Trig Graphs

What is a Trig graph ?

This is the graph of a trigonometrical function e.g. $y = \sin x$, $y = \cos x$ or $y = \tan x$

How do we draw one ?

We make a table of value using the calculator.

Try to complete the one below *(work to 2 decimal places)*.

x (degrees)	0	10	20	30	40	50	60	70	80	
sin x										

x (degrees)	90	100	110	120	130	140	150	160	170	
sin x										

x (degrees)	180	190	200	210	220	230	240	250	260	
sin x										

x (degrees)	270	280	290	300	310	320	330	340	350	360
sin x										

Below are the value for you to check:

x (degrees)	0	10	20	30	40	50	60	70	80	
sin x	0.00	0.17	0.34	0.50	0.64	0.77	0.87	0.94	0.98	

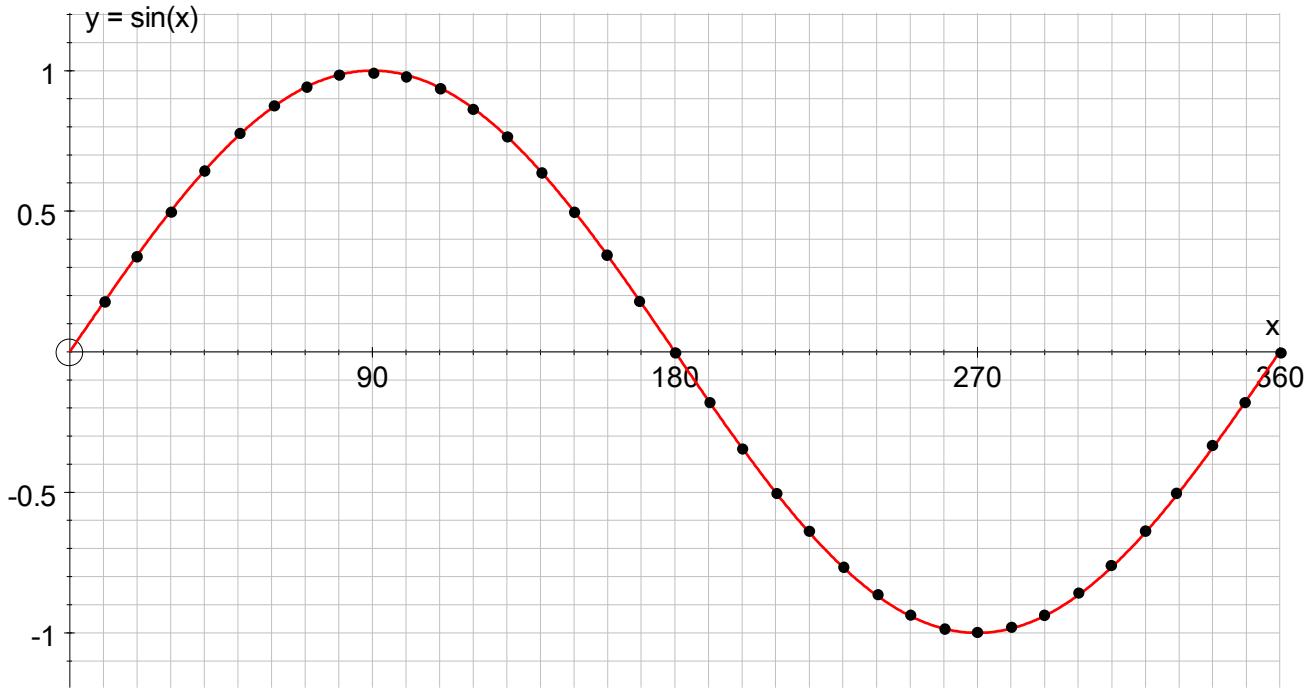
x (degrees)	90	100	110	120	130	140	150	160	170	
sin x	1.00	0.98	0.94	0.87	0.77	0.64	0.50	0.34	0.17	

x (degrees)	180	190	200	210	220	230	240	250	260	
sin x	0.00	-0.17	-0.34	-0.50	-0.64	-0.77	-0.87	-0.94	-0.98	

x (degrees)	270	280	290	300	310	320	330	340	350	360
sin x	-1.00	-0.98	-0.94	-0.87	-0.77	-0.64	-0.50	-0.34	-0.17	0.00

