The Sine Rule

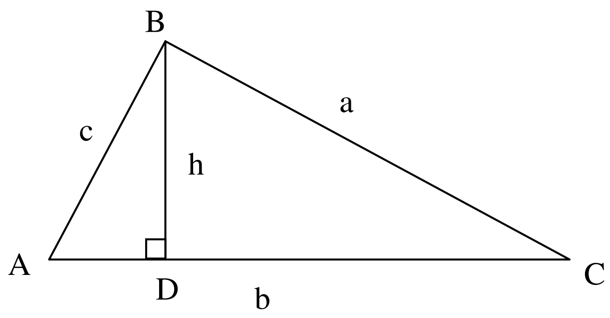
We use the sine rule for non–right angled triangles.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

We denote the angles by capital letters A, B, C

We denote the sides opposite each angle by the lower case letters a, b, c.

THEORY:



Triangle ABC is a non–right angled triangle

In triangle ABC, draw a perpendicular line from B to AC meeting AC at D.

This creates two right angled triangles **ABD** and **BDC**

In triangle ABD:

$\sin A = \frac{h}{c}$ i.e. $c \sin A = h$

In triangle BDC:

$\sin C = \frac{h}{a}$ i.e. $a \sin C = h$

Therefore: $a \sin C = c \sin A$ and re–arranging we get: $\frac{a}{\sin A} = \frac{c}{\sin C}$

By simply rotating the letters around (cyclic permutation) we get:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

This is known as the **sine rule**.

To use the sine rule, choose an appropriate pair, depending on what you know in the triangle.

e.g. $\frac{a}{\sin A} = \frac{b}{\sin B}$ or $\frac{a}{\sin A} = \frac{c}{\sin C}$ or $\frac{b}{\sin B} = \frac{c}{\sin C}$

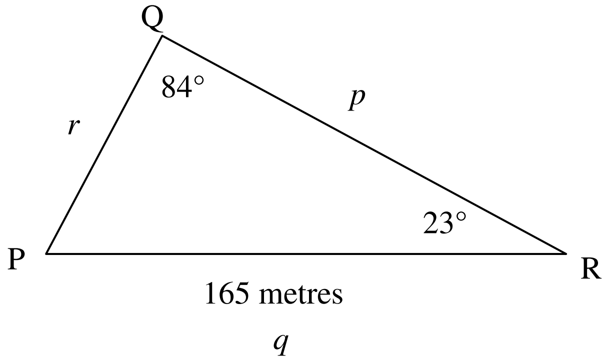
If you are finding an angle, you can invert the formulae.

e.g. . $\frac{\sin A}{a} = \frac{\sin B}{b}$ or $\frac{\sin A}{a} = \frac{\sin C}{c}$ or $\frac{\sin B}{b} = \frac{\sin C}{c}$

The sine rule

Example

Find the length of PQ in triangle PQR



Use the **sine rule**

Tick what you have and what you want just as before

$$\frac{p}{\sin P} = \frac{\overset{\checkmark}{q}}{\underset{\checkmark}{\sin Q}} = \frac{\overset{\checkmark}{r}}{\underset{\checkmark}{\sin R}}$$

Use: $$\frac{q}{\sin Q} = \frac{r}{\sin R}$$

So, $$\frac{165}{\sin 84} = \frac{r}{\sin 23}$$

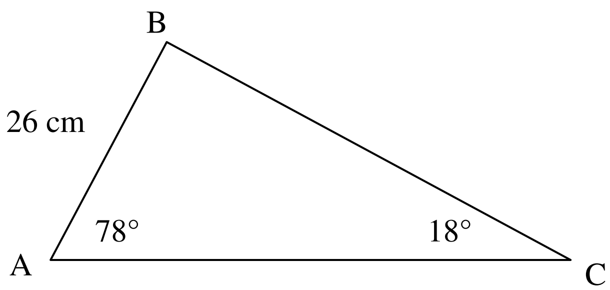
and $$\frac{165 \times \sin 23}{\sin 84} = r$$

thus $r = 64.8$ metres (1 d.p.)

Try this one:

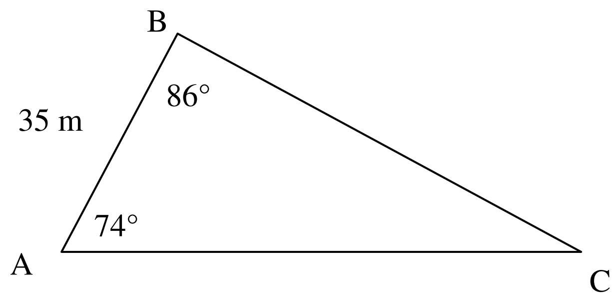
Find the length of BC in triangle ABC

[Ans: 82.3 cm]



The Sine Rule

A slight variation



Find the length of AC in triangle ABC

[Ans: 102.1 metres]

Hint:

We do not know the angle opposite AB – however, we can easily work it out since we have the other two angles in the triangle.

