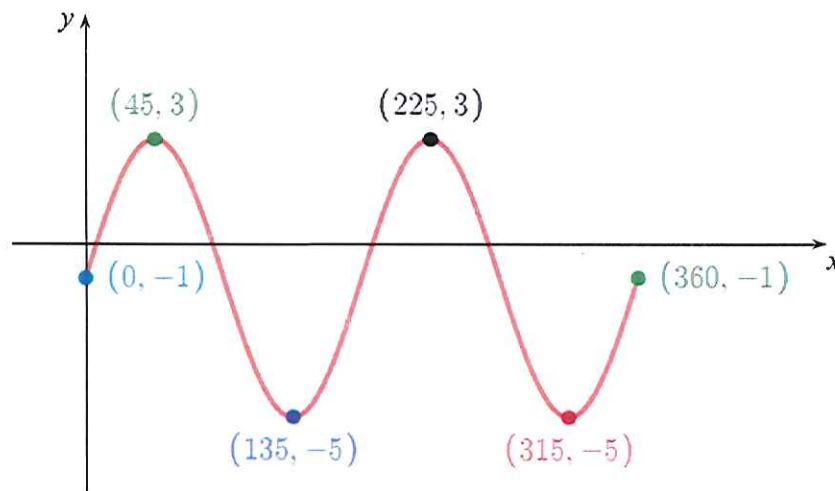
(multiply both sides by 2)

$$2x + 3 = 8x$$

$$3 = 6x$$

$$\frac{3}{6} = x$$

$$x = \frac{3}{6} = \frac{1}{2}$$

15. This graph has an equation of the form $y = a\sin bx + c$

Write down the values of a, b and c. 3

$$a = \left(\frac{3+5}{2}\right) = 4$$

$b = 2$ (number of patterns in $360°$)

$c = -1$ (graph has been moved down 1 place)

16. (a) Write down the coordinates of the turning point of the graph of $y = (x+3)^2 + 1$. 2

(moved left 3 places, up 1 place)

$(-3, 1)$

(b) Write down equation of the axis of symmetry. 1

$x = -3$

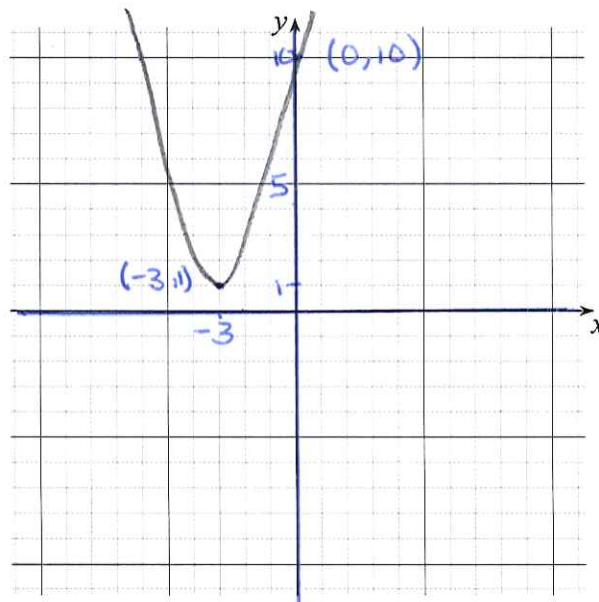(c) Write down the coordinates of the point where the graph meets the $y-$axis. 1

At the y-axis $x = 0$
$y = (0+3)^2 + 1$
$y = 9 + 1 = 10$ $(0, 10)$

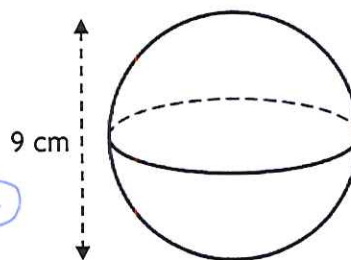(d) Sketch and annotate fully the graph of $y = (x+3)^2 + 1$. 2

17. The sphere shown has diameter 9 centimetres.

(a) Calculate the volume of this sphere.

$V = \frac{4}{3} \pi r^3$ (from formula sheet)

9 cm

$V = \frac{4}{3} \times \pi \times 4.5^3$

$= 381.7 \, cm^3$

radius = diameter ÷ 2

$= 9 \div 2$

$= 4.5$

2

The cone shown here has radius 5 centimetres and volume 209.44 cubic centimetres.

(b) Find (to the nearest centimetre) the height (h) of this cone.
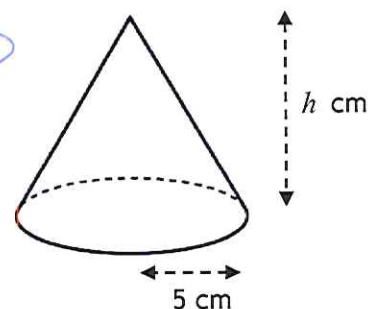
$V = \frac{1}{3} \pi r^2 h$ (from formula sheet)

$209.44 = \frac{1}{3} \times \pi \times 5^2 \times h$

$h = \dfrac{209.44 \times 3}{(\pi \times 5^2)}$

h cm

5 cm

$= 8.0000187$

$= 8 \, cm$ (to the nearest cm)

3

End of Booster Paper B2