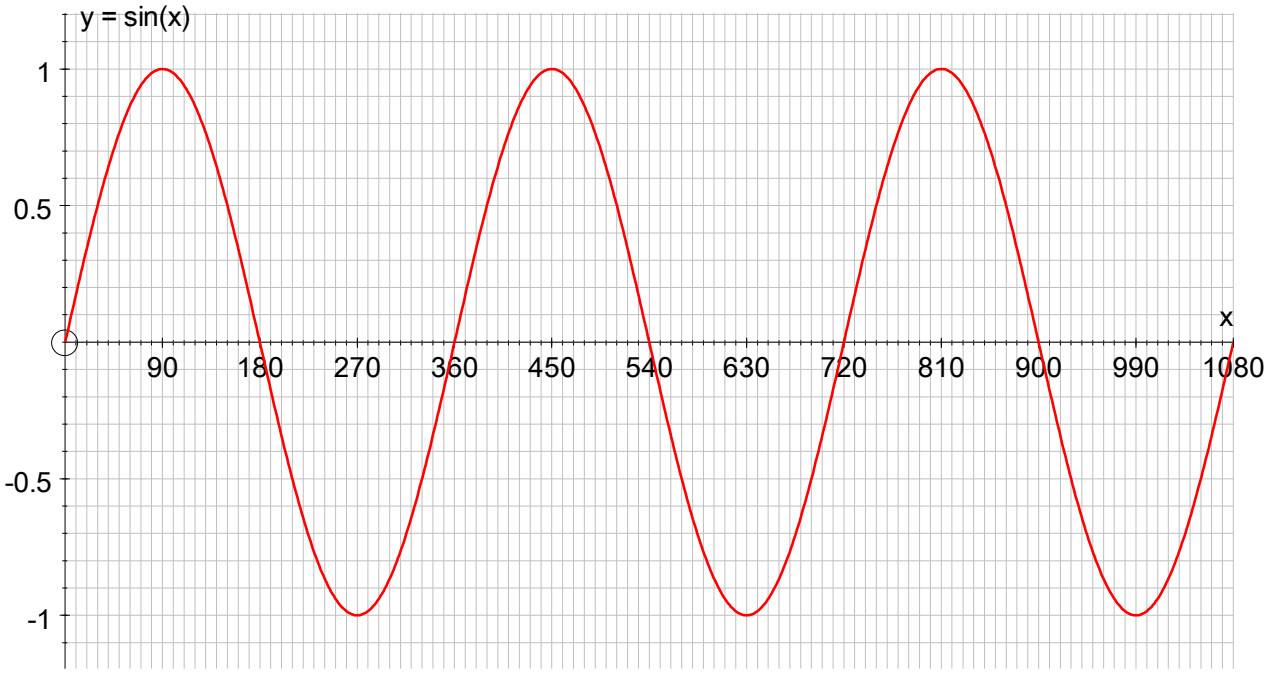
On the next sheet, we will plot the above values on a graph

Graph of $y = \sin x$



If we continue past 360°, then the graph repeats itself as below:

You can check this with your calculator, by calculating sin 450° etc and making sure it agrees with the graph.



It should be noted that:

- The maximum value of $y = \sin x$ is +1
- The minimum value of $y = \sin x$ is –1
- The graph is centred on the line $y = 0$
- The graph repeats itself every 360° – there is a one complete wave in 360°

The graph of $y = \cos x$

We can repeat the previous steps.

x (degrees)	0	10	20	30	40	50	60	70	80	
cos x										

x (degrees)	90	100	110	120	130	140	150	160	170	
cos x										

x (degrees)	180	190	200	210	220	230	240	250	260	
cos x										

x (degrees)	270	280	290	300	310	320	330	340	350	360
cos x										

Below are the value for you to check:

x (degrees)	0	10	20	30	40	50	60	70	80	
cos x	1.00	0.98	0.94	0.87	0.77	0.64	0.50	0.34	0.17	

x (degrees)	90	100	110	120	130	140	150	160	170	
cos x	0.00	-0.17	-0.34	-0.50	-0.64	-0.77	-0.87	-0.94	-0.98	

x (degrees)	180	190	200	210	220	230	240	250	260	
cos x	-1.00	-0.98	-0.94	-0.87	-0.77	-0.64	-0.50	-0.34	-0.17	

x (degrees)	270	280	290	300	310	320	330	340	350	360
cos x	0.00	0.17	0.34	0.50	0.64	0.77	0.87	0.94	0.98	1.00

