Name SOLUTIONS

Teacher

Mathematics

Paper 2

National 5 Booster Paper D2

Duration: 1 hour 50 minutes

Total Marks – 60

Attempt **ALL** questions.

You may use a calculator

To earn full marks, you must show your working in your answers.

State the units for your answer where appropriate.

Write your answers clearly in the spaces provided in this booklet.

Use **blue** or **black** ink.

Notes:

- This is a **Booster Paper**. Your May exam will be (a bit) harder than this.
- The Booster Papers get **more challenging** as you work through them.
- The final Booster Paper will be as challenging as your May exam.
- The number of marks indicated beside each question is intended as a guide and may differ slightly from SQA marking instructions.
- These original papers are **produced independently of the SQA** and are **free of charge**.
- All Booster Papers and answers can be found at www.maths180.com/BoosterPapers

FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sine Rule: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule: $$a^2 = b^2 + c^2 - 2bc\cos A \quad \text{or} \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Area of a triangle: $$A = \frac{1}{2}ab\sin C$$

Volume of a sphere: $$V = \frac{4}{3}\pi r^3$$

Volume of a cone: $$V = \frac{1}{3}\pi r^2 h$$

Volume of a pyramid: $$V = \frac{1}{3}Ah$$

Standard deviation: $$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

or $$s = \sqrt{\frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1}}$$, where n is the sample size.

MARKS

Total marks - 60

Attempt ALL questions

1. Solve $7\tan x^\circ = 5\tan x^\circ - 3$ where $0 \le x \le 360^\circ$ 3

$$2\tan x = -3$$
$$\tan x = -\frac{3}{2}$$

1st quadrant angle $= \tan^{-1}\left(\frac{3}{2}\right) = 56.309...$

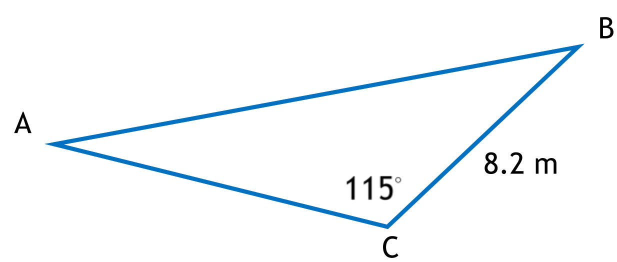
$$x = 180 - 56.309...,\ 360 - 56.309...$$
$$x = 123.690...,\ 303.690...$$
$$x = 123.7^\circ,\ 303.7^\circ$$

2. The area of this triangle is 40.3 square metres.

All lengths are in metres.

Find the length of side AC.

Round your answer to 1 decimal place.



$$\frac{1}{2}(8.2)(\text{AC})\sin 115 = 40.3$$ 2

$$4.1(\text{AC})\sin 115 = 40.3$$
$$\text{AC} = \frac{40.3}{4.1\sin 115}$$
$$\text{AC} = 10.845...$$

AC = 10.8 metres (1 decimal place)

3. Mr Campbell has been working hard on the indoor running machine. Over 5 days, he recorded how many metres he ran in 6 minutes. The results are shown below.

1200 1250 1350 1300 1350

(a) Find the mean and standard deviation of these distances. 3

$$\bar{x} = \frac{\sum x}{n} = \frac{6450}{5} = 1290 \text{ metres}$$

$$s = \sqrt{\frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1}}$$

$$s = \sqrt{\frac{8337500 - \frac{41602500}{5}}{4}}$$

$$s = \sqrt{\frac{8337500 - 8320500}{4}}$$

$$s = \sqrt{4250}$$

$$s = 65.192...$$

$$s = 65.2 \text{ metres (1 decimal place)}$$

A few weeks later, Mr Campbell changed to running outside on an athletics track. Again, he recorded how many metres he ran in 6 minutes over a period of 5 days.

His mean distance was 1450 metres and the standard deviation was 75 metres.

(b) Make two valid comparisons between the distances run indoors on the machine and those run outside on the track. 2

On average, Mr Campbell ran further per day on the track. (1450 > 1290)

The distances run by Mr Campbell on the machine were more consistent. (65.2 < 75)

4. Solve, algebraically, $\frac{4x+1}{2} - \frac{x+2}{3} = 2$ 3

$$\frac{3(4x+1)}{6} - \frac{2(x+2)}{6} = 2$$

$$\frac{12x+3}{6} - \frac{2x+4}{6} = 2$$

$$\frac{10x-1}{6} = 2$$

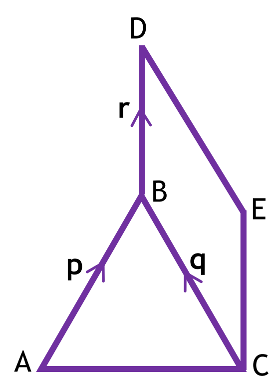
$$10x - 1 = 12$$

$$10x = 13$$

$$x = \frac{13}{10} \quad \text{or} \quad 1.3$$

5. This diagram is formed from an equilateral triangle and a parallelogram.

In the diagram, $\overrightarrow{AB}$, $\overrightarrow{CB}$ and $\overrightarrow{BD}$ represent the vectors **p**, **q** and **r** respectively.



