Appreciation & Depreciation

**Percentage Multipliers:**

If we wish to increase a quantity by a percentage to a new value, we can:

1. Find the percentage of the quantity
2. Add it on to the original amount to obtain the new value.

This requires 2 steps.

**Example:**

Peter earns a salary of £12,000 p.a., this year he gets a 10% rise, what will his new salary be?

His 10% rise is 10% of £12,000 = £1,200
His new salary will be £12,000 + £1,200 = £13,200

We can however do this calculation in a single step.

His original salary corresponds to 100%, he gets a rise of 10%, so he now has 100% + 10% = 110% of his original salary.

To find 110% we multiply by \( \frac{110}{100} \) which is 1.10

So Peter’s new salary is £12,000 \( \times 1.1 = £13,200 \)

**Multiplier** = \( \frac{100 + \% \text{ increase}}{100} \)

**Examples:**

To obtain an increase of:

5% multiply by 1.05 \( (105 \div 100) \)
7% multiply by 1.07 \( (107 \div 100) \)
20% multiply by 1.20 \( (120 \div 100) \)
2½% multiply by 1.025 \( (102.5 \div 100) \)

**Try these:**

Find the multiplier to give an increase of:

15% \[ \text{Ans: 1.15} \]
25% \[ \text{Ans: 1.25} \]
3% \[ \text{Ans: 1.03} \]
17½% \[ \text{Ans: 1.175} \]
8¾% \[ \text{Ans: 1.0875} \]
7.3% \[ \text{Ans: 1.073} \]
**Decrease**
Similarly, to find a decrease, we subtract from 100.

For example:
To find a decrease of 10%

We want 100% − 10% = 90%, so we multiply by $\frac{90}{100}$ i.e. 0.9

\[ \text{Multiplier} = \frac{100 - \% \text{ increase}}{100} \]

Examples:
To obtain an decrease of:
5% multiply by 0.95 (95 ÷ 100)
7% multiply by 0.93 (93 ÷ 100)
20% multiply by 0.80 (80 ÷ 100)
2½% multiply by 0.975 (97.5 ÷ 100)

Try these:
Find the multiplier to give an decrease of:
15% [ Ans: 0.85 ]
25% [ Ans: 0.75 ]
3% [ Ans: 0.97 ]
7½% [ Ans: 0.925 ]
8¾% [ Ans: 0.9125 ]
7.3% [ Ans: 0.927 ]

**Definitions:**
**Appreciation:**
A gain or increase in value over time. Items that appreciate in value are:
Appreciation is usually expressed as a percentage.

**Depreciation:**
A loss or decrease in value over time. Items that depreciate in value are:
Cars, Machinery, Technology *e.g.* computers, Furniture.
Depreciation is usually expressed as a percentage.
Use of multipliers:

If a house is valued today at £80,000 and is expected to appreciate by 3% p.a. (per year). Then we can calculate the value after 3 years.

We can calculate year by year.

You should note that appreciation is always calculated on the value at the start of each year, so it is compounded.

<table>
<thead>
<tr>
<th>Start Value:</th>
<th>Increase</th>
<th>Value at end of year.</th>
</tr>
</thead>
<tbody>
<tr>
<td>£80,000</td>
<td>£2,400</td>
<td>£82,400 (after 1 year)</td>
</tr>
<tr>
<td>£82,400</td>
<td>£2,472</td>
<td>£84,872 (after 2 years)</td>
</tr>
<tr>
<td>£84,872</td>
<td>£2,546.16</td>
<td>£87,418.16 (after 3 years)</td>
</tr>
</tbody>
</table>

So value after 3 years is: £87,418.16

An easier way to calculate is to use the **multiplier** instead:

An increase of 3% corresponds to a multiplier of 1.03

After 3 years, the house is worth:

\[
£80,000 \times 1.03 \times 1.03 \times 1.03 \\
\text{or } £80,000 \times 1.03^3 = £87,418.16
\]

Similarly, we can apply this to **depreciation**.

**Example:**

A car is bought new for £15,000 and **depreciates** at 20% p.a.

What is the value of the car after 4 years.

**Solution:**

The car loses 20% of its value each year, so it is worth only 80%

So, multiplier = 0.8

After 4 years, car is worth:

\[
£15,000 \times 0.8 \times 0.8 \times 0.8 \times 0.8 \\
\text{or } £15,000 \times 0.8^4 = £6,144
\]
We can have problems involving both appreciation and depreciation:

A factory is valued at £120,000 for the building and £60,000 for the machinery.
If the building appreciates by 5% p.a. and the machinery depreciates by 8% p.a.,
calculate the total value of the buildings and machinery after 5 years.

Value of building after 5 years: \( £120,000 \times 1.05^5 = £153,153.79 \)

Value of machinery after 5 years: \( £60,000 \times 0.92^5 = £39,544.89 \)

Total value of building and machinery: \( £153,153.79 + £39,544.89 = £192,698.68 \)

We can have problems involving multiple rates of depreciation:

A car is purchased for £20,000.
It is assumed to depreciate by 25% in the 1st year, 20% in the 2nd year
and 15% in each of the 3rd and 4th years.