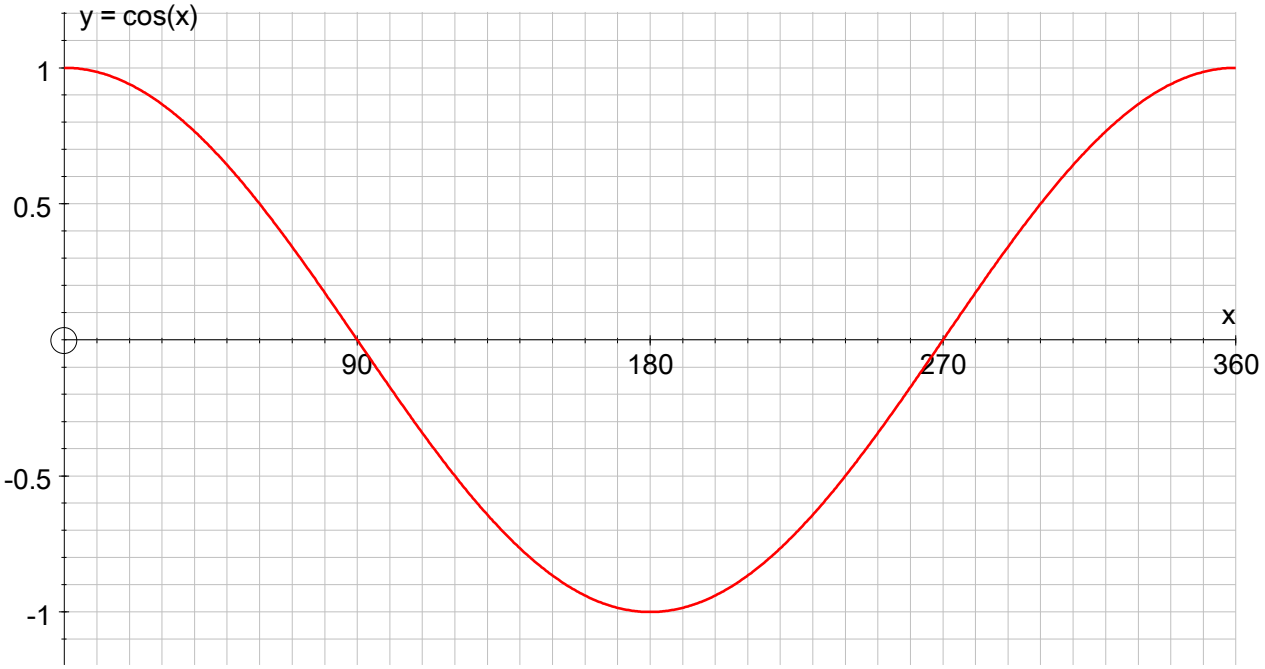
On the next sheet, we will draw the graph.

For clarity on the graph, the plotted points from above will not be visible.

However, you should check to see if you agree that the graph matches the points above.

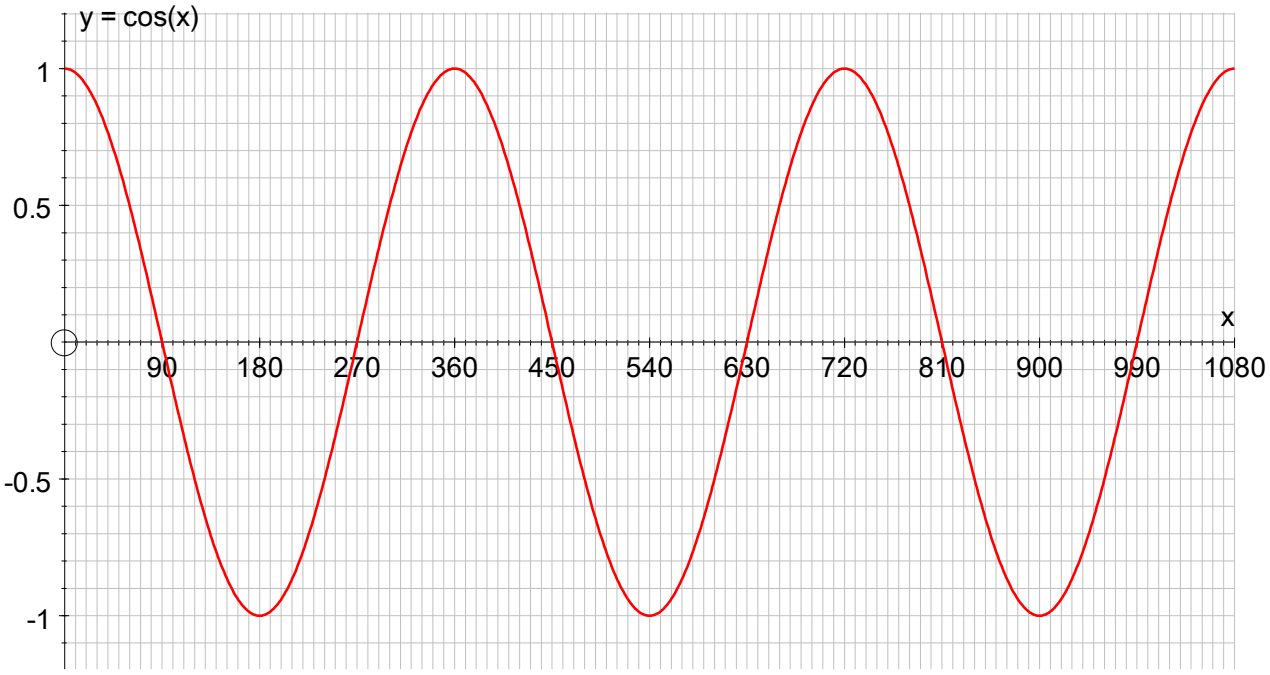
Check one or two values to be sure.

