Appreciation & Depreciation

Percentage Multipliers:

If we wish to increase a quantity by a percentage to a new value, we can:

1. Find the percentage of the quantity
2. Add it on to the original amount to obtain the new value.

This requires 2 steps.

Example:

Peter earns a salary of £12,000 p.a.,
this year he gets a 10% rise, what will his new salary be?

His 10% rise is 10% of £12,000 = £1,200
His new salary will be £12,000 + £1,200 = £13,200

We can however do this calculation in a single step.

His original salary corresponds to 100%, he gets a rise of 10%,
so he now has 100% + 10% = 110% of his original salary.

To find 110% we multiply by $\frac{110}{100}$ which is 1.10

So Peter's new salary is £12,000 × 1.1 = £13,200

$$\text{Multiplier} = \frac{100 + \%\text{ increase}}{100}$$

Examples:

To obtain an increase of:

5%	multiply by 1.05	(105 ÷ 100)
7%	multiply by 1.07	(107 ÷ 100)
20%	multiply by 1.20	(120 ÷ 100)
2½%	multiply by 1.025	(102.5 ÷ 100)

Try these:

Find the multiplier to give an increase of:

15%	[Ans: 1.15]
25%	[Ans: 1.25]
3%	[Ans: 1.03]
17½%	[Ans: 1.175]
8¾%	[Ans: 1.0875]
7.3%	[Ans: 1.073]

Decrease

Similarly, to find a decrease, we subtract from 100.

For example:

To find a decrease of 10%

We want 100% – 10% = 90%, so we multiply by $\frac{90}{100}$ i.e. 0.9

$$\text{Multiplier} = \frac{100 - \%\ \text{increase}}{100}$$

Examples:

To obtain an decrease of:

5%	multiply by 0.95	(95 ÷ 100)
7%	multiply by 0.93	(93 ÷ 100)
20%	multiply by 0.80	(80 ÷ 100)
2½%	multiply by 0.975	(97.5 ÷ 100)

Try these:

Find the multiplier to give an decrease of:

15%	[Ans: 0.85]
25%	[Ans: 0.75]
3%	[Ans: 0.97]
7½%	[Ans: 0.925]
8¾%	[Ans: 0.9125]
7.3%	[Ans: 0.927]

Definitions:

Appreciation:

A gain or increase in value over time. Items that appreciate in value are:

Buildings, Antiques, Paintings, Jewellery, Works of Art.

Appreciation is usually expressed as a percentage.

Depreciation:

A loss or decrease in value over time. Items that depreciate in value are:

Cars, Machinery, Technology *e.g. computers*, Furniture.

Depreciation is usually expressed as a percentage.

Use of multipliers:

If a house is valued today at £80,000 and is expected to appreciate by 3% p.a. *(per year)*.
Then we can calculate the value after 3 years.

We can calculate year by year.

You should note that appreciation is always calculated on the value at the start of each year, so it is compounded.

Start Value:	Increase	Value at end of year.	
£80,000	£2,400	£82,400	(after 1 year)
£82,400	£2,472	£84,872	(after 2 years)
£84,872	£2,546.16	£87,418.16	(after 3 years)

So value after 3 years is: £ 87, 418.16

An easier way to calculate is to use the multiplier instead:

An increase of 3% corresponds to a multiplier of 1.03

After 3 years, the house is worth:

$£80{,}000 \times 1.03 \times 1.03 \times 1.03$

or $£80{,}000 \times 1.03^3 = £87{,}418.16$

Similarly, we can apply this to depreciation.

Example: A car is bought new for £15,000 and depreciates at 20% p.a.
What is the value of the car after 4 years.

Solution: The car loses 20% of its value each year, so it is worth only 80%
So, multiplier = 0.8
After 4 years, car is worth:

$£15{,}000 \times 0.8 \times 0.8 \times 0.8 \times 0.8$

or $£15{,}000 \times 0.8^4 = £6{,}144$

We can have problems involving both appreciation and depreciation:

A factory is valued at £120,000 for the building and £60,000 for the machinery.

If the building appreciates by 5% p.a. and the machinery depreciates by 8% p.a., calculate the total value of the buildings and machinery after 5 years.

Value of building after 5 years: $£120,000 \times 1.05^5 = £153,153.79$

Value of machinery after 5 years: $£\ 60,000 \times 0.92^5 = £\ 39,544.89$

Total value of building and machinery: $£153,153.79 + £39,544.89 = £192,698.68$

We can have problems involving multiple rates of depreciation:

A car is purchased for £20,000.

It is assumed to depreciate by 25% in the 1st year, 20% in the 2nd year and 15% in each of the 3rd and 4th years.

Calculate the value of the car after 4 years.