(a) Express $\overrightarrow{AD}$ in terms of **p** and **r**. 1

AD = **p** + **r**

(b) Express $\overrightarrow{AE}$ in terms of **p**, **q** and **r**. 2

AE = **p** + **r** - **q**

6. A house was valued in April 2015. By April 2016, the value had fallen by 13%.

Between April 2016 and April 2017, the house value increased by 5% to £45675.

Find the value of the house in April 2015. 3

$$= 45675 \div 1.05 \div 0.87$$

= £50000

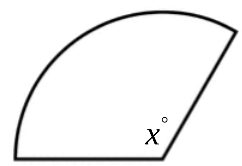
7. Simplify $2x^{\frac{3}{2}}\left(x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}\right)$ 2

$$= 2x^{\left(\frac{3}{2}+\frac{1}{2}\right)} + 6x^{\left(\frac{3}{2}-\frac{1}{2}\right)}$$

$$= 2x^2 + 6x$$

8. This sector has an arc length of 95 centimetres and radius 45 centimetres.

The sector angle is labelled x°



Calculate the area of the sector.

Round your answer to the nearest whole number. 3

$$\frac{\text{Sector Area}}{\pi \times 45^2} = \frac{95}{\pi \times 45 \times 2}$$

$$\text{Sector Area } = \frac{95 \times \pi \times 45^2}{\pi \times 45 \times 2}$$

Sector Area = 2137.5 = 2138 cm^2 (nearest whole number)

Alternative method

$$\frac{95}{\pi \times 45 \times 2} \times 360 = 120.957...^\circ$$

$$\frac{120.957...}{360} \times \pi \times 45^2 = 2137.5$$ = 2138 cm^2 (nearest whole number)

9. Find the range of values of k so that $y = kx^2 - 6x + 1$ has no real roots. 3

$b^2 - 4ac < 0$ for real roots

$(-6)^2 - 4(k)(1) < 0$

$36 - 4k < 0$

$-4k < -36$

$k > 9$

10. These two tins are mathematically similar in shape.

V = 72 cm^3

V = 243 cm^3

The volume of the smaller tin is 72 cubic centimetres.

The volume of the larger tin is 243 cubic centimetres.

The surface area of the smaller tin is 140 square centimetres.

Find the surface area of the larger tin. 3

Volume scale factor $= \dfrac{243}{72} = \dfrac{27}{8}$

Linear scale factor $= \sqrt[3]{\dfrac{27}{8}} = \dfrac{3}{2}$

Area scale factor = $\left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}$

Surface area of larger tin $= \dfrac{9}{4} \times 140 =$ 315 square centimetres

11. Mr Smith has made some chocolate dipped biscuits.

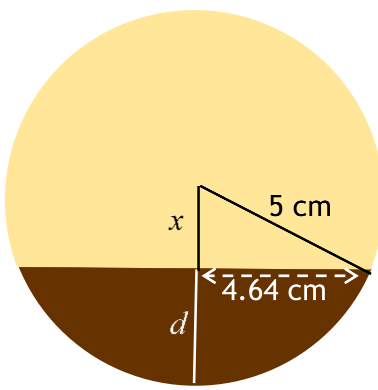
Each biscuit has diameter 10 centimetres.

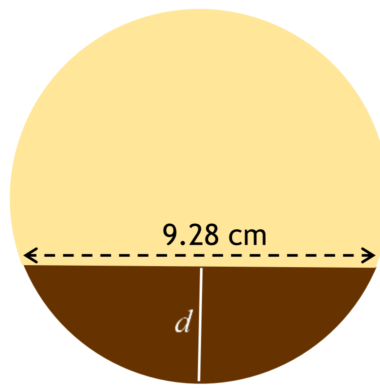
One biscuit is shown in the diagram below.

The width of chocolate at its widest point is 9.28 centimetres.

Calculate the depth (d) to which the biscuit was dipped.