Graph of $y = \cos x$



If we continue past 360°, then the graph repeats itself as below:

You can check this with your calculator, by calculating cos 450° etc and making sure it agrees with the graph.

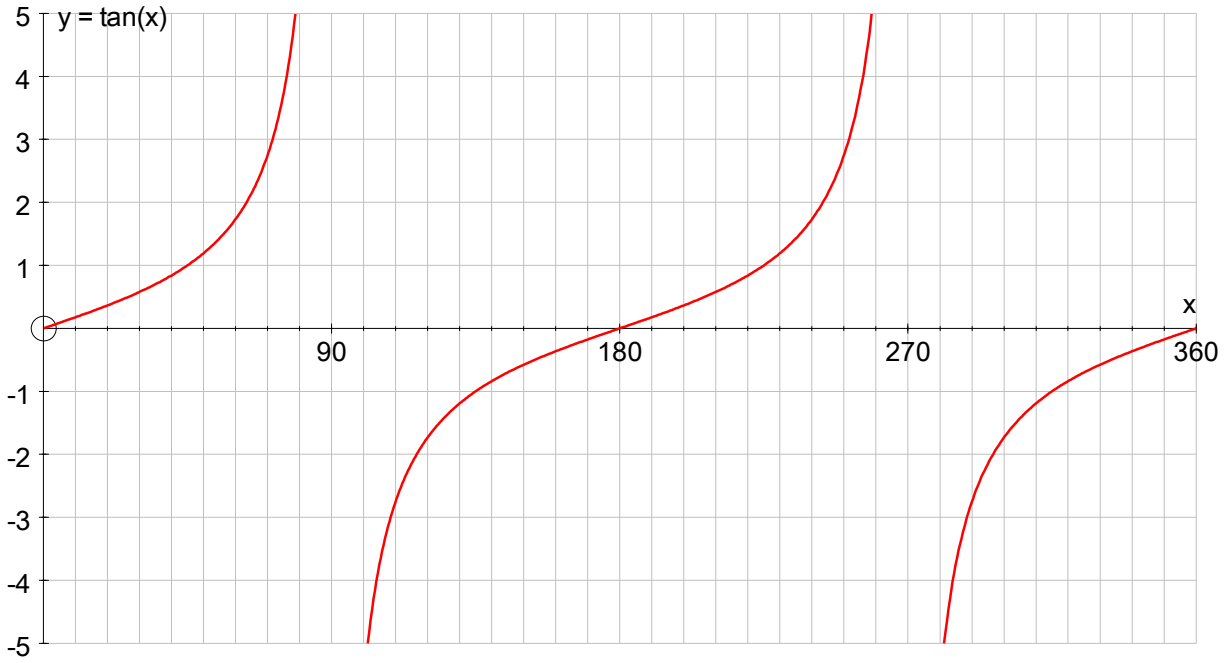


It should be noted that:

- The maximum value of $y = \cos x$ is +1
- The minimum value of $y = \cos x$ is –1
- The graph is centred on the line $y = 0$
- The graph repeats itself every 360° – there is a one complete wave in 360°

