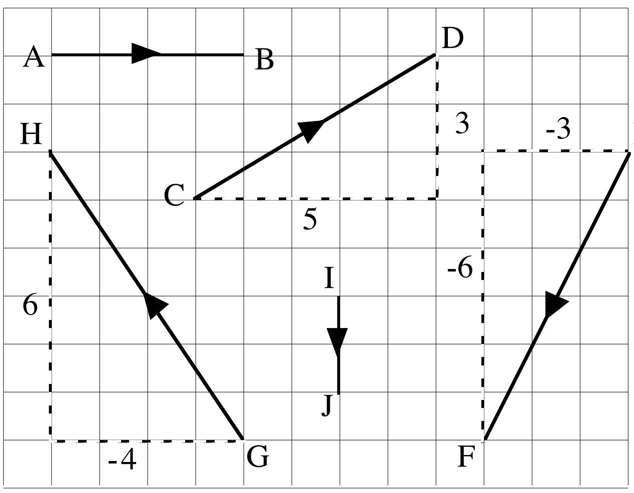
CHAPTER 19: VECTORS

SCALAR quantities have size(magnitude). eg. time, speed, volume

VECTOR quantities have **size** and **direction**. eg. force, velocity

A **directed line segment** represents a vector.

Vectors can be written in component form as **column vectors**



$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ $\overrightarrow{CD} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ $\overrightarrow{EF} = \begin{pmatrix} -3 \\ -6 \end{pmatrix}$

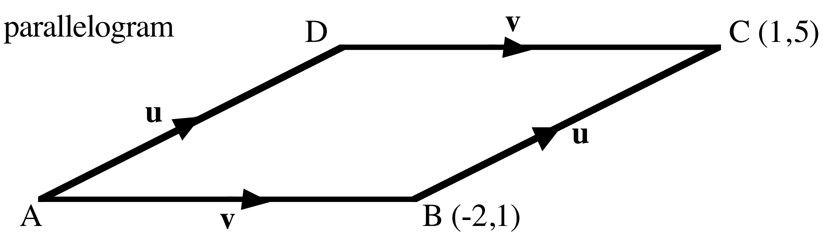
$\overrightarrow{GH} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$ $\overrightarrow{IJ} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$

negative vector, *opposite direction*

$\overrightarrow{HG} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$

SIZE follows from Pyth. Thm

$\mathbf{u} = \begin{pmatrix} a \\ b \end{pmatrix}$ $|\mathbf{u}| = \sqrt{a^2 + b^2}$ $|\overrightarrow{GH}| = \sqrt{(-4)^2 + 6^2} = \sqrt{52} = 2\sqrt{13}$ *units*



$\overrightarrow{AD} = \overrightarrow{BC} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

same size and direction
same vector **u**
same component form

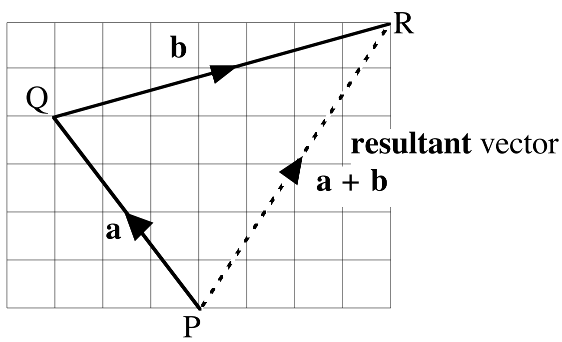
ADD/SUBTRACT

by column vectors
add or subtract components.

$$\begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} 7 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

by diagram
"head-to-tail" addition

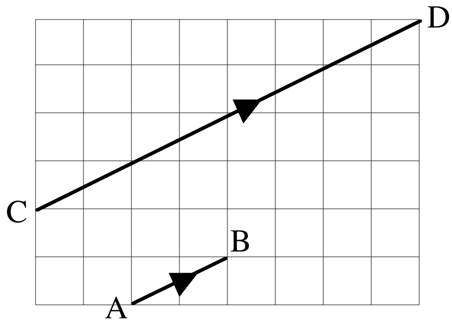
$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$



MULTIPLY by a number

$\mathbf{u} = \begin{pmatrix} a \\ b \end{pmatrix}$ $\quad$ $\mathrm{k}\mathbf{u} = \begin{pmatrix} \mathrm{k}a \\ \mathrm{k}b \end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\quad$ $\overrightarrow{CD} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$



if $\quad \mathbf{v} = \mathrm{k}\mathbf{u}$

then $\quad$ **u** *and* **v** *are parallel*

$\overrightarrow{CD} = 4\,\overrightarrow{AB}$

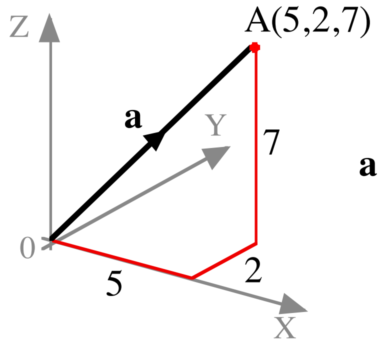
CD is parallel to AB

3D

Vectors in 3D operate in the same way as vectors in 2D.

Points are plotted on 3 mutually perpendicular axes.

P(x,y,z) has **position vector** $\mathbf{p} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$



$\mathbf{a} = \begin{pmatrix} 5 \\ 2 \\ 7 \end{pmatrix}$

If $\mathbf{u} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ find the value of $|\mathbf{v} - 2\mathbf{u}|$.

$$\mathbf{v} - 2\mathbf{u} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} - 2\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ -5 \end{pmatrix}$$

$$|\mathbf{v} - 2\mathbf{u}| = \sqrt{(-3)^2 + 4^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2}$$