Give your answer to two decimal places. 3





$$x^2 = 5^2 - 4.64^2$$
$$x^2 = 3.4704$$
$$x = \sqrt{3.4704}$$
$$x = 1.862\ldots$$

$$d = 5 - x$$
$$d = 5 - 1.862\ldots$$
$$d = 3.137\ldots$$

$d =$ 3.14 cm (to two decimal places)

12. The diagram shows the position of three oil rigs.

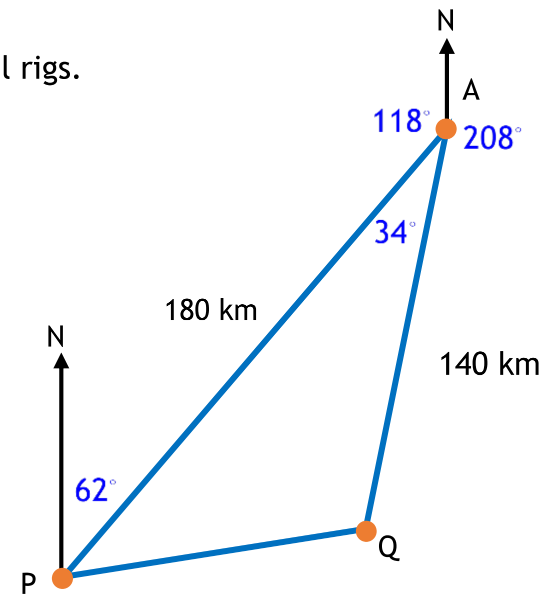
P is 180 kilometres from A

Q is 140 kilometres from A

From P, the bearing of A is 062°

From A, the bearing of Q is 208°

How far apart are oil rigs P and Q?



$180^\circ - 62^\circ = 118^\circ$

$118^\circ + 208^\circ = 326^\circ$

$360^\circ - 326^\circ = 34^\circ$

$PQ^2 = 180^2 + 140^2 - 2(180)(140)\cos 34^\circ$

$PQ^2 = 10216.506..$

$PQ = 101.076...$

$PQ = 101.1$ kilometres

4

13. Solve the equation

$$7x^2 + 2x - 3 = 2x^2 + 5x - 2$$

Give your answer(s) correct to three significant figures. 4

$$7x^2 + 2x - 3 = 2x^2 + 5x - 2$$

$$5x^2 - 3x - 1 = 0$$

$$a = 5, \quad b = -3, \quad c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(5)(-1)}}{2(5)}$$

$$x = \frac{3 \pm \sqrt{29}}{10}$$

$$x = \frac{3 + \sqrt{29}}{10}, \quad x = \frac{3 - \sqrt{29}}{10}$$

$$x = 0.83851\ldots, \quad x = -0.2385\ldots$$

$x = 0.839, \quad x = -0.239$ (to 3 significant figures)

14. Solve the equation $\frac{5}{2x}-\frac{1}{4x}=6$ Give your answer in its simplest form. 3

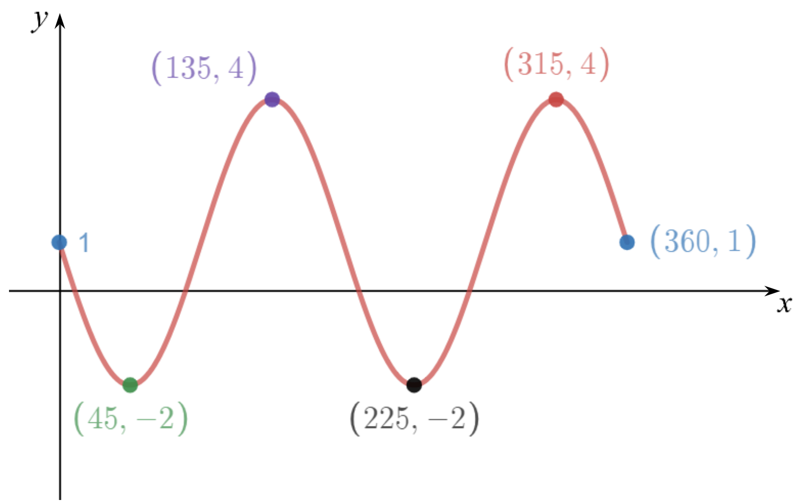
$$\frac{10}{4x}-\frac{1}{4x}=6$$

$$\frac{9}{4x}=6$$

$$9=24x$$

$$x=\frac{9}{24} \qquad x=\frac{3}{8}$$

15. This graph has an equation of the form $y=a\sin bx+c$



Write down the values of a, b and c. 3

$$a=-\frac{\text{max}-\text{min}}{2}=-\frac{4-(-2)}{2}=-3$$

$b=2$ (number of complete waves between 0 and 360°)

$c=1$ (vertical shift)

16. (a) Write down the coordinates of the turning point of the graph of $y = 5 - (x+3)^2$. 2