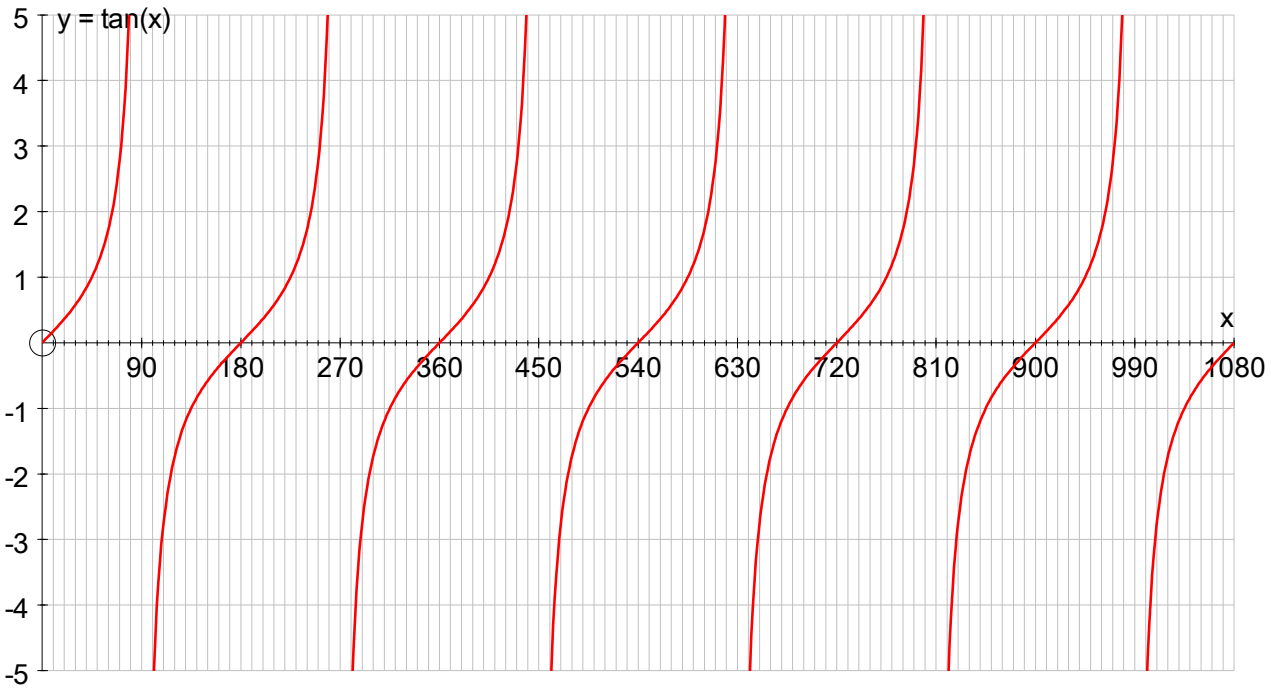
Graph of $y = \tan x$

Similarly, we can draw this graph also. The results are shown below.



If we continue past 360°, then the graph repeats itself as below:

You can check this with your calculator, by calculating tan 540° etc and making sure it agrees with the graph.

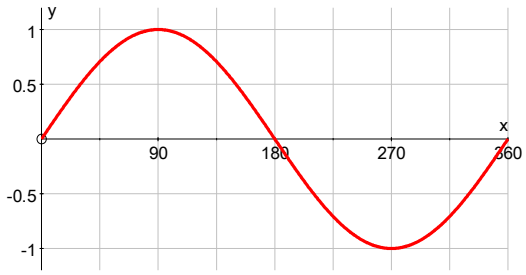


It should be noted that:

- There is no maximum or minimum value of $y = \tan x$
- The graph is centred on the line $y = 0$
- The graph repeats itself every 180°

Amplitude

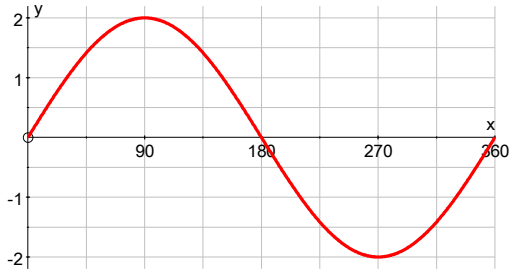
We can use the graphs to make observations about trig functions.