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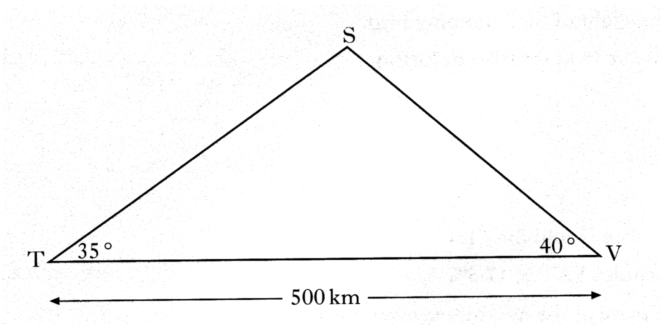
1.   A past paper question



A TV signal is sent from a transmitter T,
via a satellite S, to a village V,
as shown in the diagram.

The village is 500 kilometres from the transmitter.
The signal is sent out at an angle of 35°
and is received in the village at an angle of 40°.

Calculate the height of the satellite above the ground.

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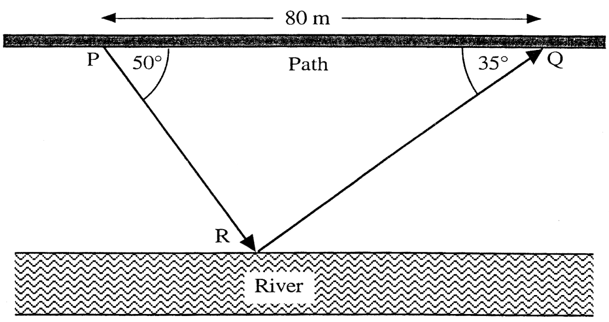
2.   Another past paper question



The path in the diagram opposite runs
parallel to the river.

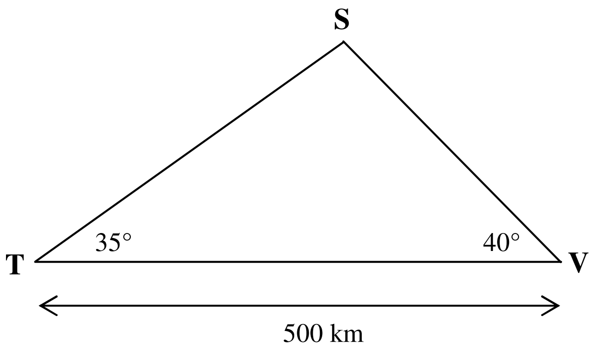
Jennifer leaves the path at P, walks to the river
to bathe her feet at R and rejoins the path
further on at Q.

Calculate the distance between the river and the path.

Solutions to the above questions are on the following sheet.

Solutions to past paper questions

1.



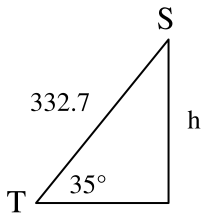
Use Sine Rule to find either side ST or SV

Then use SOH-CAH-TOA to find perpendicular height.

First find angle at S = 180° – (35° + 40°) S is 105°

$$\frac{ST}{\sin 40} = \frac{500}{\sin 105}$$

$$ST = \frac{500 \sin 40}{\sin 105} \Rightarrow \quad ST = 332.731...$$



$$\sin 35 = \frac{h}{332.7}$$

$$h = 332.7 \times \sin 35 \ = 190.828...$$

height of satellite = 190 km

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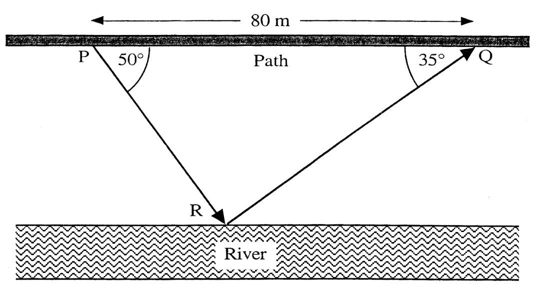
2. Basically same as previous question

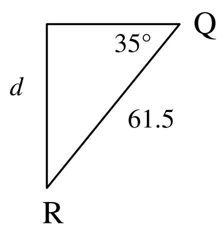


$\angle PRQ = 95°$ Find RQ using sine rule

$$\frac{RQ}{\sin 50} = \frac{80}{\sin 95}$$ RQ = 61.5 metres

Now use SOH-CAH-TOA to find distance

Let distance between river and path be *d* metres.



$$\sin 35 = \frac{d}{61.5}$$ hence, d = 35.3 metres
