Similar Shapes
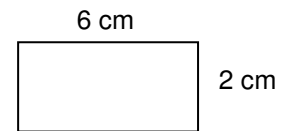
Area and Volume Scale Factors

Area scale factor:
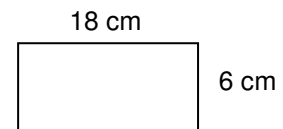
If we take a rectangle of length 6 cm and breadth 2 cm then the

Area = 12 cm^2.

If we now scale this rectangle, with a scale factor of 3,

then the length will be 18 cm and the breadth, 6 cm. and now

Area = 18 × 6 = 108 cm^2.

We have used a length (linear) scale factor of 3

but the area has a scale factor of 9 i.e. 3^2

This will always apply.

<div align="center">

Area scale factor = linear scale factor^2

</div>

Example:

A box has a surface area of 3.5 m^2

If the length of each side is increased by a scale factor of 4.

What will the new surface area be ?

Solution:

Linear scale factor = 4   Area scale factor = 4^2 (= 16)

New surface area: 3.5 × 16 = 56 m^2

Volume scale factor:

If we take a cuboid of length 3 cm and breadth 2 cm
and height 4 cm

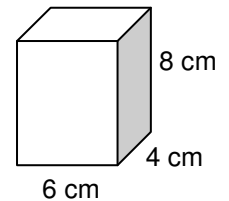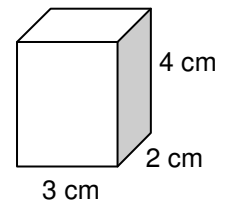then the

  Volume = 3 × 2 × 4 = 24 cm^3.

4 cm
2 cm
3 cm

If we now scale this cuboid, with a scale factor of 2,

then the length will be 6 cm and the breadth, 4 cm.
and height 8 cm

and now

  Volume = 6 × 4 × 8 = 192 cm^3.

8 cm
4 cm
6 cm

We have used a length (linear) scale factor of 2
but the volume has a scale factor of 8 i.e. 2^3

This will always apply.

  Volume scale factor = linear scale factor^3

Example:

A bottle has a volume of 200 ml and a height of 12 cm.

Another bottle is mathematically similar, and has a height of 18 cm.

What will be the volume of this bottle ?

Solution:

  Linear scale factor = 18 ÷ 12 = 1.5   Volume scale factor = 1.5^3
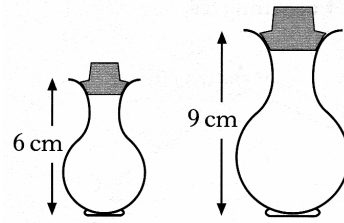
  Volume of 2nd bottle: 200 × 1.5^3 = 675 ml

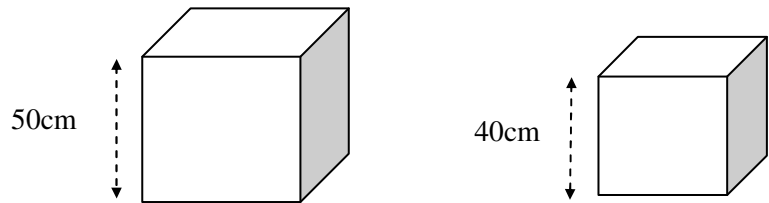1. Two perfume bottles are mathematically similar in shape.

    The smaller one is 6 centimetres high and
    holds 30 millilitres of perfume.

    The larger one is 9 centimetres high.

    What volume of perfume will the larger one hold.

2. The two boxes below are
    mathematically similar
    and both have to be wrapped
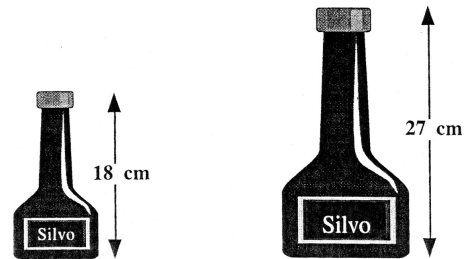   with decorative paper.

    If it requires 3.27 m^2 of paper to cover
    the large box, calculate the amount of paper needed to cover the smaller box.
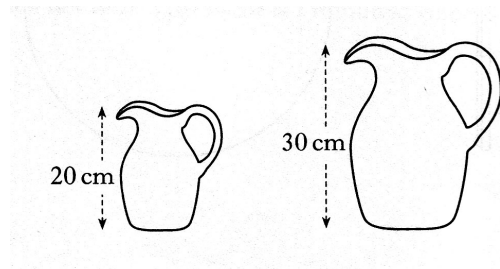
3. The diagram shows two bottles of
    Silvo Shampoo.

    The two bottles are **mathematically
    similar**, and the cost of the shampoo
    depends only on the volume of liquid
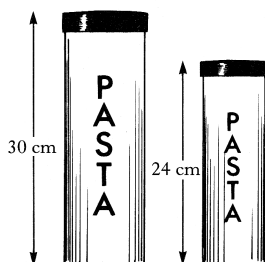   in the bottle.

    If the small one costs 80p, what should the large one cost ?

4. The diagram shows two jugs
    which are mathematically similar.

    The volume of the smaller jug is 0.8 litres.

    Find the volume of the larger jug.

5. The diagram shows two storage jars which are mathematically similar.

    The volume of the large jar is 1.2 litres.

    Find the volume of the smaller jar.

    **Give your answer in litres
    correct to 2 significant figures.**

6. The diagram shows two tubes of toothpaste.

    Assuming that the tubes are mathematically
    similar, and that the price of toothpaste depends
    only on the volume of toothpaste in the tube,
    what would be the cost of the large tube when
    the small one costs £1.12?
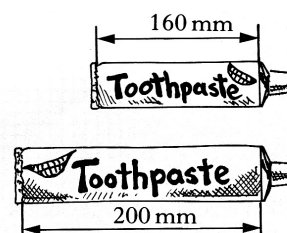
Solutions:

1. Linear scale factor $= \dfrac{9}{6} \rightarrow \dfrac{3}{2}$

   Volume: $= 30 \times \dfrac{3}{2} \times \dfrac{3}{2} \times \dfrac{3}{2} = 101.25$ *mls*

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2. Linear scale factor $= \dfrac{40}{50} \rightarrow \dfrac{4}{5}$

   Area: $= 3.27 \times \dfrac{4}{5} \times \dfrac{4}{5} = 2.0928$ m^2 $= 2.09$ m^2

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3. Linear scale factor $= \dfrac{27}{18} \rightarrow \dfrac{3}{2}$

   Cost: $= 80 \times \dfrac{3}{2} \times \dfrac{3}{2} \times \dfrac{3}{2} = £\,2.70$

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4. Linear scale factor $= \dfrac{30}{20} \rightarrow \dfrac{3}{2}$

   Volume: $= 0.8 \times \dfrac{3}{2} \times \dfrac{3}{2} \times \dfrac{3}{2} = 2.7$ *litres*

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5. Linear scale factor $= \dfrac{24}{30} \rightarrow \dfrac{4}{5}$

   Volume: $= 1.2 \times \dfrac{4}{5} \times \dfrac{4}{5} \times \dfrac{4}{5} = 0.6144$ *litres*

   Volume $= 0.61$ litres (2 sig figs)

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6. Linear scale factor $= \dfrac{200}{160} \rightarrow \dfrac{5}{4}$

   Cost: $= £1.12 \times \dfrac{5}{4} \times \dfrac{5}{4} \times \dfrac{5}{4} = £\,2.1875$

   Cost $= £\,2.19$