Solving Quadratic Equations

Introduction: What is a Quadratic Equation

We know how to solve simple equations

e.g. $3x+7=13$ or $5x-3=2x+6$

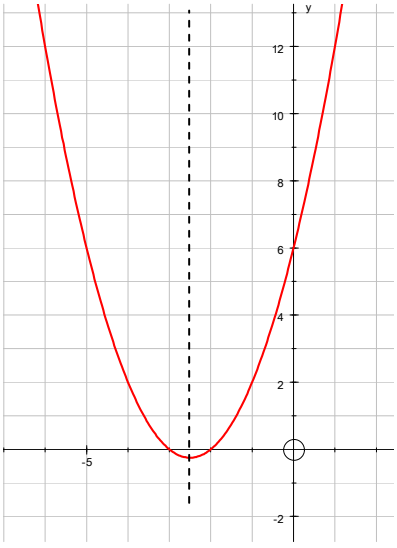
But how would we solve this one: $x^2+5x+6=0$. This is called a quadratic equation.

The general form of the graph is: $y=x^2+5x+6$ and we are looking for when y = 0
i.e. when the graph crosses the x–axis.

Graphs of Quadratic Equations – what do they look like ?

If we were to draw the graph of $y=x^2+5x+6$ we would see it looks like the graph below:

x	–4	–3	–2	–1	0
x^2	16	9	4	1	0
+ 5x	–20	–15	–10	–5	0
+6	6	6	6	6	6
y	2	0	0	2	6



This graph is called a **parabola**.

Note that it is **symmetrical** about the dotted line.

Where the graph cuts the *x*–axis we call the **roots** of the

$x^2+5x+6=0$ because this is where *y* = 0

The roots of this equation are: *x* = –2 and *x* = –3

Note that we can see this in the table and using the graph.

The **axis of symmetry** lies **mid–way** between the **roots**.

If the graph is always above the *x*–axis, we say there are **no real roots**.

We can find the solution to any quadratic by drawing a graph, however it would be easier if we could solve the equation algebraically.

An Algebraic Method

Let us look at the equation again: $x^2+5x+6=0$ This should remind us of trinomials.

We can put the quadratic expression into two brackets: $(x+3)(x+2)=0$

The only way that two expressions multiply together to give zero, is if one or the other is 0.

So, either $x+3=0$ or $x+2=0$ This means that $x = -2$ or $x = -3$.

This is a lot quicker.

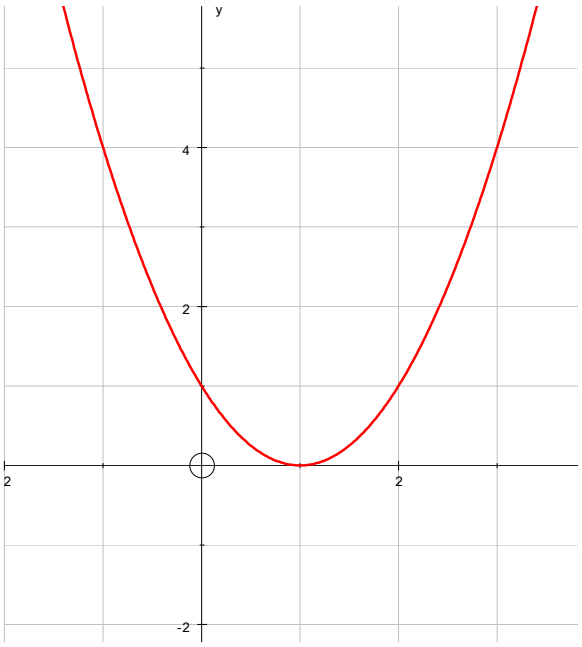
Solving Quadratic Equations (Method)

1. Put into two brackets (check you are right by using FOIL)
2. Put each bracket = 0 and solve the simpler equations to get two solutions.

   (Note if both brackets are the same – you will get **two equal roots**).

Equal roots shown graphically

Graphically, this means that the graph just touches the x–axis at one point only.



This is the graph of $y=x^2-2x+1$
So we will solve $x^2-2x+1=0$

Factorising gives:
$(x-1)(x-1)=0$

So, $x-1=0$ or $x-1=0$
and hence $x = 1$ or $x = 1$

Note equal roots, touches at one point only.

The Quadratic Formula

Sometimes the trinomial will not factorise.

This could be because there are no real roots.
or it could be that the roots are not whole numbers or simple fractions (rational numbers)

In this case there is a **formula** that we can use.
It is called the **quadratic formula** and is at the front of your examination paper.

The formula

For the quadratic equation: $ax^2+bx+c=0$, where *a*, *b* and *c* are simply numbers,

The solution is given by $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

This will generally be a decimal solution.

A clue to **use the formula** is given in the question. It will ask you to solve the quadratic equation and "give your answer **to 1 decimal place**" or some other degree of accuracy.

Using the formula – A method

Solve $x^2-5x-3=0$ and give your answer correct to 1 decimal place.

First identify *a*, *b* and *c*. $a = 1$, $b = -5$, $c = -3$

Now substitute into the formula – do not take shortcuts.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad\rightarrow\quad x=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(-3)}}{2(1)}$$

Simplify bit by bit. $\rightarrow\quad x=\dfrac{5\pm\sqrt{25+12}}{2}\quad\rightarrow\quad x=\dfrac{5\pm\sqrt{37}}{2}$

The $\pm$ is where you get the two roots from.

So our two solutions are: $\rightarrow\quad x=\dfrac{5+\sqrt{37}}{2}\ \ or\ \ x=\dfrac{5-\sqrt{37}}{2}$

Using the calculator, we find: $\rightarrow\quad x=\dfrac{11.083}{2}\ \ or\ \ x=\dfrac{-1.083}{2}$

So, solutions are: $x=5.541...\ \ or\ \ x=-0.541...$

Thus solutions are $x = 5.5$ or $x = -0.5$ to l d.p.

Examples to try.

Solve the following quadratic equations by factorisation:

1. $x^2 - 2x - 3 = 0$ [Ans. $x = 3$, $x = -1$]

2. $x^2 - 6x + 8 = 0$ [Ans. $x = 4$, $x = 2$]

3. $x^2 + 8x + 15 = 0$ [Ans. $x = -3$, $x = -5$]

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4. $x^2 - 4x + 4 = 0$ [Ans. $x = 2$, $x = 2$]

5. $2x^2 - x - 1 = 0$ [Ans. $x = 1$, $x = -\frac{1}{2}$]

Solve the following quadratic equations using the formula, to 1 decimal place.

6. $x^2 + 2x - 1 = 0$ [Ans. $x = -2.4$, $x = 0.4$]

7. $3x^2 - 4x - 5 = 0$ [Ans. $x = -0.8$, $x = 2.1$]

8. $2x^2 + 3x - 4 = 0$ [Ans. $x = -2.4$, $x = 0.9$]