$$y = \sin x$$

maximum = 1

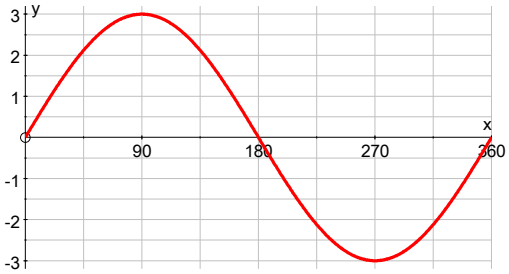
minimum = –1



$$y = 2\sin x$$

maximum = 2

minimum = –2



$$y = 3\sin x$$

maximum = 3

minimum = –3

In general, for the graph $y = a\sin x$

- the maximum value will be a
- the minimum value will be –a

Where a is the distance of the top of the wave from the centre line.

We call a the amplitude of the wave *(how big it is)*.

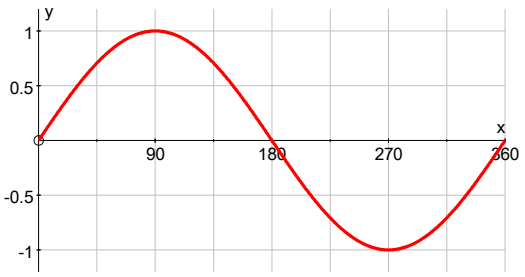
amplitude = 5

This same definition applies to the cosine wave.

We shall not be concerned very much with the tangent waveform at Standard Grade.

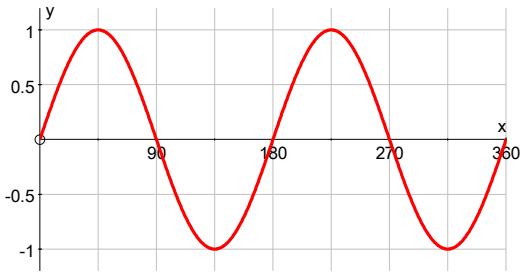
Periodicity

We can use the graphs to make observations about trig functions.



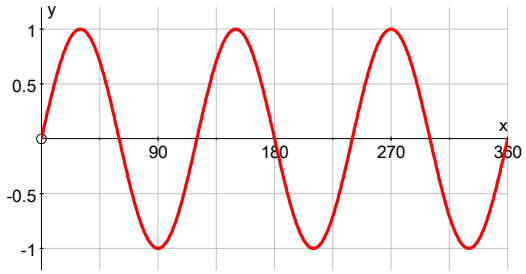
$$y = \sin x$$

One waveform in 360°



$$y = \sin 2x$$

Two waveforms in 360°



$$y = \sin 3x$$

Three waveforms in 360°

In general, for the graph $y = \sin bx$

- there will be b complete waves in 360°

We say that the period of the waveform, is the number of degrees for one complete wave.

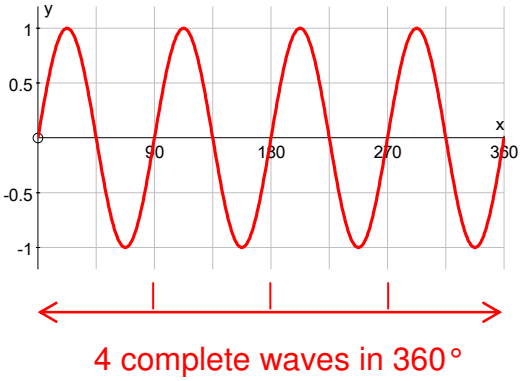
The period will be given by: $\dfrac{360°}{b}$

We determine b by determining how many complete waves there are in 360°

The equation of this graph is:

$$y = \sin 4x$$

because there are 4 complete waveforms in 360°

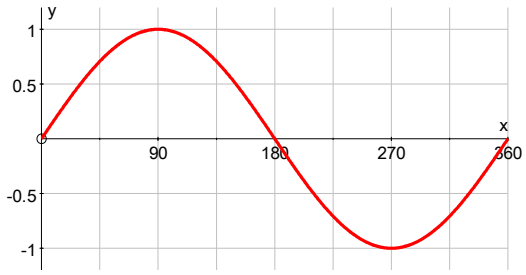


4 complete waves in 360°

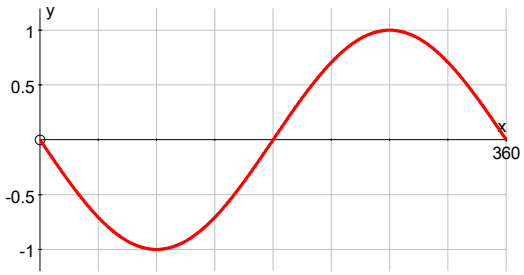
This also applies to the cosine wave.

