Indices

Indices:

We are already familiar with squares and cubes: e.g. 4^2, 2^3 etc and their meaning.
We are also used to dealing with powers of 10 in standard form. e.g. 2.5×10^6.

Intuitively we know that: a^3 means $a \times a \times a$

Rules of indices:

Three basic rules:

$a^m \times a^n = a^{m+n}$ when multiplying – add the indices

e.g. $a^2 \times a^3 \rightarrow (a \times a) \times (a \times a \times a) \rightarrow a^5$

$a^m \div a^n = a^{m-n}$ when dividing – subtract the indices

e.g. $a^5 \div a^3 \rightarrow \frac{a^5}{a^3} \rightarrow \frac{\cancel{a} \times \cancel{a} \times \cancel{a} \times a \times a}{\cancel{a} \times \cancel{a} \times \cancel{a}} \rightarrow a^2$

$(a^m)^n = a^{mn}$ when raising to a power – multiply the indices

e.g. $(a^3)^2 \rightarrow (a \times a \times a)^2 \rightarrow (a \times a \times a) \times (a \times a \times a) \rightarrow a^6$

Zero Indices:

What does a^0 mean ?

Using the above rule for multiplying: $a^m \times a^n = a^{m+n}$

then $a^m \times a^0 = a^{m+0} \rightarrow a^m$

So, $a^0 = 1$

Negative Indices:

What does a^{-m} mean ?

Using the above rule for multiplying: $a^m \times a^n = a^{m+n}$

then $a^m \times a^{-m} = a^{m-m} \rightarrow a^0 \rightarrow 1$

So, $a^{-m} = \frac{1}{a^m}$

In general think of a negative index as meaning '1 over'

Fractional Indices:

What does $a^{\frac{m}{n}}$ mean ?

Using the rule for raising powers: $\left(a^m\right)^n = a^{mn}$

In particular: $\left(a^{\frac{1}{2}}\right)^2 = a^1 = a$

so $a^{\frac{1}{2}} = \sqrt{a}$

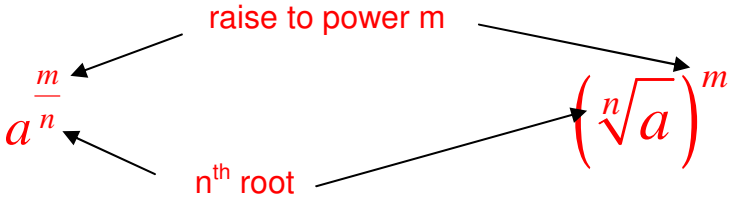
similarly, $a^{\frac{1}{3}} = \sqrt[3]{a}$ (the cube root of a)

In general: $a^{\frac{1}{n}} = \sqrt[n]{a}$

again using the rule: $\left(a^m\right)^n = a^{mn}$

we find: $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

So, $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$



Applications:

Simplify: $2a^2 \times a^3 \rightarrow 2 \times a^{2+3} \rightarrow 2a^5$

Simplify: $\left(c^5\right)^2 \rightarrow c^{5\times 2} \rightarrow c^{10}$

Simplify: $\dfrac{x^3 \times x^4}{x^2} \rightarrow x^{3+4-2} \rightarrow x^5$

Simplify: $x^3\left(x^2 + x^4\right) \rightarrow x^3 \times x^2 + x^3 \times x^4 \rightarrow x^5 + x^7$

Simplify: $n^{-2}\left(n^3 + n\right) \rightarrow n^{-2} \times n^3 + n^{-2} \times n \rightarrow n + n^{-1} \rightarrow n + \dfrac{1}{n}$

Evaluate: $8^{\frac{2}{3}} \rightarrow \left(\sqrt[3]{8}\right)^2 \rightarrow (2)^2 \rightarrow 4$

Evaluate: $16^{-\frac{3}{4}} \rightarrow \dfrac{1}{\left(\sqrt[4]{16}\right)^3} \rightarrow \dfrac{1}{(2)^3} \rightarrow \dfrac{1}{8}$

Past Paper Questions:

1. Evaluate $27^{\frac{2}{3}}$

   [Ans. $\left(\sqrt[3]{27}\right)^2 \rightarrow 3^2 \rightarrow 9$]

2. Express in its simplest form $y^{10} \times \left(y^4\right)^{-2}$

   [Ans. $y^{10} \times y^{-8} \rightarrow y^2$]

3. Simplify $a^3(a^{-7} + 5)$

   [Ans. $a^3 \times a^{-7} + 5a^3 \rightarrow a^{-4} + 5a^3$]

4. Express $\dfrac{3y^5 \times 4y^{-1}}{6y}$ in its simplest form.

   [Ans. $\dfrac{3 \times 4 \times y^5 \times y^{-1}}{6y} \rightarrow \dfrac{12 \times y^{5-1}}{6y} \rightarrow \dfrac{\cancel{12}^{2} \times y^4}{\cancel{6}^{1} y} \rightarrow 2y^3$]

5. Express $\frac{y^4 \times y}{y^{-2}}$ in its simplest form.

[Ans. $\frac{y^{4+1}}{y^{-2}} \rightarrow y^{5-(-2)} \rightarrow y^7$]

6. Express $\frac{b^{\frac{1}{2}} \times b^{\frac{3}{2}}}{b}$ in its simplest form.

[Ans. $\frac{b^{\frac{1}{2}} \times b^{\frac{3}{2}}}{b} \rightarrow \frac{b^{\frac{1}{2}+\frac{3}{2}}}{b} \rightarrow \frac{b^2}{b} \rightarrow b$]

7. Remove the brackets and simplify $b^{\frac{1}{2}}\left(b^{\frac{1}{2}} + b^{-\frac{1}{2}}\right)$

[Ans. $b^{\frac{1}{2}}\left(b^{\frac{1}{2}} + b^{-\frac{1}{2}}\right) \rightarrow b^{\frac{1}{2}+\frac{1}{2}} + b^{\frac{1}{2}+\left(-\frac{1}{2}\right)} \rightarrow b^1 + b^0 \rightarrow b+1$]

8. Remove the brackets and simplify $a^{\frac{1}{2}}\left(a + \frac{1}{a}\right)$

[Ans. $a^{\frac{1}{2}}\left(a + \frac{1}{a}\right) \rightarrow a^{\frac{1}{2}} \times a + a^{\frac{1}{2}} \times a^{-1} \rightarrow a^{\frac{3}{2}} + a^{-\frac{1}{2}}$]