Calculate the value of the car after 4 years.

Value of car after 4 years: \( £20,000 \times 0.75 \times 0.8 \times 0.85 \times 0.85 = £8,670 \)

The same principles apply to growth (increase) and decay (decrease) problems:

Example of growth:

A colony of bacteria initially contain 25,000 bacteria.
It is found that the colony grows at a rate of 35% per hour.
What will be the size of the colony after 3 hours.

Size of colony after 3 hours: \( 25,000 \times 1.35^3 = 61509.375 = 61509 \) bacteria.

Example of decay:

A flask contains 5 litres of a chemical.
If it is left open to the air, it is found that the chemical evaporates at a rate of 15% per hour.
How much chemical will be left after 5 hours.

After 3 hours: \( 5,000 \times 0.85^5 = 2218.5 \) millilitres.
Past Paper Questions

1. Bacteria in a test tube increase at the rate of 0.9% per hour. 
At 12 noon there are 4500 bacteria. 
At 3 pm, how many bacteria will be present? 
Give your answer to 3 significant figures.

2. In January 2001, it was estimated that the number of flamingos in a colony was 7000. 
The number of flamingos is decreasing at the rate of 14% per year. 
How many flamingos are expected to be in this colony in January 2005? 
Give your answer to the nearest 10.

3. In 1999, a house was valued at £70,000 and the contents were valued at £45,000. 
The value of the house appreciates by 7% each year. 
The value of the contents depreciates by 9% each year. 
What will be the total value of the house and contents in 2002?

4. A factory was put on the market in January 2001. 
The site was in an excellent location so the value of the building has appreciated since then by 5.3% per year. 
Unfortunately the plant & machinery were poorly maintained and have depreciated by 8.5% per year. 
The value of the building was £435 000 and the value of the plant & machinery was £156 000 in January 2001. 
What would be the expected value of the complete factory in January 2003?

5. How much would the Strachans pay for a new iron, priced £16.50 at Watsons?

Solutions:

1. \[4500 \times 1.009^3 \approx 4622.59678... \quad 4620 \text{ (3 sf)}\]
2. \[7000 \times 0.86^4 \approx 3829.0571... \quad 3830 \text{ (nst 10)}\]
3. House: \[£70 000 \times 1.07^3 = £ 85 753.01\]
   Contents: \[£45 000 \times 0.91^3 = £ 33 910.70\]
   Total value: \[= £ 119 663.71\]
4. Factory: \[£435 000 \times 1.053^2 = £ 482 331.92\]
   Plant & Mcy: \[£156 000 \times 0.915^2 = £ 130 607.10\]
   Total value: \[= £ 612 939.02\]
5. \[66\frac{2}{3} \% = \frac{2}{3}\]
   So, \[\frac{2}{3}\] off means you pay \[\frac{1}{3}\]
   They pay \[\frac{1}{3}\] of £16.50 = £ 5.50
Reversing the change:

Quite often we are given the result after a percentage change has been applied, and asked to calculate the original value.

Example:

A ticket is on sale at 40% discount.
Paul paid £9.00 for the ticket.
What was the original price before the discount.

Solution:

A 40% discount means that the ticket was sold for 60% of its price.

i.e. 60% is equivalent to £9.00

So, 1% is equivalent to £9.00 \( \div \) 60

and 100% is equivalent to £9.00 \( \div \) 60 \( \times \) 100 = £15

Original price of ticket was £15. (You can check this by taking 40% off it)

An alternative (algebraic) solution:

Let the original price be £ P

Then reduce the price by 40%. \( \rightarrow \) \( P \times 0.6 \)

So: \( \rightarrow \) \( P \times 0.6 = 9.00 \)

Divide both sides by 0.6 \( \rightarrow \) \( P = £15.00 \)

Examples:

7. A computer is sold for £695. This price includes VAT at 17.5%, Calculate the price of the computer without VAT.

8. During the Christmas Sales a shopkeeper sold 60% of his “Santa Claus Dolls” He then found he was left with 50 dolls. How many dolls had he in stock to begin with?

9. Kerry bought a new car in 1996. When she sold it four years later, she found that it had reduced in value by 60% and she received only £4640. How much had Kerry paid for the car in 1996?

10. James bought a car last year. It has lost 12.5% of its value since then. It is now valued at £14 875. How much did James pay for his car.

Solutions:

7. Ex-VAT Price \( \times \) 1.175 = £695
   Ex-VAT Price = £695 \( \div \) 1.175 = £591.49

8. Stock \( \times \) 0.4 = 50 (60% sold = 40% left)
   Stock = 50 \( \div \) 0.4 = 125

9. Original Price \( \times \) 0.4 = £4 640
   Original Price = £4 640 \( \div \) 0.4 = £11 600

10. Original Price \( \times \) 0.875 = £14 875
   Original Price = £14 875 \( \div \) 0.875 = £17 000