$(-3, 5)$

(b) Write down equation of the axis of symmetry. 1

$x = -3$

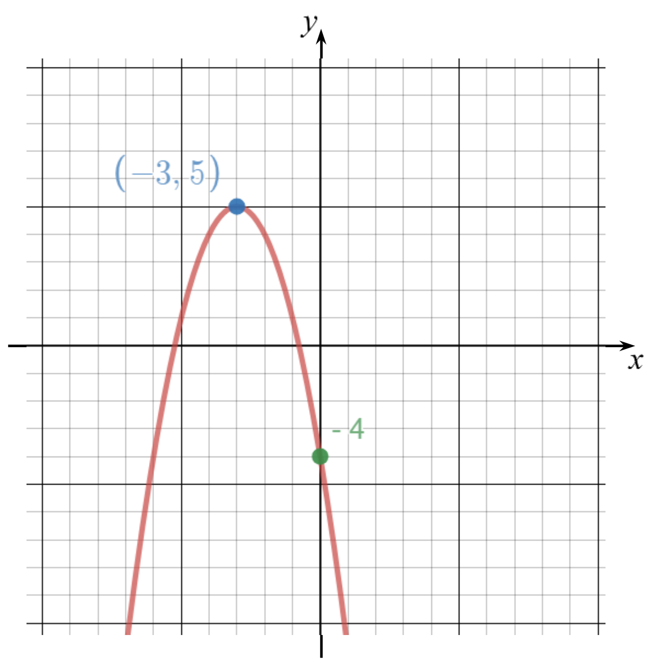
(c) Write down the coordinates of the point where the graph meets the $y-$axis. 1

Crosses $y-$axis when $x = 0$

$y = 5 - (0+3)^2 = -4$

$(0, -4)$

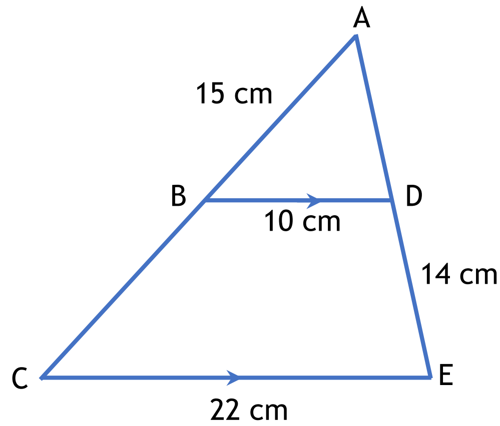
(d) Sketch the graph of $y = 5 - (x+3)^2$. Mark on the coordinates of the turning point and the point of intersection with the $y-$axis. 2



17. In the figure opposite, BD = 10, CE = 22, AB = 15 and DE = 14.

Find the length of BC.

All lengths are in centimetres.



3

Linear scale factor = $\frac{22}{10} = \frac{11}{5}$

AC $= \frac{11}{5} \times 15 = 33$

BC $= 33 - 15 = 18$ centimetres

18. Simplify $\sin x(\sin x+1)+\cos x(\cos x+1)-1$ 2

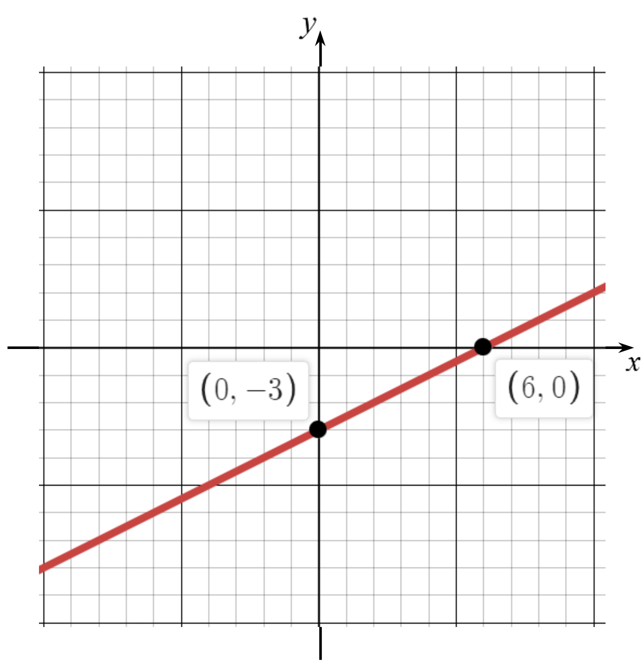
$= \sin^2 x + \sin x + \cos^2 x + \cos x - 1$

$= \sin x + \cos x + 1$

since $\sin^2 x + \cos^2 x = 1$

19. On the axes provided, draw the straight line with equation $2y = x - 6$.

Mark on the coordinates of the points where the line crosses the axes. 2



End of Booster Paper D2