HSoG N5 Model Papers ① A1

Paper A : 1

(1) $3\frac{2}{5} - 1\frac{3}{4}$

$= 2\left(\frac{2}{5} - \frac{3}{4}\right)$ [LCM 4,5 = 20]

$= 2\left(\frac{8}{20} - \frac{15}{20}\right)$ $(8 - 15 \rightarrow \text{No!})$

$= 1\left(\frac{20}{20} + \frac{8}{20} - \frac{15}{20}\right) = 1\frac{13}{20}$

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(2) $x^2 + 2x - 15$ [Factors of −15]

$= \underline{(x+5)(x-3)}$

$$\begin{array}{rr} -5 & +5 \\ +3 & -3 \\ \hline -2 \times & +2 \checkmark \end{array}$$

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(3) (y-intercept = $(0, 5)$, also $(2, 25)$)

$5 = c$

Gradient $(m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{25-5}{2-0} = \frac{20}{2} = 10$

So $y = mx + c \longrightarrow \underline{y = 10x + 5}$

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(4) $x^2 + 8x - 7 \rightarrow (x+4)^2 - 4^2 - 7$

$8 \div 2 = 4$

[$(x+4)^2 = x^2 + 8x + 16$

needs to be "got rid of"]

$= (x+4)^2 - 16 - 7$

$\underline{y = (x+4)^2 - 23}$

(Min. turn. pt) when $(x+4) = 0 \Rightarrow x = -4$

$y = (0)^2 - 23 \Rightarrow y = -23$

$\underline{(-4, -23) \rightarrow \text{turn pt}}$

(5) $P = R^2b - 5$ or $R^2b - 5 = P$

$R^2b = P + 5$ (+5)

$R^2 = \frac{(P+5)}{b}$ ($\div b$)

$R = \sqrt{\frac{(P+5)}{b}}$ (Take $\sqrt{\ }$)

(6) a) $\underline{u} + 3\underline{v} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} + 3 \times \begin{pmatrix} -4 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ -5 \end{pmatrix} + \begin{pmatrix} -12 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 + (-12) \\ (-5) + 9 \end{pmatrix}$

$= \begin{pmatrix} -10 \\ 4 \end{pmatrix}$

b) $|\underline{u} + 3\underline{v}| = \sqrt{(-10)^2 + (4)^2} = \sqrt{100 + 16}$

$= \sqrt{116} = 10 \cdot 8$ units (1 dec. pl.)

(7) $b = 3$ (1 complete period : $120°$, so $\underline{3}$ complete periods $0 \to 360°$).

(8) $2x + y = 5$ ($\times 3$) $\longrightarrow$ $6x + 3y = 15$ (Eliminate y)

$x - 3y = 6$ $\qquad$ (+) $x - 3y = 6$

$7x = 21$

$\Rightarrow x = 3$

Now substitute $x = 3$:

$2x + y = 5$

$2 \times 3 + y = 5$

$6 + y = 5$ (−6)

$y = 5 - 6 = -1$.

Lines intersect at $(3, -1)$

(9) $y = x^2 - 3x + 7$

$a=1$ $b=-3$ $c=7$

Disc: $\sqrt{b^2-4ac}$

$= \sqrt{(-3)^2 - 4\times1\times7}$

$= \sqrt{9-28} = \sqrt{-19}$

As $\sqrt{\ } < 0$, there are <u>NO real roots</u> (cannot find $\sqrt{\ }$ of negative number)

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10) $3x - y = 9$ $(+y)$

$\Leftrightarrow 3x = 9 + y$ (-9)

$\Leftrightarrow 3x - 9 = y$

$y = 3x - 9$ so gradient $(m) = 3$ [parallel lines: same m]

Line through $(5, -3)$: $y - (-3) = 3(x-5)$ $[(y-b) = m(x-a)]$

a b

$y + 3 = 3x - 15$ (-3)