Inversion – reflected or negative graphs

We can use the graphs to make observations about trig functions.



$$y = \sin x$$



$$y = -\sin x$$

Note the reflection in the x–axis

$$y = \cos x$$

$$y = -\cos x$$

Note the reflection in the x–axis

We are now in a position to be able to write down the equation of a sine or cosine graph simply by looking for the amplitude and the periodicity.

e.g. The graph is of the form $y = a\sin bx$
Find a and b.

Amplitude is 2

There are 2 waveforms in 360°

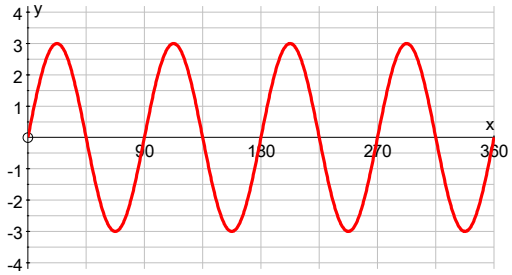
Equation is: $y = 2\sin 2x$

e.g. The graph is of the form $y = a\sin bx$
Find a and b.

Amplitude is 3

There are 4 waveforms in 360°

Equation is: $y = 3\sin 4x$



e.g. The graph is of the form $y = a\cos bx$
Find a and b.

Amplitude is 2

There are 3 waveforms in 360°

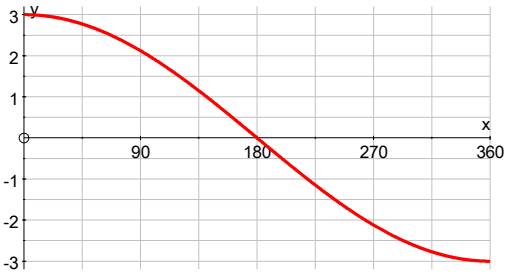
Equation is: $y = 2\cos 3x$

e.g. The graph is of the form $y = a\cos bx$
Find a and b.

Amplitude is 3

There is only ½ a waveform in 360°

Equation is: $y = 3\cos\frac{1}{2}x$

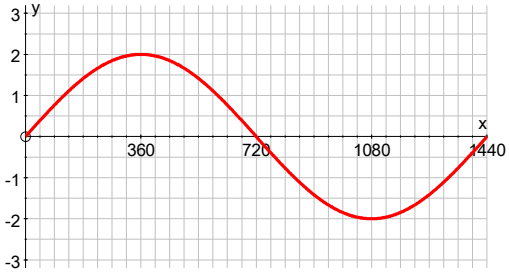


e.g. The graph is of the form $y = a\sin bx$
Find a and b.

Amplitude is 2

There is only ¼ a waveform in 360°

Equation is: $y = 2\cos\frac{1}{4}x$



e.g. The graph is of the form $y = a\sin bx$

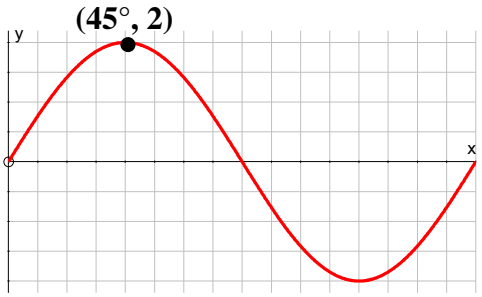
Find a and b.

Amplitude is 2

¼ of the wave is in 45°, whole wave in 180°

So, two waves in 360°

Equation is: $y = 2\sin 2x$



e.g. The graph is of the form $y = a\cos bx$

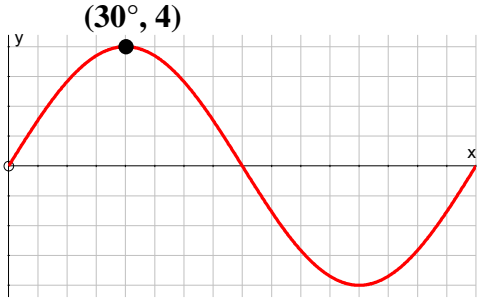
Find a and b.