Value of car after 4 years: $£20,000 \times 0.75 \times 0.8 \times 0.85 \times 0.85 = £8,670$

The same principles apply to growth *(increase)* and decay *(decrease)* problems:

Example of growth:

A colony of bacteria initially contain 25,000 bacteria.

It is found that the colony grows at a rate of 35% per hour.

What will be the size of the colony after 3 hours.

Size of colony after 3 hours: $25,000 \times 1.35^3 = 61509.375 = 61509$ bacteria.

Example of decay:

A flask contains 5 litres of a chemical.

If it is left open to the air, it is found that the chemical evaporates at a rate of 15% per hour.

How much chemical will be left after 5 hours.

After 3 hours: $5,000 \times 0.85^5 = 2218.5$ millilitres.

Past Paper Questions

1. Bacteria in a test tube increase at the rate of 0.9% per hour.
At 12 noon there are 4500 bacteria.
At 3 pm, how many bacteria will be present?
Give your answer **to 3 significant figures**.

2. In January 2001, it was estimated that the number of flamingos in a colony was 7000.
The number of flamingos is decreasing at the rate of 14% per year.
How many flamingos are expected to be in this colony in January 2005 ?
Give your answer **to the nearest 10.**

3. In 1999, a house was valued at £70,000 and the contents were valued at £45,000.
The value of the house **appreciates** by 7% each year.
The value of the contents **depreciates** by 9% each year.
What will be the **total** value of the house **and** contents in 2002 ?

4. A factory was put on the market in January 2001.

   The site was in an excellent location so the value of the building has appreciated since then by 5.3% per year.

   Unfortunately the plant & machinery were poorly maintained and have depreciated by 8·5% per year.

   The value of the building was £435 000 and the value of the plant & machinery was £156 000 in January 2001.

   What would be the expected value of the complete factory in January 2003 ?

5. How much would the Strachans pay for a new iron, priced £16.50 at Watsons ?

WATSON'S SALE
$66\frac{2}{3}$ % off everything

Solutions:

1. 4500×1.009^3 = 4622.59678… **4620 (3 sf)**

2. 7000×0.86^4 = 3829.0571… **3830 (nst 10)**

3. House: £ 70 000 $\times$ 1.07^3 = £ 85 753.01
Contents: £ 45 000 $\times$ 0.91^3 = £ 33 910.70
Total value: = **£ 119 663.71**

4. Factory: £ 435 000 $\times$ 1.053^2 = £ 482 331.92
Plant & Mcy: £ 156 000 $\times$ 0.915^2 = £ 130 607.10
Total value: = **£ 612 939.02**

5. $66\,^2/_3$ % = $^2/_3$ So, $^2/_3$ off means you pay $^1/_3$

   They pay $^1/_3$ of £ 16.50 = £ 5.50

Reversing the change:

Quite often we are given the result after a percentage change has been applied, and asked to calculate the original value.

Example:

A ticket is on sale at 40% discount.

Paul paid £9.00 for the ticket.

What was the original price before the discount.

Solution:

A 40% discount means that the ticket was sold for 60% of its price.

i.e.		60%	is equivalent to	£9.00
	So,	1%	is equivalent to	£9.00 $\div$ 60
	and	100%	is equivalent to	£9.00 $\div$ 60 $\times$ 100 = £15

Original price of ticket was £15. *(You can check this by taking 40% off it)*

An alternative (algebraic) solution:

Let the original price be £ P

Then reduce the price by 40%. $\rightarrow$ $P \times 0.6$

So: $\rightarrow$ $P \times 0.6 = 9.00$

Divide both sides by 0.6 $\rightarrow$ P = £15.00

Examples:

7. A computer is sold for £695. This price includes VAT at 17.5%
   Calculate the price of the computer **without** VAT.

8. During the Christmas Sales a shopkeeper sold 60% of his “Santa Claus Dolls”
   He then found he was left with 50 dolls.
   How many dolls had he in stock to begin with ?

9. Kerry bought a new car in 1996. When she sold it four years later, she found that it had reduced in value by 60% and she received only £4640.
   How much had Kerry paid for the car in 1996 ?

10. James bought a car last year. It has lost 12.5 % of its value since then.
    It is now valued at £14 875.
    How much did James pay for his car.

Solutions:

7. Ex-VAT Price $\times$ 1.175 = £695
   Ex-VAT Price = £695 $\div$ 1.175 = **£ 591.49**

8. Stock $\times$ 0.4 = 50 (60% sold = 40% left)
   Stock = 50 $\div$ 0.4 = 125

9. Original Price $\times$ 0.4 = £ 4640
   Original Price = £ 4640 $\div$ 0.4 = £ 11 600

10. Original Price $\times$ 0.875 = £ 14 875
    Original Price = £ 14 875 $\div$ 0.875 = £ 17 000