$\underline{\underline{y = 3x - 18}}$ $(3x - y = 18)$

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(11) a) $\underline{\underline{(2, -9)}}$ [$y = (x-2)^2 - 9$: Min. turn. pt. when $(x-2)^2 = 0 \Rightarrow x = 2$

$y = (0)^2 - 9 = -9$ Hence $(2, -9)$]

b) At C: $x = 0$ so $y = (0-2)^2 - 9$

$= (-2)^2 - 9 = 4 - 9 = -5$

$\underline{\underline{C = (0, -5)}}$

c) $\underline{\underline{(5, 0)}}$ [Curve symmetrical around $x = 2$ → see turning pt.

$2 - (-1) = 3$ so A is 3 units left of axis of symm.

→ B is 3 units right; $2 + 3 = \underline{5}$]

N5 Model Papers (HSoG)

(12) Perimeter (Square) $= 4(2x+2)$

$= 8x+8$

Perimeter (Rectangle) $= 2(x+3) + 2(\text{length})$ $\left[= 8x+8\right]$

$2x+6 + 2(\text{length}) = 8x+8$ $(-2x)$

$6 + 2(\text{length}) = 6x+8$ (-6)

$2(\text{length}) = 6x+2$

$6x+2 = 2(3x+1) = 2(\text{length})$

So length $= 3x+1$ proved

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(13) a) $\frac{3}{x} - \frac{5}{x+2}$ $\quad (\text{LCM} = x(x+2))$

$= \frac{3(x+2)}{x(x+2)} - \frac{5x}{x(x+2)} = \frac{3x+6-5x}{x(x+2)} = \frac{-2x+6}{x(x+2)}$

or $\frac{2(3-x)}{x(x+2)}$ common factor = 2

b) $\sqrt{18} - \sqrt{2} + \sqrt{72}$ (Factorise using "perfect squares")

$= \sqrt{\underline{9} \times 2} - \sqrt{2} + \sqrt{\underline{36} \times 2}$

$= \sqrt{9} \times \sqrt{2} - \sqrt{2} + \sqrt{36} \times \sqrt{2}$

$= 3\sqrt{2} - \sqrt{2} + 6\sqrt{2}$

$= \underline{8\sqrt{2}}$ $\quad (3-1+6=8)$

Paper A:2

(1) $528\,000 \times (1{\cdot}024)^4$ ← 4 years

[$2{\cdot}4\% = 0{\cdot}024$
$1 + 0{\cdot}024$
$= 1{\cdot}024$]

$= 580542{\cdot}1\ldots$

$= \underline{\underline{580542}}$ (nearest ~~whole~~ thousand no.)

581000

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(2) a) Mean $= \dfrac{(\text{Total})}{6} = \dfrac{360}{6} = \underline{\underline{60 \text{ (marks)}}}$

(called $\bar{x}$)

data items
$n = 6$
→ 6 items

x	$(x-\bar{x})$	$(x-\bar{x})^2$
73	13	169
47	(-13)	169
59	(-1)	1
71	11	121
48	(-12)	144
62	2	4
	(TOTAL) →	608 (Σ)

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{608}{5}}$$

$$= \sqrt{121{\cdot}6} = \underline{\underline{11{\cdot}0}}$$

b) Group B have the same average ~~(mean)~~, but their spread of marks is much greater than Group A's

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(3) $(x+4)(2x^2+3x-1) = \underbrace{2x^3+3x^2-x}_{(\times x)} + \underbrace{8x^2+12x-4}_{(\times 4)}$