Amplitude is 4

¼ of the wave is in 30°, whole wave in 120°

So, three waves in 360°

Equation is: $y = 4\sin 3x$



e.g. The graph is of the form $y = a\cos bx$

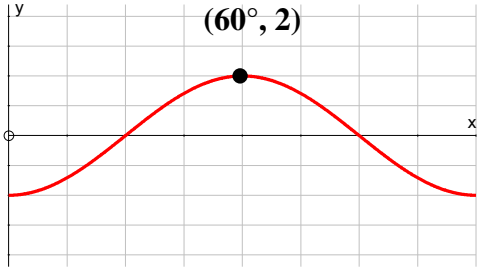
Find a and b.

Amplitude is 2, note this is: –cos x wave

½ of the wave is in 60°, whole wave in 120°

So, three waves in 360°

Equation is: $y = -2\cos 3x$



e.g. The graph is of the form $y = a\sin bx$

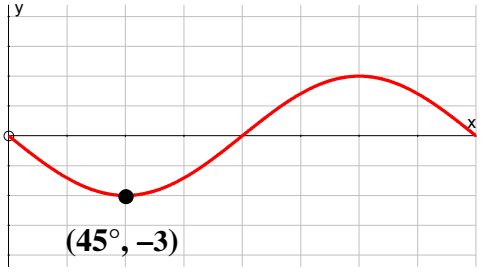
Find a and b.

Amplitude is 3, note this is: –sin x wave

¼ of the wave is in 45°, whole wave in 180°

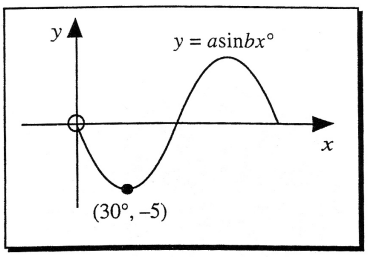
So, two waves in 360°

Equation is: $y = -3\sin 2x$



Some past paper questions:

1. Shown is the graph of $y = a \sin bx°$



Write down the values of *a* and *b*.

note –sin x wave;

Amplitude = 5; ¼ of wave in 30°, whole wave in 120°, 3 waves in 360°

a = –5, b = 3

3. On a certain day the depth, *D* metres, of water at a fishing port, *t* hours after midnight, is given by the formula

$$D = 12.5 + 9.5\sin(30t)°$$

a) Find the depth of water at 1.30 pm

b) The depth of water in the harbour is recorded each hour.
What is the maximum difference in the depths of water in the harbour, over the 24 hour period ?

Show clearly all your working.

a) 1.30 pm is 13.5 hours after midnight

$D = 12.5 + 9.5\sin(30 \times 13.5)°$ = → $12.5 + 9.5\sin(405)°$ → 19.217... = 19.2 metres.

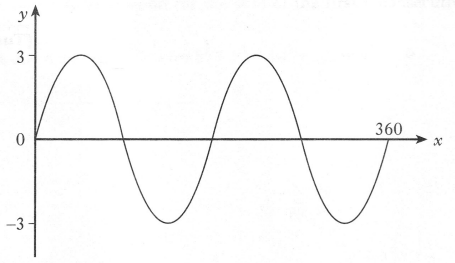
b) The maximum value of sin is 1; so max value of D is 12.5 + 9.5 = 22 metres

The minimum value of sin is –1; so min value of D is 12.5 – 9.5 = 3 metres.

Hence maximum difference in depths of water = 22 – 3 = 19 metres.

4. The diagram shows the graph of

$$y = k\sin ax°, \quad 0 \le x \le 360$$

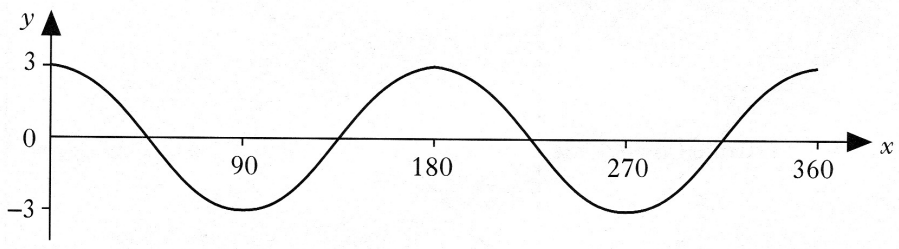


Find the values of *a* and *k*.

Amplitude = 3; 2 waves in 360°

a = 3, k = 2

5.



The diagram shows the graph of $y = a\cos bx°, \quad 0 \le x \le 360$
Find the values of *a* and *b*.

Amplitude = 3; 2 waves in 360° a = 3, b = 2