The Straight Line

**Gradient:**

The gradient of a straight line is: \( \frac{\text{Rise}}{\text{Run}} \) and

The gradient is usually denoted by \( m \)

**y–intercept:**

The y–intercept of a straight line where the line crosses the y–axis.

The y–intercept is usually denoted by \( c \)

**Equation of a straight line:**

The equation of a straight line is given by:

\[ y = mx + c \]

where \( m \) is the gradient and \( c \) is the y–intercept.

**Finding the equation of a line.**

We need the gradient \( m \) and the y–intercept \( c \).

**Example:**

What is the equation of the straight line with gradient 2 and y–intercept 3 ?

**Solution:**

\( m = 2, \quad c = 3 \)

Hence, since \( y = mx + c \) then, \( y = 2x + 3 \)

**Two Points:**

We may be given two points on a graph, one of which is the y–intercept.

From the two points we can work out the gradient – so we have \( m \).

If one point is the y–intercept then we also have \( c \).

So we can now write down the equation, substituting for \( m \) and \( c \) in \( y = mx + c \)
Using the Equation of the Straight Line.

If we are given the equation of a line, then we can find out where it cuts the x and y axes.

\[ y \text{–intercept:} \]
If we put \( x = 0 \) into the equation, we can find where the line cuts the y–axis.
\[ y = 3x - 1 \] putting \( x = 0 \) gives \( y = 3(0) - 1 \) \( y = -1 \)
so, \( y \)–intercept = \( -1 \)

\[ x \text{–intercept:} \]
If we put \( y = 0 \) into the equation, we can find where the line cuts the x–axis.
\[ y = 2x - 4 \] putting \( y = 0 \) gives \( 0 = 2x - 4 \) \( \rightarrow \) \( 2x = 4 \) \( \rightarrow \) \( x = 2 \)
so, \( x \)–intercept = \( 2 \)

A point lies on a line, if it satisfies the equation of the line.
If we substitute the coordinates of the point into the equation of the line, then the equation will be true i.e. satisfied.

\[ \text{e.g. } \]
Does the point \( (1, 2) \) lie on the line \( y = 3x - 1 \)
Substitute into the equation – is it true?
\[ 2 = 3(1) - 1 \] \( \rightarrow \) \( 2 = 3 - 1 \) \( \rightarrow \) \( 2 = 2 \)
this is true, so point does lie on the line.

\[ \text{e.g. } \]
Does the point \( (3, 4) \) lie on the line \( y = 3x - 1 \)
Substitute into the equation – is it true?
\[ 4 = 3(3) - 1 \] \( \rightarrow \) \( 4 = 9 - 1 \) \( \rightarrow \) \( 4 = 8 \)
this is \textbf{NOT} true,
so point does \textbf{NOT} lie on the line.

On the next page are some past paper questions.
Past Paper Questions:

1. In the diagram, A is the point (-1, 7) and B is the point (4, 3).
   a) Find the gradient of the line AB.
   b) AB cuts the y-axis at the point (0, -5).
      Write down the equation of the line AB
   c) The point (3k, k) lies on AB
      Find the value of k.

Solution

1. a) Gradient AB = \(\frac{3 - (-7)}{4 - (-1)} = \frac{10}{5} = 2\)
   
   b) Use \(y = mx + c\) Eqn is: \(y = 2x - 5\)
   
   c) (3k, k) lies on AB, so it will satisfy the equation
      Hence, \(k = 2(3k) - 5\)
      \(k = 6k - 5\)
      \(5 = 5k\)
      \(k = 1\)

2. The straight line through the points A(2, 4) and B(6, 6) is shown in the diagram.
   The point M is where the line AB cuts the x-axis.
   a) Find the equation of the straight line AB.
   b) Use this equation to find the coordinates of the point M.

Solution

2. a) Gradient AB = \(\frac{6 - 4}{6 - 2} = \frac{2}{4} = \frac{1}{2}\)
   
   Use \(y = mx + c\), so \(y = \frac{1}{2} x + c\)
   
   Need to find c, so use point (2, 4)
   \(4 = \frac{1}{2} (2) + c\)
   \(4 = 1 + c\)
   \(c = 3\)
   
   Equation is \(y = \frac{1}{2} x + 3\)
   
   b) To find M, we know that \(y = 0\)
      Hence \(0 = \frac{1}{2} x + 3\) solving gives \(x = -6\)