$= \underline{\underline{2x^3 + 11x^2 + 11x - 4}}$

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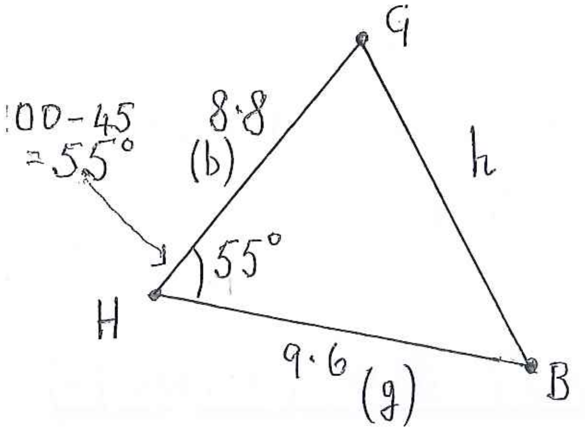
(4) (Gordon walks $2 \times 4{\cdot}4 = 8{\cdot}8$ km; Brian $2 \times 4{\cdot}8 = 9{\cdot}6$ km)



(Cosine rule):

$h^2 = b^2 + g^2 - 2bg\cos\hat{H}$

$= 8{\cdot}8^2 + 9{\cdot}6^2 - 2 \times 8{\cdot}8 \times 9{\cdot}6 \times \cos 55°$

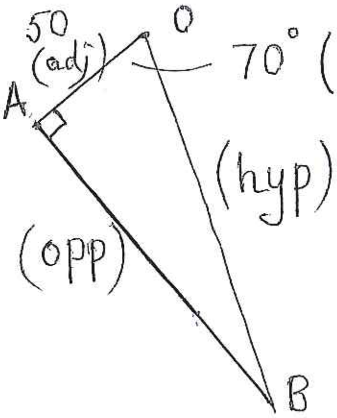
$= 77{\cdot}44 + 92{\cdot}16 - 96{\cdot}911$

$= 72{\cdot}689$ So $h = \sqrt{72{\cdot}689}$

$= \underline{\underline{8{\cdot}53 \text{ km (to 2 dec. pl)}}}$

N5 Model Papers (HSoG)

(5)



$\text{Tan } 70^\circ = \frac{\text{opp}}{\text{adj}} = \frac{AB}{50}$

So $AB = 50 \times \text{Tan } 70^\circ$

$= 137{\cdot}4\text{cm}$ (and $BC = AB$)

Circle

(Major arc) AC

$= \frac{220}{360} \times \pi \times (2 \times 50)$

$= 192{\cdot}0 \text{ cm}$

Perimeter $= 137{\cdot}4 + 137{\cdot}4 + 192{\cdot}0 = \underline{\underline{466{\cdot}8 \text{ cm}}}$

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(6) a) $V = \pi r^2 h$ $(r = 20, h = 50)$

$= \pi \times 20 \times 20 \times 50$

$= 62831{\cdot}85\ldots = \underline{\underline{63{,}000 \text{ cm}^3 \text{ (to 2sf)}}}$

b) V (each cup) $= 63{,}000 \div 800 = 78{\cdot}75\text{cm}^3$

$V = \frac{1}{3}\pi r^2 h$ (solve for h)

$78{\cdot}75 = \frac{1}{3}\pi \times 3^2 \times h$ $(\times 3)$

$236{\cdot}25 = (\pi \times 9) \times h$

$h = \frac{236{\cdot}25}{9\pi} = \underline{\underline{8{\cdot}36 \text{ cm (to 2 dec. pl)}}}$

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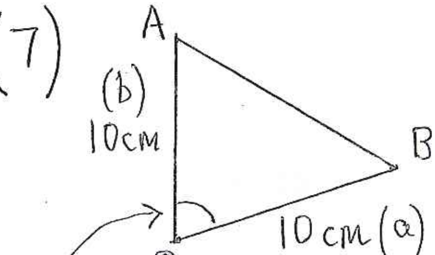
(7)



Triangle $A = \frac{1}{2}ab \sin \hat{O}$

$= \frac{1}{2} \times 10 \times 10 \times \sin 72^\circ$

$= 47{\cdot}55 \text{cm}^2$

Pentagon $A = 47{\cdot}55 \times 5$

$= \underline{\underline{237{\cdot}8 \text{ cm}^2 \text{ (to 1 dec. pl)}}}$

(8) a) $a^2\left(2a^{-1/2} + a\right) = 2a^{3/2} + a^3$ $\quad \left(2 + \left(-\tfrac{1}{2}\right) = \tfrac{4}{2} - \tfrac{1}{2} = \tfrac{3}{2}\right.$

$= \underline{\underline{2\sqrt{a^3} + a^3}}$ $\quad$ <u>AND</u> $\left. a = a^1 \right)$

b) $3x^2 + 3x - 7 = 0$ $\qquad a = 3$, $b = 3$, $c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4\times3\times(-7)}}{2\times3} = \frac{-3 \pm \sqrt{9-(-84)}}{6} \longrightarrow \sqrt{9\ldots} = 9\cdot64$$

$$= \frac{-3 + 9\cdot64}{6} \text{ or } \frac{-3 - 9\cdot64}{6} \Rightarrow \underline{\underline{x = 1\cdot1 \text{ or } -2\cdot1}}$$

(1 dec. pl)

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(9) a) $4\tan x^\circ + 5 = 0$ (-5) $\Rightarrow 4\tan x^\circ = -5$ $(\div 4)$

$\Rightarrow \tan x^\circ = -\frac{5}{4}$ $(= -1\cdot25)$ $\qquad$ For $\tan x^\circ = +1\cdot25$,

$x^\circ = 51\cdot3^\circ$ (acute angle)

S ✓	A
T	C ✓

(x is in Quadrants 2 or 4 as its tan is $-$ve) $\longrightarrow$

$x = 180 - 51\cdot3^\circ$ (Q2)

$x = 360 - 51\cdot3^\circ$ (Q4)

$\Rightarrow \underline{\underline{x = 128\cdot7 \text{ or } 308\cdot7}}$

b) [Prove $\tan x \cos x = \sin x$]

Left side: $\tan x \cos x = \dfrac{\sin x}{\cancel{\cos x}} \times \dfrac{\cancel{\cos x}}{1}$ $\quad \left(\text{since } \tan x = \dfrac{\sin x}{\cos x}\right)$

$= \underline{\sin x}$

<u>Left side = right side → Proved</u>

(10) a) New length $= (30+x)$ [or $(x+30)$]

New breadth $= (x+10)$

Hence new area $= (x+30)(x+10) \leftarrow$ (length × breadth)

$= x^2 + 10x + 30x + 300$

$A = x^2 + 40x + 300$ Proved

b) New area $= 300 + (75\% \text{ of } 300) = 300 + 225$

$= 525 \text{ cm}^2$

$A \; (= x^2 + 40x + 300) \geqslant 525$

$x^2 + 40x - 225 \geqslant 0 \quad (-525)$

Solve: $x^2 + 40x - 225 = 0$ [Factors of -225: 45, -5, $\overline{40}$ ✓]

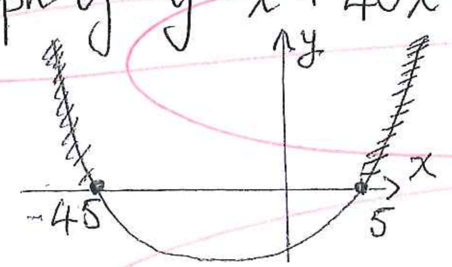
[N.B. Can also use Quadratic Formula]

$(x+45)(x-5) = 0$

$x + 45 = 0 \Rightarrow x = -45$ ← IGNORE AS LENGTH

$x - 5 = 0 \Rightarrow x = 5$

Graph of $y = x^2 + 40x - 225$: ← Not needed at Nat 5.



We need

$x^2 + 40x - 225 \geqslant 0$

$\Rightarrow x \leqslant -45$ or $x \geqslant 5$

(i.e. above x-axis)

Ignore -ve values (since x is a length)

so $x \geqslant 5$

Minimum value for x is 5, giving:

New length $= 35$ cm $\leftarrow (x+30)$

New breadth $= 15$ cm $\leftarrow (x+10)$ 4